Spoon Feeding - Indefinite Integrals Survival Guide

Simplified Knowledge Management Classes Bangalore

My name is Subhashish Chattopadhyay. I have been teaching for IIT-JEE, Various International Exams (such as IMO [International Mathematics Olympiad], IPhO [International Physics Olympiad], IChO [International Chemistry Olympiad]), IGCSE (IB), CBSE, I.Sc, Indian State Board exams such as WB-Board, Karnataka PU-II etc since 1989. As I write this book in 2016, it is my 27 th year of teaching. I was a Visiting Professor to BARC Mankhurd, Chembur, Mumbai, Homi Bhabha Centre for Science Education (HBCSE) Physics Olympics camp BARC Campus.

I am Life Member of …

- IAPT (Indian Association of Physics Teachers)

- IPA (Indian Physics Association)

- AMTI (Association of Mathematics Teachers of India)

- National Human Rights Association

- Men's Rights Movement (India and International)

- MGTOW Movement (India and International)

And also of

IACT (Indian Association of Chemistry Teachers)

The selection for National Camp (for Official Science Olympiads - Physics, Chemistry, Biology, Astronomy) happens in the following steps ….

1) **NSEP** (National Standard Exam in Physics) and **NSEC** (National Standard Exam in Chemistry) held around 24 rth November. Approx 35,000 students appear for these exams every year. The exam fees is Rs 100 each. Since 1998 the IIT JEE toppers have been topping these exams and they get to know their rank / performance ahead of others.

2) **INPhO** (Indian National Physics Olympiad) and **INChO** (Indian National Chemistry Olympiad). Around 300 students in each subject are allowed to take these exams. Students coming from outside cities are paid fair from the Govt of India.

3) The Top 35 students of each subject are invited at HBCSE (Homi Bhabha Center for Science Education) Mankhurd, near Chembur, BARC, Mumbai. After a 2-3 weeks camp the top 5 are selected to represent India. The flight tickets and many other expenses are taken care by Govt of India.

Since last 50 years there has been no dearth of "Good Books". Those who are interested in studies have been always doing well. This e-Book does not intend to replace any standard text book. These topics are very old and already standardized.

There are 3 kinds of Text Books

- The thin Books - Good students who want more details are not happy with these. Average students who need more examples are not happy with these. Most students who want to "Cram" quickly and pass somehow find the thin books "good" as they have to read less !!

- **The Thick Books - Most students do not like these, as they want to read as less as possible. Average students are "busy" with many other things and have no time to read all these.**

- The Average sized Books - Good students do not get all details in any one book. Most bad students do not want to read books of "this much thickness" also !!

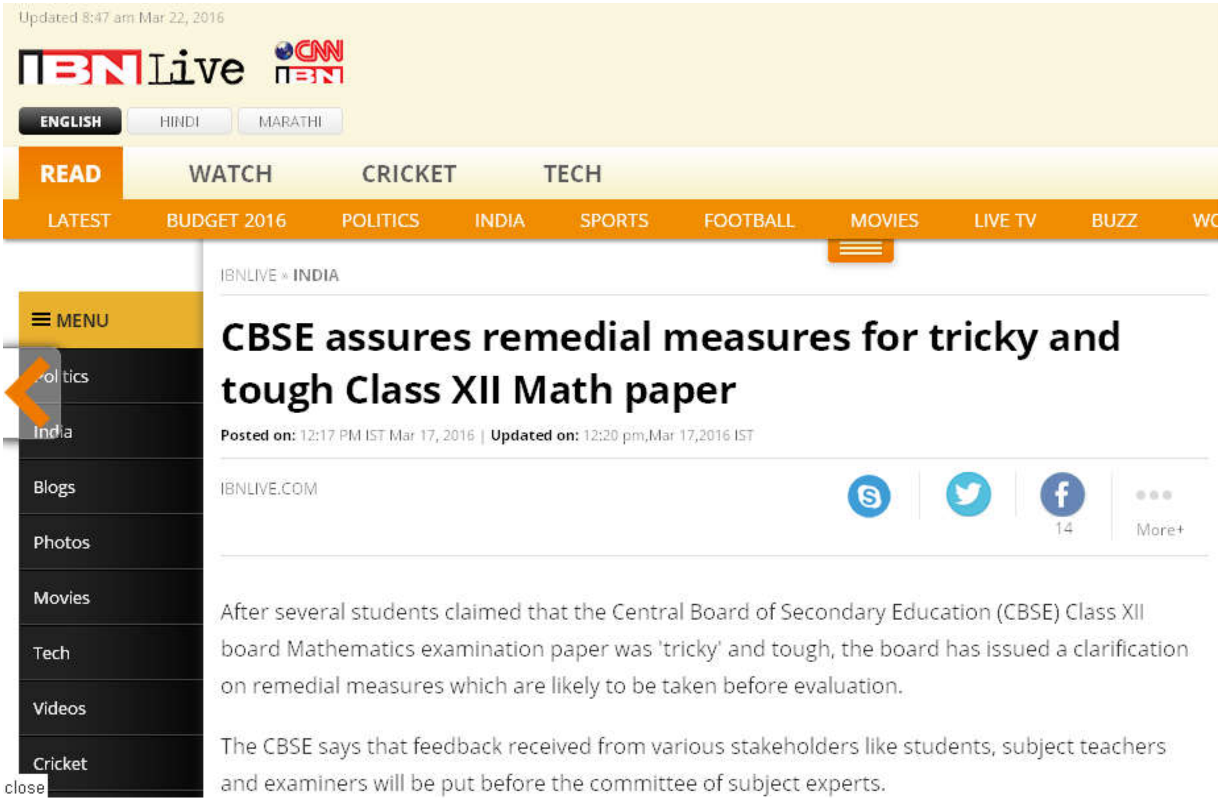
We know there can be no shoe that's fits in all.

Printed books are not e-Books! Can't be downloaded and kept in hard-disc for reading "later" ………

So if you read this book later, you will get all kinds of examples in a single place. This becomes a very good "Reference Material". I sincerely wish that all find this "very useful".

Students who do not practice lots of problems, do not do well. The rules of "doing well" had never changed …. Will never change !

After 2016 CBSE Mathematics exam, lots of students complained that the paper was tough!



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CBSE assures remedial measures for tricky and tough Class XII Math paper

Posted on: 12:17 PM IST Mar 17, 2016 | **Updated on:** 12:20 pm,Mar 17,2016 IST

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After several students claimed that the Central Board of Secondary Education (CBSE) Class XII board Mathematics examination paper was 'tricky' and tough, the board has issued a clarification on remedial measures which are likely to be taken before evaluation.

The CBSE says that feedback received from various stakeholders like students, subject teachers and examiners will be put before the committee of subject experts.

On 21 st May 2016 the CBSE standard 12 result was declared. I loved the headline

INDIATODAY.IN NEW DELHI, MAY 21, 2016 | UPDATED 16:40 IST

CBSE Class 12 Results out: No leniency in Maths paper, high paper standard to be maintained in future

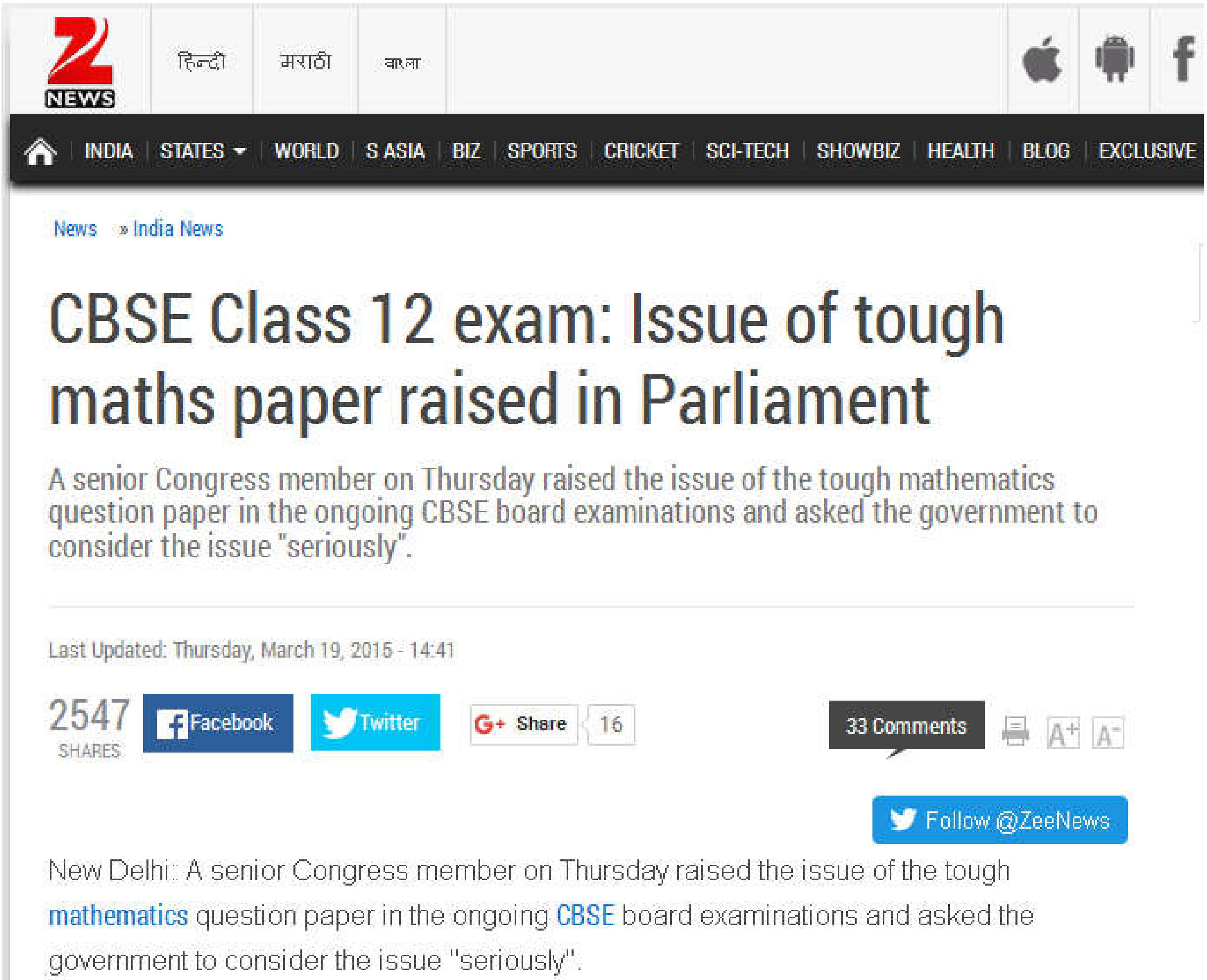
The CBSE Class 12 Mathematics board exam on March 14 reduced many students to tears as they found the paper quite lengthy and tough and many couldn't finish it on time. The results show an overall lowering of marks received in the Maths paper.

RELATED STORIES

- CBSE Board result 2016 declared! Thiruvanathpuram obtains the highest part percentage, check how your region scored
- Meet CBSE topper Sukriti Gupta: Check her percentage here!
- CBSE Class 12 Boards 2016: Results announced ahead of time!
- CBSE results declared at www.cbse.nic.in: Steps to check online
- Exclusive! CBSE declares Class 12 Results at www.cbseresults.nic.in and cbse.nic.in

The CBSE (Central Board of Secondary Education) Class 12 Board exam results have been announced today, i.e on May 21, around 10:30 am ahead of time. Students may check their scores at the official website, *www.cbseresults.nic.in*. (**Read: CBSE Class 12 Boards 2016: Results announced ahead of time! Check your score at cbseresults.nic.in**)

In 2015 also the same complain was there by many students



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News » India News

CBSE Class 12 exam: Issue of tough maths paper raised in Parliament

A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

Last Updated: Thursday, March 19, 2015 - 14:41

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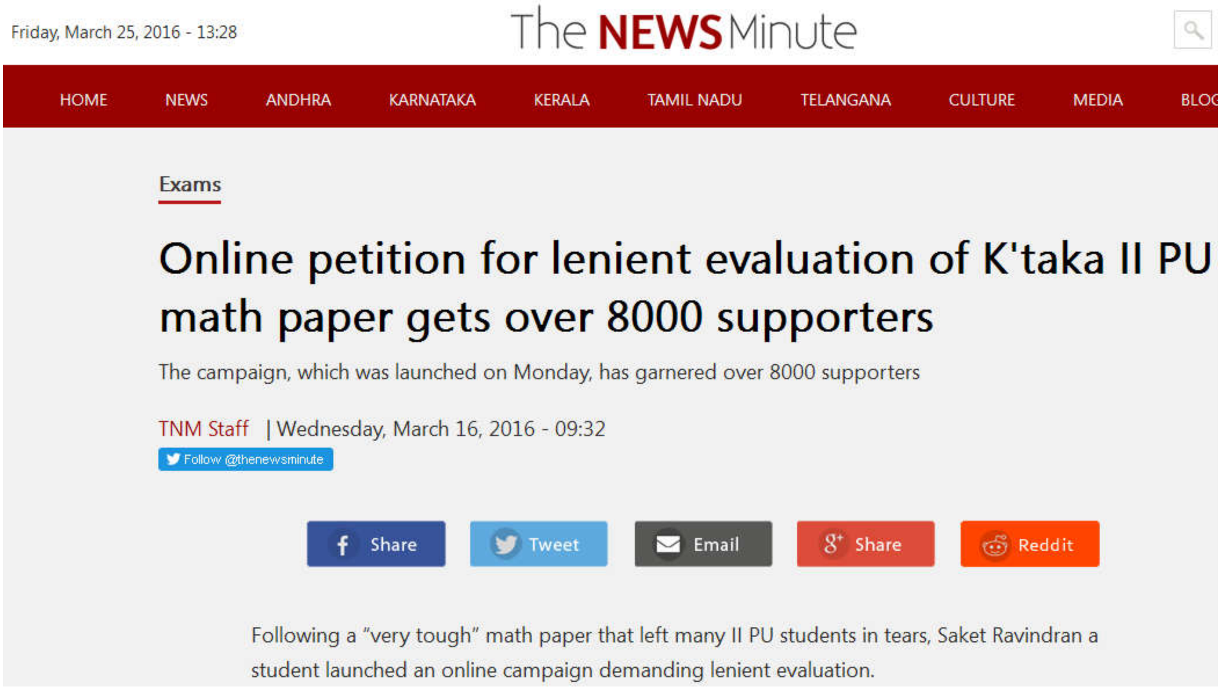
New Delhi: A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

So we see that by raising frivolous requests, even upto parliament; actually does not help. Many times requests from several quarters have been put to CBSE, or Parliament etc for easy Math Paper. These kinds of requests actually can-not be entertained, never will be.

In March 2016, students of Karnataka PU-II also complained the same, regarding standard 12 (PU-II Mathematics Exam). Even though the Math Paper was identical to previous year, most students had not even solved the 2015 Question Paper.



Friday, March 25, 2016 - 13:28

The NEWS Minute

HOME NEWS ANDHRA KARNATAKA KERALA TAMIL NADU TELANGANA CULTURE MEDIA BLOG

Exams

Online petition for lenient evaluation of K'taka II PU math paper gets over 8000 supporters

The campaign, which was launched on Monday, has garnered over 8000 supporters

TNM Staff | Wednesday, March 16, 2016 - 09:32

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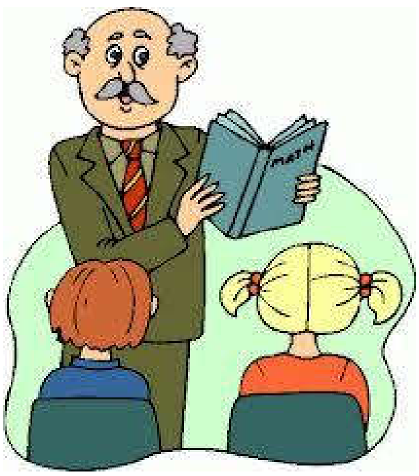
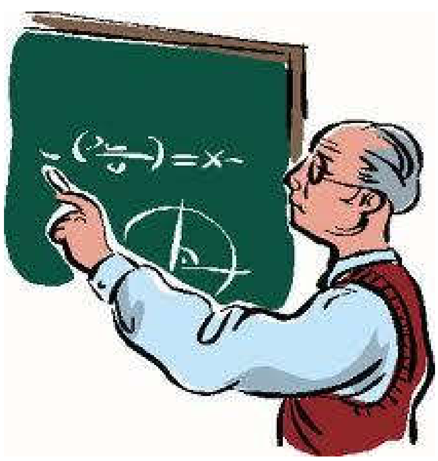
Following a "very tough" math paper that left many II PU students in tears, Saket Ravindran a student launched an online campaign demanding lenient evaluation.

These complains are not new. In fact since last 40 years, (since my childhood), I always see this; every year the same setback, same complain!

In this e-Book I am trying to solve this problem. Those students who practice can learn.

No one can help those who are not studying, or practicing.





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A very polite request :

I wish these e-Books are read only by Boys and Men. Girls and Women, better read something else; learn from somewhere else.

Foreword for the Book, by Dr. Navsky Gupta

Director and Consultant, Shankar Netrika Eye Center, Mumbai

Studied at University of California, Irvine, and Volgograd Medical Academy

-

On human apes by the storytelling chimpanzee

My view of human apes

Let me be clear at the onset of my view. **I am not proud of my species which calls itself Homo sapiens.**

You just need to look our **sorry history** of **violence, warring and massacres over power, resources** and **religion**.

I think, for the most part, the human ape thinks, acts and reproduces as do his great ape cousins. (they mate, have family, have culture etc. as shown by studies of Jane Goodall, Desmond Morris and many more)

Our evolution of higher faculties

Yet, for an ape, we have come a long way forward. The journey has been slow and arduous.

The first ape like humans probably arose (quiet literally) on their two feet some 5 to 7 million years ago (that is 50,000 to 70,000 centuries ago).

The great apes as a family go back 15 million years.

Somewhere down the line we developed imagination, curiosity, and the ability to consider "What if ?"

These qualities of imagination, curiosity and abstract thinking are vital components of storytelling so that when developed, a mere mention or even the thought of a word can evoke artificial, imaginative or real worlds in the mind.

Other animals too have traits of intelligence

We are not certain if our cousin great apes have it or not, and if they have, to what extent it is developed.

Curiosity is certainly very common in animal kingdom.

It is a human hubris to think that we are sole possessor of this facility.

Other animals are as curious as us including our cousin apes, cats, rodents to name a few?

Curiosity is an inquisitive thinking that involves observation, exploration, investigation, learning and finally changes in behavior.

Curiosity has survival and reproductive value which is essential for success of DNA transmission, the raison d'etre for any kind of life based on carbon and DNA.

Curiosity involves several neurological aspects such as motivation and reward, attention, memory and learning.

Our crippling shortcomings

The other thing that we humans need to be aware is that we are in the end apes and very flawed apes at that.

No doubt we have higher intelligence and contemplate abstract thinking.

Yet, our evolutionary mind uses principles that had served us well when we were hunter-gatherers in the African savannas but now do us grave injustice.

They are termed cognitive fallacies.

The list of these heuristics (mental shortcuts), biases, is devastatingly huge and long.

They become a fertile ground for the breeding of irrationality in human apes.

Worse, irrationality is highly contagious.

Classification of cognitive biases

These cognitive biases are divided into three categories:

1. Decision making and belief biases:

There are more than 80 of these.

One good example is the **bandwagon effect** or the **herd mentality**. This explains how easily a temple, or church or a statue gets tagged as “lucky”.

2. Social biases

There are at least 25 of these.

The classic one being, the just-world hypothesis also known as the moral luck. It is a belief that good stuff happens to virtuous and ill happens to the diabolical, deservingly of course.

Another good example is the **Barnum effect** (closely related to subjective validation) wherein an individual considers a general and a vague statement highly specific to his or her own personality.

Example: Disciplined and self controlled outside, you tend to be worrisome and insecure inside.

Entire chicanery of astrology, palmistry and astrology are based on this one bias.

3. Memory errors and biases

There are at least 60 of them

The peak-end rule is a suitable example. It is the assessment of any experience by an individual largely on how they felt it at its peak and at its termination. This has a special significance for medical procedures and surgeries.

Limitations of curiosity, logic and abstract thinking

You will realize that just being curious and having the ability of abstract thinking is not enough.

These two generally end up in giving rise to either philosophy or worse, religion.

These traits alone would very likely have us end up in creating a world view that is largely hopeful, helpful and endearing but factually incorrect.

This in fact did happen for most of the time in human history.

Added with these two, if one begins to apply logic and proofs, the brain is capable of generating powerful mathematics.

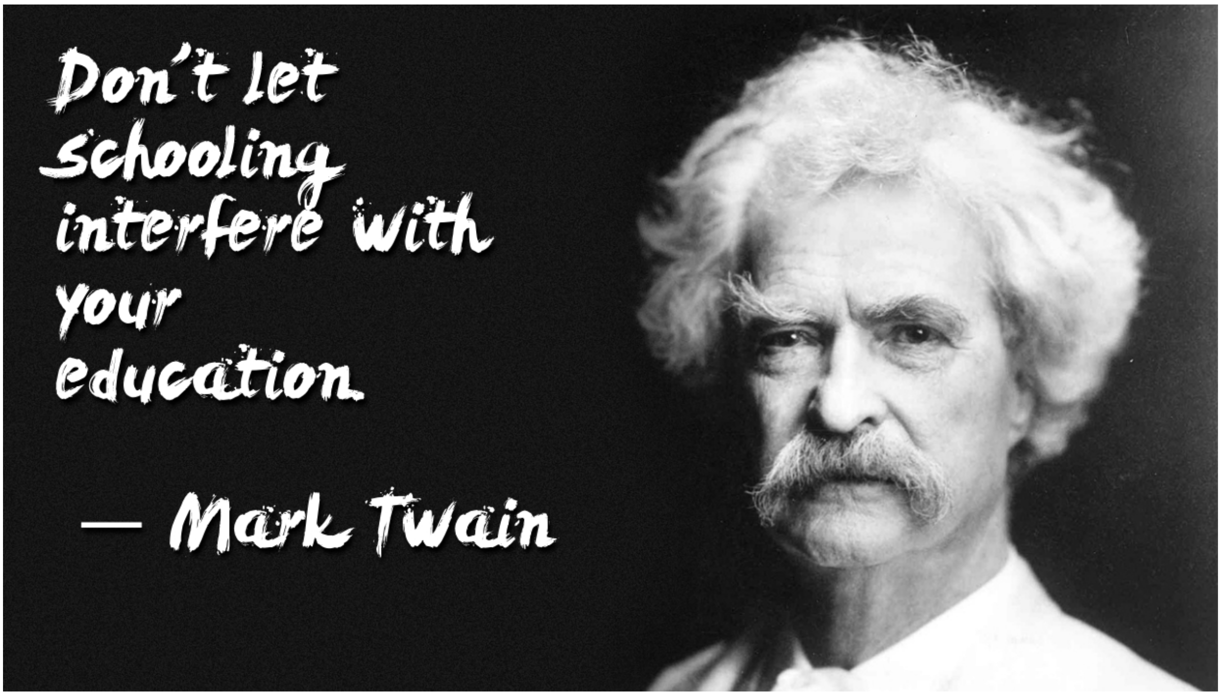
Yet, all these devises and tools namely curiosity, imagination, logic and mathematical proofs have proved themselves deficient in curbing our remarkable ability to fool ourselves.

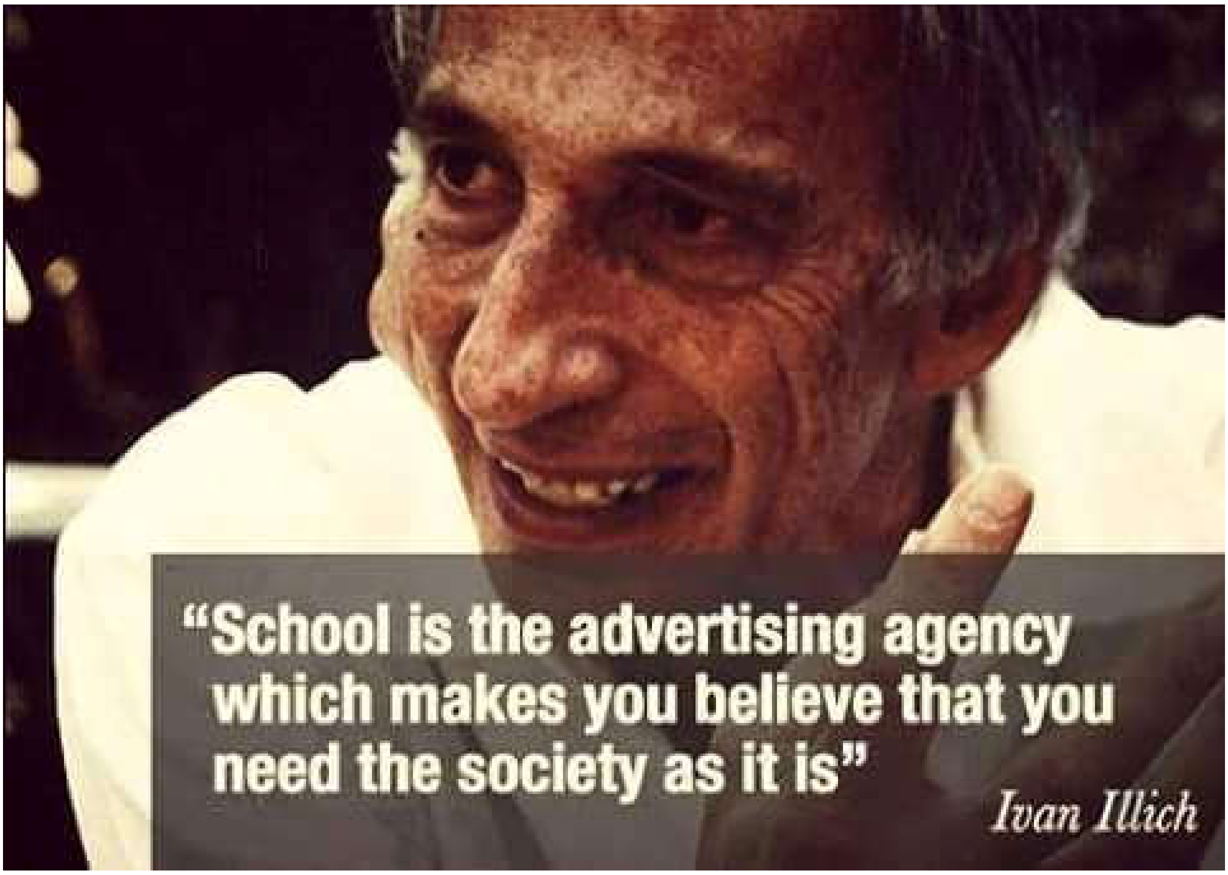
Experimental Science is the best tool ever devised to understand reality

The only tool and the best method that we humans came up with understanding reality is experimentation, particularly well controlled, repeatable verifiable experiments that can minimize the experimenter's bias.

In medicine, the gold standard of drug testing for its efficacy and safety is the placebo controlled double blind clinical trial.

It is not an easy task to conduct an original experiment.





Education's Biggest Failure

Our school education's profoundest failure is exactly this.

It does not inculcate either questioning or original thinking or more specifically critical thinking.

We fail to teach our students the idea of how to propose a hypothesis and go about testing it.

Our schooling fails to provide to even the best outgoing student the notion of conceiving an original experiment to prove or disprove an idea.

Only few people are good experimentalists, meaning they take care to isolate their study from events that can undue influence its outcome.

The most important aspect about the experimental findings is that it should be repeatable, verifiable by other people who repeat them under similar conditions in other places.

It is the one biggest universal failure of education system all over the world.

Education is currently seen as a way to attain professional career and job security which is not bad per se.

But something very important has been lost.

Do we encourage a student to write an original paper?

Do we encourage a student to ever lay out a plan for considering an original experiment?

In fact, in our education, do we even mention that so many unknown things remain to discover.

May be it is so that there is now so much to know that it overwhelms a young mind.

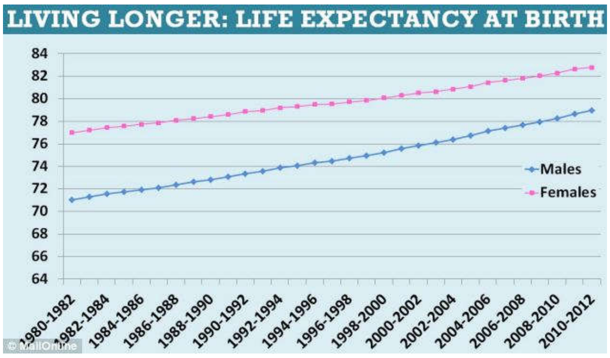
At least most young minds.

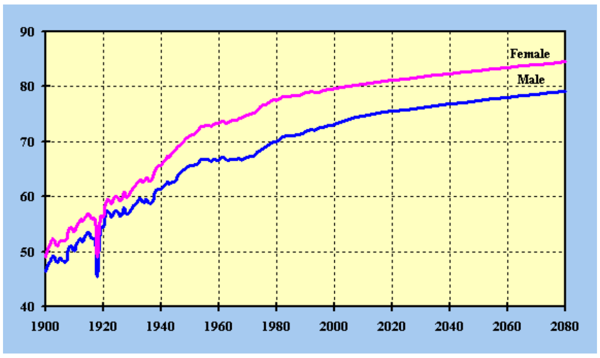
The reason for the failure of education

What prevents us from imparting the type of education we often know about, speak about but fail to carry out?

You will be surprised at the answer.

It is overpopulation; too many of us human apes.





(**Did you notice Female Life expectancy is always higher than Men ! Do you know why ?**)

If someone were to ask me what is the key problem today, I would say that we are simply too many of us today.

India or South Asia is an extreme example but almost all the nations face this hideous calamity.

Are nation states able to provide clean air and water to their citizens?

Are they able to provide a basic housing to their citizens?

Are they able to provide even basic level healthcare to their citizens?

What about jobs?

Many argue between capitalism, socialism, mixed system and so on and so forth.

I think they keep missing the key issue.

Denial is probably the right word.

Such a populace simply cannot be given the fundamental rights as enshrined in the constitutions of most nation states.

Most would not sit to listen to this and may get up and leave in protest.

Stating the problem

But let me make my case.

Just feeding, giving clean water and jobs is not the way we should be looking at the citizens of the world; though even that itself is a herculean task and even the most developed nation states are grappling with the problem.

I want to go beyond this.

Why has education, the process of acquiring knowledge become such a painful task for most young people?

Let us see this step by step.

For starters, every child right from a day she is born needs a decent health care and nutrition.

The idea is to get very good schooling.

Good schools are few and the race starts right here.

Only very few percentage of humans born will get good schooling.

Second step, after the school, it is the college.

The idea of scoring top percentages is to get into the best colleges.

We all know that in general in any country, including the United States, only a tiny percentage of colleges or universities offer a life enhancing and transforming program.

Good education needs great teachers.

Great and dedicated teachers are a rarity as a society can afford to pay and reward only a handful of good teachers, professors.

Following that, we have the problem of jobs or a professional career.

Here again one encounters a cut throat competition and only a few will land up with a satisfactory job.

As it is, most of us humans are average and really not very productive for a society.

In fact, most of us can be or turn out to be a burden for the society.

A planet that has fewer people, can be better educated, can be given better lives, and can be given better policing /security and a speedier and effective justice.

Crime itself will come down.

The lesser we are, the more we will care for each other.

Moreover, more productive and educated people are more likely to contribute funds not only for the resources needed to run a society but to higher pursuits of sciences and mathematics.

This idea is extremely repulsive and disgusting to nearly everybody as it goes against our biological drive, our most primal instinct.

But what needs to be done must be done.

Otherwise we will be doomed to mediocrity and worse, nightmarish suffering that is visible all around us.

Someone asked me the one biggest mistake we have made.

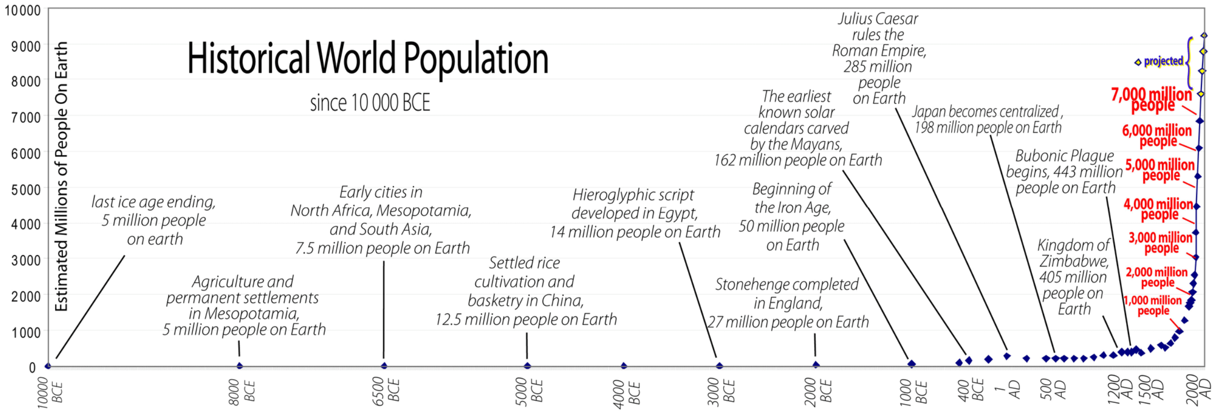
I think it is this.

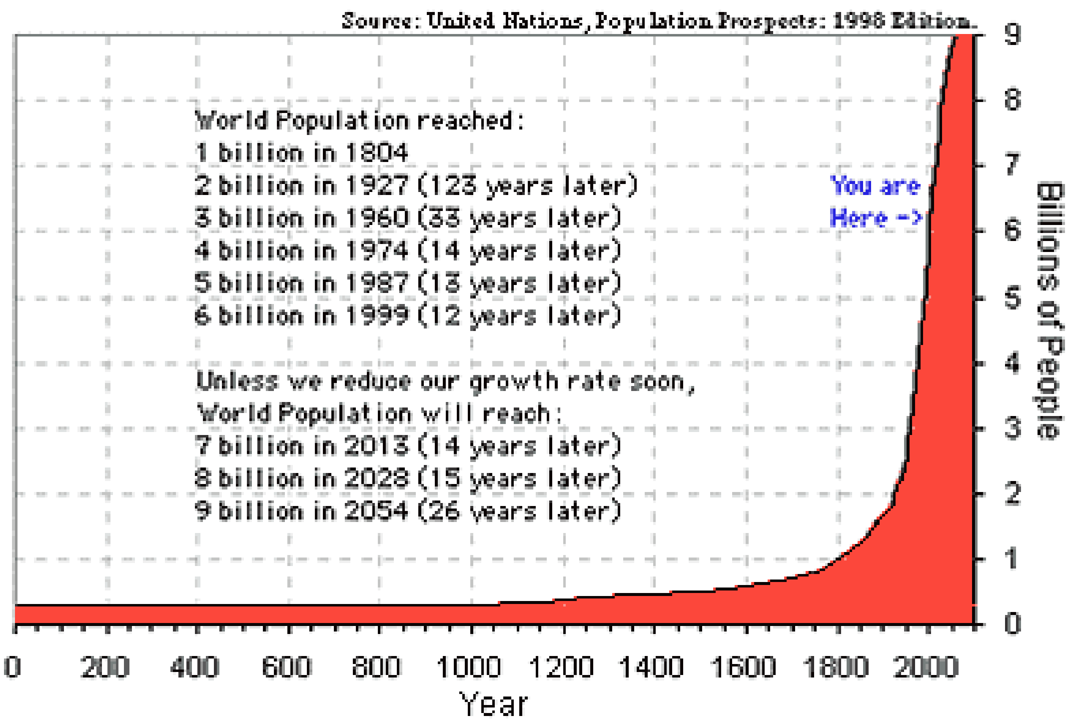
We have allowed runaway breeding of ourselves.



If we wish all schools to impart scientific teaching and inculcate scientific methods, we need to have fewer of them very good ones with better facilities with fewer pupils to care after.

Just being a few would increase love and tolerance for each other and further our cooperation.





Going one step further

In this context, another important pops up.

We are aware that resources are scare, may it be for education, for health, for research, for fuels, for energy.

We, if are intelligent, and rational enough; must plan our death once we realize that our contribution to the society is nil.

After that, we become a parasite and a hindrance for the younger generation who exist and who are to come.

This is one of the biggest prices we are paying for the success of medicine.

Ageing and geriatric diseases are taking a huge toll on the national economies, especially of the developed world where the state bears the expenses of the early to a large extent.

Finally when the time comes, one needs to embrace death by making death peaceful, planned and curbing our greedy desire to go on and on.

Story Telling Chimpanzee

See http://panarrans.blogspot.in/

A Dose on Teaching Methodologies

Often Ideas, opinion, concepts and / or "Point of views" are better explained by contrasting examples. Here I will explain "Teaching Methodologies" with contrasting facts, to invoke logic and thoughts.

Thought Provoke 1 -

Certain facts about Stock Market are known to many, but not to all. The "Blue Chip and / or Large cap" stocks are traded the most. More people want to own pieces of these Big / Successful companies. The high trading volume, and the Lots of money into these stocks confirm this. Next in interest are Medium Sized companies, followed by Mid Caps, Small caps, Penny stocks ... so on.

Even in Mutual Funds, more money is in Large Cap Funds or Blue chip Funds. The least is in Penny stocks. Most days there is no trading in Penny stocks. To buy stocks that are not being traded, someone has to contact and request brokers specifically. There are millions of examples where someone 's money got "locked" into non-traded stocked and became very difficult to exit.

Now think why is this ? Market as overall is "extremely intelligent". The Market as a whole rewards or punishes performance, trends, future Growth / Profit / Prospects Ruthlessly. People in general want Stability, Liquidity, Quicker and Steady Profit. Investing in trees which will grow and give you return after 25 years is hardly acceptable in the world where computers trade in seconds for every arbitrage advantage. Blue chip, Big companies are huge, are around for long time with lots of data with their ups and downs, so many performance analysis and graphs ... in contrast to IPOs or startups! Is it interesting that 90% startups Vanish within 5 years ?

Replace Companies with students, in the above discussion. Which is more riskier to bet on (for future results / Success in life / Results / Money Earned etc) on toddlers ? or on students in Standard 8 ? Or on Students of Standard 10 ? students of standard 12 ? Students in Famous colleges ? (such as NITs or IITs etc) ? Guys with IIT + IIM combination ? etc. If you or someone else meets 10,000 students of age 4 to 6 years, what can be concluded about any individuals performance ? What can we guess about group performance ? What can we predict about all of them ?

[I expect people to know that NOTHING can be predicted with any group or individuals. Read Nassim Taleb's book The Black Swan]

- -

Thought Provoke 2 -

Meet a random group of people and tell them to name some Famous Physicists. (Doesn't Matter Alive or Dead). Most probably you will get the names Albert Einstein, Newton, Galileo Galilei, ... (You can have fun assuming that I don't know any other names !). Well ... You

will get some names say take atleast 10 - 15 names. [Most probably the list will not have names of John Stewart Bell, Alain Aspect, William George Unruh, John Bardeen or say Hendrik Casimir in the list. Even though these guys are best of the bests, general People do not know their names !]

Next tell the random group to name some (at least 15) Famous Chemistry Guys ... This is will tough. Most probably the names you will get are Mendeleev, Dalton, Lavoisier, Joseph Priestly, Fritz Haber, Frederick Sanger, etc. I am sure this list will vary widely, from group to group. Most people will not know that Frederick Sanger is the only Person with Two Nobel Prizes in Chemistry, and Linus Pauling once in Chemistry and once in Peace. Almost everyone knows that Marie Curie got Nobel Prize once in Physics and once in Chemistry. While most people will not know that John Bardeen in the only Person to Nobel Physics Prize twice !

If you tell the group to make a list of top 10 (or famous) Botanists ? or Zoologists ? ... Hardly any group will able to tell you a few names.

What about name of 10 Psychologists ? Most probably the only name you ever get is Sigmund Freud. No one will tell you names of Gordon Allport and S. Odbert.

[All explanations given by Freud are wrong, and crap. Modern Psychologists, call Freud worst than a quack. See how Professor Bloom, from Yale laugh at Freud, (and I agree with Prof. Bloom), in the class...

https://www.youtube.com/watch?v=P3FKHH2RzjI&list=PL6A08EB4EEFF3E91F]

If the random group is told to make a list of 10 Famous Persons in general, then most will try to put names of movie Stars or say Music Legends. I have conducted these discussions with many groups, and seen that if Movie Stars, or Music stars are not allowed to be named, then it really becomes difficult for the guys in the group to name 10 Famous persons. Though some will simply say ... "There are too many .... " ... start with Mahatma Gandhi ....

What about a list of Famous Atheists ? Will people in India readily name Nobel Laureates C. V. Raman, Subrahmanyan Chandrasekhar etc as Atheists ?

My Personal list of Famous and Successful people is Nicholas Winton, Dean Radin, Luca Turin, Satyen Bose, Gertrude Elion, Dr. Harrison Schmitt, Emmy Noether, Kurt Godel, Desmond Morris, Alan Turing, Irena Sendler, Andreas Vesalius, Richard Stallman, Roman Polanski, Christopher Alexander, Carl Sagan, Perelman, Arno Penzias, Ilya Prigogyne, Nadia Comaneci, Marcel Marceau, Augusto Boal, Anthony nesty, Pele, Roger Milla, Vaclav Havel, Jim Jarmusch etc. This is because of various reasons, and with lot of searching, thoughts, pondering ...

By now it is already 2-3 minutes of long boring harangue ... is it ? So the Bomb Question ...

Which schools were all these guys from ?

Do you realize that success of each of these guys are due to huge randomness, lots of hard work, luck, and time specific. Do you realize that thousands of Billionaires, Millions

of Millionaires have revealed all details of their hard work, but the "Success sequence" can not be repeated.

Go to any school which is say 60 or 70 years old; you will find 1 or 2 ex students as scientists in NASA, very senior guy in some large cap company etc. [My school KMPM High School Bistupur also "boasts" of 3-4 guys in Nasa etc. Personally I have never heard any school talking of ex-student being in ISRO. I wonder does ISRO have Scientists ? Are Schools happy about them ? Is it more prestigious to be in NASA or in ISRO ? Does guys from NIT or IIT join ISRO ? Why does "Prestigious Schools" in India send students for "Summer Tour" to NASA but NOT to ISRO ?

So many interesting questions ... No answers. No one agrees with any answers ! These were only thought provoking discussions ...

When there is NO consensus about "Good Schools" then is there any consensus on "Teaching Methodology" ?

What are various Teaching Methodologies ?

- -

Thought Provoke 3 –

Behind my Home, in an Independent House, a Lady has put up a board. She runs a toddler play-school. In the Board about her, and about the school, she says ... "Montessori education Certified from Europe .... From some 'Famous' certification agency ". Well around my home, within 3 km there are more than 15 toddler playschools. Each distinguish themselves from "others" in some way or other. Each say they are better because of some Certification, or some teaching methodology.

Now no one talks of teaching Methodology of "Famous Educationist", the first Nobel Laureate of Asia, Robindranath Thakur. Surely what ever Robindranath had said or advocated is "very old" and should be scrapped ! Who cares of old things ? [except of course if it is Vintage car or painting selling opportunity !]

I personally don't care about what Robindranath had advocated. I did not try to find out. I am busy with many other things. These will the words of many or most people!

One of the drawbacks of common Human beings is "not to search and compare" but to get influenced by many Marketing / advertisement methods. People get influenced by suggestions, word of mouth, advices, and Modern Technical experiences such as Mobile Apps. These are huge business opportunities.

We have vedanta way of teaching by Swami Dayananda Saraswati. Very big group of institutions, who are convinced that "their method of teaching" is the best.

PSBB Millennium Group of Schools, say in their website ... combined strength of over a decade of 'thought leadership' in best pedagogic practices of the Learning Leadership Foundation and more than five decades of academic excellence...

BGS group of schools say in their website ... Fostering independent thinking, thoughtful decision-making, critical analysis, appreciation with intellectual humility to accept difference in opinion. Helping the student to discover what it is to live and grow with clarity of thought, with harmony in Nature, with beauty and freedom in the world. Inculcating the best of Indian culture and tradition among the pupils. Creating responsible, disciplined and secular citizens, who are fully aware of their social, moral and cultural obligations and commitments, with a desire for unbounded service to humanity.

Aurobindo schools, Ashrams, follow .... Integral Education regards the child as a growing soul and helps him to bring out all that is best, most powerful, most innate and living in his nature. It helps the child develop all facets of his personality and awaken his latent possibilities so that he acquires. They say ... Rupantar, one of our special initiatives, is a strategically designed initiative that targets the highest impact areas in Education with innovative solutions to transform an entire state education system in India.

A guy named Gadadhar Chattopadhyay (not related to me), became very Famous. Or should I say, yet he is famous ? I see his photograph in many houses, randomly; as I visit. There are many ashrams, in various parts of the world ... even in Bangalore, named as Ramkrishna Ashram, or Ramkrishna Paramhansa Ashram ... He also tried something on education reforms. His advice were also there for those who want to listen. The Ramkrishna Schools do follow their own "Teaching methodologies". Bhakti, Love, Kritya, Yoga .... the list is long.

Lots of kids go to Abacus classes. All the above techniques were surely enough, for teaching Maths. To become "good at Maths" the parents donate in Abacus classes.

Since when did you start assuming that Vedic Maths, and Abacus is enough to make all students good ?

Kumon, created by Toru Kumon, is a private tutoring organization. The Kumon Method is the mathematics and reading educational method which is practiced in franchised Kumon centers. Lots of Parents are donating in this method as well, so that Children can become whiz kids in Maths.

Little Einsteins Pre-School Branding is another money making venture. They also claim to be better than others. They use "Multiple Intelligence" framework. Now this is a framework; while others were using mere methodologies. Howard Gardner's theory of Multiple Intelligences utilizes aspects of cognitive and developmental psychology, anthropology, and sociology to explain the human intellect. Although Gardner had been working towards the concept of Multiple Intelligence's for many years prior, the theory was introduced in 1983, with Gardner's book, Frames of Mind. These are Research Backed theories. In contrast Indian

Gurus never talk of any research backing. Gardner's theory challenges traditional, narrower views of intelligence. Previously accepted ideas of human intellectual capacity contend that an individual's intelligence is a fixed entity throughout his lifetime and that intelligence can be measured through an individual's logical and language abilities. According to Gardner's theory, an intelligence encompasses the ability to create and solve problems, create products or provide services that are valued within a culture or society. Originally, the theory accounted for seven separate intelligence's. Subsequently, with the publishing of Gardner's Intelligence Re-framed in 1999, two more intelligence's were added to the list.

Curry's onion model (Curry, 1983) was developed with four layers -- personality learning theories, information processing theories, social learning theories, and multidimensional and instructional theories.

Personality learning theories define the influences of basic personality on preferences to acquiring and integrating information. Models used in this theory include Myers-Briggs Type Indicator, which measures personality in dichotomous terms - extroversion versus introversion, sensing versus intuition, thinking versus feeling, and judging versus perception, and the Keirsey Temperament Sorter, which classifies people as rationals, idealists, artisans, or guardians.

Information processing theories encompass individuals' preferred intellectual approach to assimilating information, and includes David Kolb's model of information processing, which identifies two separate learning activities: perception and processing.

Social learning theories determine how students interact in the classroom and include Reichmann's and Grasha's types of learners: independent, dependent, collaborative, competitive, participant, and avoidant.

Multidimensional and instructional theories address the student's environmental preference for learning and includes the Learning Style Model of Dunn and Dunn and the multiple intelligence's theory of Howard Gardner.

The World with 7 Billion people, and growing, gives opportunity to so many, to make their own share of money.

Kidzee another revolution in branded schools, say ... Regular seminars and workshops are held to align parents with Kidzee's approach and enable them to develop a safe, healthy, hygienic and developmentally appropriate environment, even at home. iLLUME kit, which is a part of every Kidzee, is chosen by Kidzee team of experts to ensure that it stimulates all the intelligence's of a child and provides her with multiple pathways to enhance learning. The focus is on providing learning aids that help the child to explore and learn in ways that interest her. Feedback is shared with the parents on regular intervals wherein areas for further development are identified and mutually agreed upon, thereby supporting the child in multiple ways.

ICF.com provides program and policy services designed to enable positive student and teacher outcomes in early childhood, K–12, postsecondary, and adult education. They say ... ICF provides training and technical assistance on education initiatives that drive positive and long-lasting change at the national, state, and local level. ICF specializes in their own methodology or approach of MDA (Multiple Dimension Approach).

cfrce.com Centre for Fundamental Research and Creative Education, says in their website ...

(CFRCE) is an organization dedicated to positive change and self-actualization and is at once a platform for untrammelled Inquiry and Research and a Talent Hotspot espousing Accelerated Learning in its deepest sense.

CFRCE levels the playing field for individuals and students by empowering them to take active and independent, systemic and systematic charge of their learning and education, inquiry and research, entrepreneurial and financial potential, driven primarily by intrinsic motivation, meaning and purpose, irrespective of extrinsic incentives or patronage.

CFRCE challenges the status quo in educational theory and practice -that narrowly classifies individuals as achievers or failures, bright or dull, talented or non-talented -and leverages individual learning to an extraordinary level of deep practice, mastery and creativity. It thrives in making learning a tremendously evocative, exhilarating and ennobling optimal experience or flow. Thereby, learning resolves itself into its natural role as an instinct, or more precisely, as an implicate order or neuro-cognitive potential that develops and expresses itself by spontaneous self-organization once the hindrances and obstacles to its unfoldment are dissolved, removed or overcome.

In the CFRCE programs earnest students and inquiring individuals at diverse stages starting from primary through high school, undergraduate and postgraduate levels are empowered to take years and sometimes even decades, off their learning curve by a unique combination of personal development, domain mastery and professional eminence, and attain world class levels of excellence and achievement.

Tablet and Mobile Apps teaching methodology ...revolution... by idiots, for the Idiots. In this methodology every parent presenting the student a Tablet, a smart Phone (Dumb phones wont do !); transforms every kid to a whiz-kid. Costlier Tablets, and Jazzy Phones will make a better Whiz-kid! Just by press of a button (sorry the icon of the App), the Whiz-kids can learn any subject in the world. By chance if they come to know that something is missing, they can google it !

Dr. Rajendra Prasad topped in Many subjects in various schools and colleges. What was the teaching methodology in the schools and colleges ?

We yet enjoy leave on Birthday of Dr Sarvepalli Radhakrishnan. He also gave his take in Teaching methodologies. If someone who is not bothered about his "teachings" then should he be allowed to celebrate Teachers Day ?

[http://www.researchinformation.org/files/Dr.-Santosh-Kumar-Behera.pdf]

Never ask which school did Srinivasa Ramanujan go ? What teaching methodology did his teachers follow ?

I have read many articles which argue that ability to Play Chess; is the best measure for IQ. If I believe in these kind of crap; should I reject students who do not play chess, or say doesn't play well ? When a student approaches me, should I ask the first question... **"Did you go to Abacus classes in childhood" ? "Do you play chess well" ?**

Let us assume only top 100 rank holders of IIT-JEE are only smart guys in this world. So in 60 years we got only 60 X 100 = 6000 unknowns. Let it be loud and clear that from every random school and colleges rarely a smart guy shines, we only get Millions of Unknowns. There is no point in asking what happened to school batch-mates or college friends of Erwin Schrödinger.

I can write many more pages on these "Teaching Methodologies". Better I ask some hard questions ....

When we were naming Famous / Successful people did we name any India or IITan ? IIT Kharagpur is around since 1951. How many guys from IIT could become famous ?

[Now don't jump and quickly tell me names of Sundar Pichai, Nandan Nilekani, or Narayana Murthy. Sundar is famous since very recently. What happened to all the IIT guys since last 60 years ? Also Nandan or Narayana are famous for Business reasons or for Money; NOT for technical reasons, or any inventions. Bjarne Stroustrup, James Gosling are more important; more famous than Nandan or Murthy.]

Vinod Dham famously known as Father of Pentium Chip was randomly from DCE Delhi College of Engineering.

It is well known that Professors at IIT are 100 times smarter than the students. Most IIT students find it difficult to cope up at college. A large percent (someone told me close to 50%) of the IIT students get a back in some subject some year.

Well if the Professors are so smart, then how many famous Professors were named in the above discussions ?

In contrast it is well known widely discussed opinon that Students in IIM are far better / superior than the IIM Professors. So no question of naming any famous professor of IIM as Sucessful or role model. We never named any ... did we ?

I shouldn't ask how many IIM Alumni became famous in so many decades.

China has 568 billionaires versus the United States 535 as of 2016. Had seen a headline in Bloomberg ... "Chinese eat so much pork that the sellers are Billionaires!"

Does each and every Billionaire become my role model ?

Just because they made lot of money, each of my students should venerate them ?

How many People know name of Aliko Dangote – Net worth: $15.7 Billion - The Richest Man of Africa ?

Which school were these people from ? If we do not care of Aliko Dangote's school, then why should we bother about Nandan's School ? Did Mr. Murthy go to school ? I don't think everyone is eager to go to that school !

The United States has had the most Nobel Prize winners, with 336 winners overall. It has been most successful in the area of Physiology or Medicine, with 94 laureates since 1901. Similarly, the United Kingdom that majority of its 117 Nobel laureates winning in Chemistry and Physiology or Medicine. The top five countries with the most Nobel laureates are all western nations - with the United States, the United Kingdom, Germany, France and Sweden topping the rankings for the best minds in peace, literature, science and economics.

Recall the concepts of Determinism vs Predictability. Randomly as I meet my ex-Students, each say some story of their life or other. Someone is a Doctor from some college, so someone is in Navy. Someone is an Engineer, while someone is running his own business, or studied "Hotel Management". In general people want to feel good of themselves, and justify the outcome as "good". Each and every person see his own outcome as the "very good". Whatever he is doing is termed as success, and achievement. No one believes or agrees with external definitions of success or achievement given by someone else!

Who is more successful ... or achieved more ... amongst Sam Walton and Anjezë Gonxhe Bojaxhiu ?

[Now most people will say Apples and Oranges can not and should not be compared ... Well we could have asked Potatoes and Pomegranates, which are better ? Though I named Sam because he wrote a book regarding success mantras; how to make money!]

Did the parents, friends or close associates knew that the guy will become Billionaire ?

What about Reading Books, Being Humble, Ready to learn, Choose a Mentor, Understand your Dreams clearly, Persevere, Seeing Videos .....

https://www.youtube.com/watch?v=7bB_fVDlvhc

What "Teaching Methodologies" were followed in the schools in which all these guys went ?

Let it be Loud and clear ... School or College does not matter. Now how can the teaching methodology of the School and College matter ? which methodology for what ?

There is no consensus regarding "Thinking Techniques" or Should I say, "methods". Now a days we have to do "out of the Box thinking", normal thinking, or just "thinking" is undefined. Someone who is not doing "out of the box thinking" is termed as, not so smart. Edward De Bono taught us Po, 6 Thinking Hats, Lateral Thinking ... etc. I am not sure if my Boss will appreciate me if I say I am trying these methods. For most Bosses, "out of the Box ... " is

enough and only acceptable technique. How are the Gurus, and Practitioners of Mind Mapping, "The Checklist", Picture Association, Change Perspective, "Get Up and Go Out", Brainstorming, Random Input, Reversal, SCAMPER, Reframing, Morphological Analysis, Storyboarding, Synectics, Metaphorical thinking, Lotus Blossum Technique, NLP (Neuro-Linguistic Programming) Techniques, Assumption Smashing, LARC Method, Simplex, TRIZ method, Fuzzy Thinking, Breakthrough Thinking ... doing ?

For some people now a days, plan is known as Hack. Growth Plan is Growth Hack. Coding is Hacking ...

Is there any consensus on Management Techniques ? Management by Goal Setting, Management by Objective, Management by Profit Centers, Management by Micro Profit, Management by exception, Management by Tactics, Management by Quality Control, Management by Total Quality, Management by Customer Focus, Management by Customer Delight, Management by Planning, Management by Forecasting, Management by Organizing, Management by Commanding, Management by "coordinating", Management by cost benefit Analysis, Management by Zero Base budgeting, Management by Log-Frame Analysis, Management by Current State Assessment, ...

All these are most commonly replaced with "Management by Meetings", Management by Wondering Around, Management by Shouting, Management by Con-calls, Management by Continuous Reminders, Management by Bribing ... Actually all these are Management by ..."**the technique and terms the Boss wants** ! "

Moms use only one technique ... Management by continuous Nagging, Chiding, Scolding, Pushing, Threatening, Ashaming, Beating ...

Key Concepts in Science as Recommended by Professor Subhashish Chattopadhyay

In youtube we have several thousand videos; where Science is discussed at sufficiently higher levels, than normally educated Engineers know. I have seen thousands of these Science videos because of my Bias towards spending time with Science (or my Hobby being Science). The information density; meaning the new things taught or discussed in the Video is very low in general. So if I tell any student or friend to see these thousands of videos, surely they will not see. Really all of us do not have so much time. Every person has different priorities, and truthfully so many things to do. While these words may be known to many; I also observed that in Panel discussions the reverent Persons are unaware of quite a few proven / well understood facts. Lot of time is wasted when a Panel member makes wrong statement or uses wrong words, and another member corrects him to say the right words. A Scientist as a Panel member sitting in the dais, in Science talk shows; is expected to know all the facts and use exact right words which should not be wrong or have any multiple meaning.

Let me quote some examples ...

1) Eienstein 100 years ago was not aware of Dark Matter, Dark Energy, or say "expansion of Universe is accelerating". In the context of stars, Galaxies, Celestial events (such as a Supernova explosion) he correctly said that if someone travels at a very high speed towards a Galaxy he will see the events earlier than the people who remain back on Earth. The word "now" has different meanings in different parts of the world. With resepect to People at Earth we can travel into the future, at very long distances. So an Astronaut can see a Supernova explosion before People in Earth see it.

But yet we see Panelists / Scientists changing this context to near distances, in Earth; and confused about flow of time from Past to future.

2) Since last 80 years (Approx) of Quantum Mechanics it is well known, (well understood, and Mathematically well formulated; Backed up or confirmed by several experiments) that smaller Particles can tunnel easily. So an electron or Neutrino can tunnel more easily than a Proton, Neutron or Mesons. As we have group of Particles or as Complexity increases such as a Folded Protein or a Ball in the Macro world, then then the Wave Phases randomly cancel out. The Properties of Tunneling, Interference, Diffraction etc does not hold. So Balls thrown through bars in a cage will bounce or pass through. Diffraction of a Ball or Interference of Balls is a meaningless Question or Waste of time to be discussed. We don 't have to take such big objects as Balls or Human Beings. If we take Molecules or Amino Acids; the Quantum World discussions are not relevant any more. In the Quantum World "Calculations"; we only have Probabilities, not Deterministic or Predictable. The Quantum state collapses; when observed. The entangled particles also get affected. These Quantum world concepts are not needed or not extendable to macro world.
But yet we see Panelists / Scientists changing this context to Bigger Objects and discussing about time reversal, Time Travel etc.

3) Second Law of Thermodynamics is understood well since Last 100 years. According to the Laws of Thermodynamics, entropy, the measure of the disorder in a closed system. It is about Statistical Laws of Randomness, Organization, elastic collisions, Entropy, Temperature etc. The Entropy of the Universe is almost always increasing, because the Universe is expanding. There can be small local fluctuations in Entropy and disorderliness randomly and due to attractive forces such as Gravity or Strong force etc. Photosynthesis, formation of Molecules, formation of Polymers or sugars or Proteins from monomers, Secondary Structures, Tertiary structures such as folded Proteins joining up mechanically and increasing order are understood in context of "open systems" and stability Laws which as more prevailing than second law. In an open system, there can be an influx of energy into the system capable of reinvigorating the structure; in full accord with the Second Law of Thermodynamics. Energy input can decrease entropy, and can simultaneously increase order. So a tree can grow by "bunching up" carbohydrates, Animals can grow by digesting carbohydrate chains, etc. Self-organization is a natural property of complex genetic systems. There is a spontaneous crystallization of order out of complex systems, and that this spontaneity can occur with no need for natural selection or any other external force. Dynamic systems, have a tendency to become more concentrated and heterogeneous as they evolve.

But yet we see Panelists / Scientists changes this context and tries to apply a lower version of 2nd law of thermodynamics only; in every situation. Seeing the holistic picture is not in the

good habit of many. It is expected Thermodynamic Asymmetry in Time; should be well known and well understood by everyone.

[May be, I am assuming a world where the Panelists of Science Discussion forums will not contradict or correct one another in Public. They can argue and compromise in private discussions, and in public all say the same correct words.]

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I recommend students to know the following Key Concepts

- The paradox of predictability
- Kolmogorov complexity
- Chaos Versus Complexity
- Dynamic networks or complex systems
- Concept of emergence
- Patterns Amid Complexity
- Red Queen effect
- Determinism VS Predictability
- Poincare fluctuations
- epistemic uncertainty
- Aumann's agreement theorem
- LQG (Loop Quantum Gravity)
- Occam's Razor
- Ology
- Ontology
- Nomology
- Bohmian quantum theories or Bohmian mechanics
- Planck length, Space, time etc
- Stability of Solar System
- Thermodynamic Asymmetry in Time
- How Prabability is distorted in Human Mind by Prospect Theory
- Anna Karenina principle

Many years ago Laplace made an error. Laplace assumed an Universe, in which all of the rules of the are fixed. In this type of universe, as Laplace pointed out, if we knew enough information about the current state of the universe in addition to all of its fundamental and unchanging laws, we would be able both to calculate the entire history of the universe and to predict its entire future. There would be no room for free will, which would be seen merely as an illusion. The actual solar system contains eight planets, six of which were known to Newton, Millions of Asteroids and each planet and rock exerts small, periodically varying, gravitational forces on all the other. The puzzle posed by Newton is whether the net effect of these periodic forces on the planetary orbits averages to zero over long times, so that the planets continue to follow orbits similar to the ones they have today, or whether these small mutual interactions gradually degrade the regular arrangement of the orbits in the solar system, leading eventually to a collision between two planets, the ejection of a planet to interstellar space, or perhaps the incineration of a planet by the Sun. Even though, the interplanetary gravitational interactions are very small, the force on Earth from Jupiter, the largest planet, is only about ten parts per million of the force from the Sun—but the time available for their effects to accumulate is even longer: over four billion years since the solar

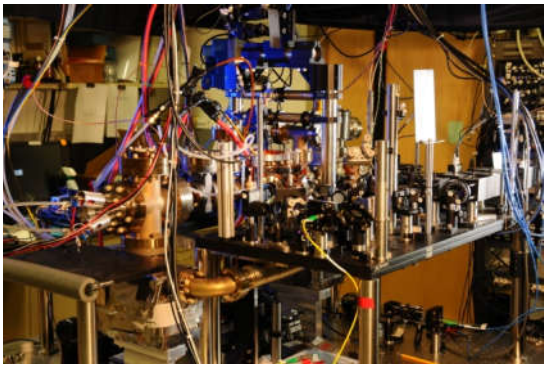
system was formed, and almost eight billion years until the death of the Sun. The effects of various forces, stability or instability with various possible random initial conditions, were tried in computers.

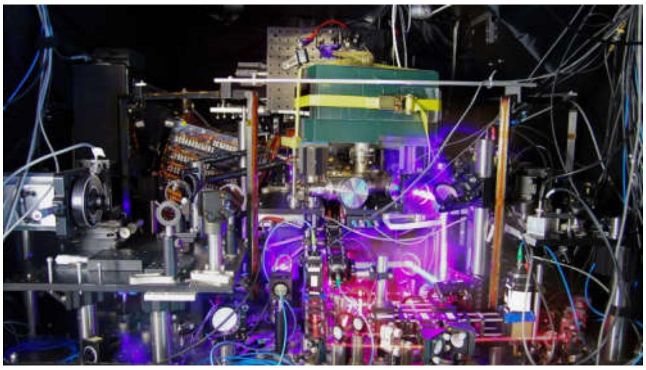
[Compound Pendulum with LED showing Chaotic movement is shown at

https://www.youtube.com/watch?v=GFxPMMkhHuA]

BUT ... Chaos theory studies these mechanistic types of systems but it tends to emphasise the principle of feedback whereby two variables are influenced by each other: this can lead to non-linearity and the variables behaving in seemingly chaotic ways. An important insight of Chaos Theory is the sensitivity of a chaotic system to initial conditions due to the non-linearity of the system. What this means is that if the initial conditions of a chaotic system were changed microscopically, then over a long enough period of time the outcome of the whole system will be completely different. This is often referred to as The Butterfly Effect. However, it is important to emphasize that if the initial conditions of the chaotic system were unchanged between two simulations to an infinite degree of precision, the outcome of the two will be the same over any period of time. So the butterfly effect really only serves to contrast the outcomes in two marginally different systems that are still deterministic i.e. machine-like. In one simulation, the butterfly flapped its wings, in the other it did not.

The science of Complexity happens somewhere between totally ordered and totally random systems. Complex systems are denoted by the fact that they may be generated by a relatively simple set of subprocesses; a few things interacting, but producing tremendously divergent behaviour. As Nobel laureate Murray Gell-Mann phrased it: “Surface complexity arising out of deep simplicity.” One might also call this: deterministic chaos; in other words, it appears random but isn’t. In complex systems, there is a concept known as a global cascade, which is similar to what people often mean by the butterfly effect but it is in fact fundamentally different. A global cascade is basically a network-wide domino effect that occurs in a dynamic network, made famous by Duncan Watts in 2002. Watts showed that sometimes a complex system proved robust in the face of a modest shock (it might just wobble slightly); but in other instances, the same shock might cascade across the system, showing it to be fragile.





Whatever we measure, there is a factor of error. Atomic clocks measuring time interval upto 17 decimal places, have error factors at the 18 th place. We know "time interval" ticks slower near more gravity, compared to less gravitational field. So time interval at the roof of the lab

will tick quicker, and record more number of ticks, compared to ground floor of the Lab. The atomic clocks with 17 decimal places Precision, can see the difference in Time interval ticks with a height difference of 40 cms. Now in normal real world we hardly work or do things with 2 to 3 decimal places of Precision. Meaning things are not exactly repeatable. If we keep hitting a ball with 2.345 Newton force repeatedly, at a decided angle, each time there will be a different ball, different angle, different value of the force, within various error factors. If we imagine a slightly different initial direction, the trajectory will at first be only slightly different. And collisions with the straight walls will not tend to increase very rapidly the difference between trajectories. But collisions with the convex object will have the effect of amplifying the differences. After several collisions with the convex body or bodies, trajectories that started out very close to one another will have become wildly different. So a student should know that the future is not repeatable. With a ball itself if so much of Chaos, complexity etc, then imagine what happens for people, future, success and fame of persons, Careers, accidents, disease, lottery .... Nothing is predictable in the Trillion random incidences.

In Quantum world the complexity or chaos of repeating is more. Diffraction, entangled particles, Interference, interaction with virtual particles that pop up, various decays and transformations, etc creates a probability soup. At the microscopic level the world is ultimately mysterious and chancy.

So both in micro world and macro world events are not repeatable. Further it goes, with more interactions, outcomes may or may not fall into boundaries, or envelopes. In some cases there are fractal outcomes, some cases Gaussian, some cases long tail, the list can go on.

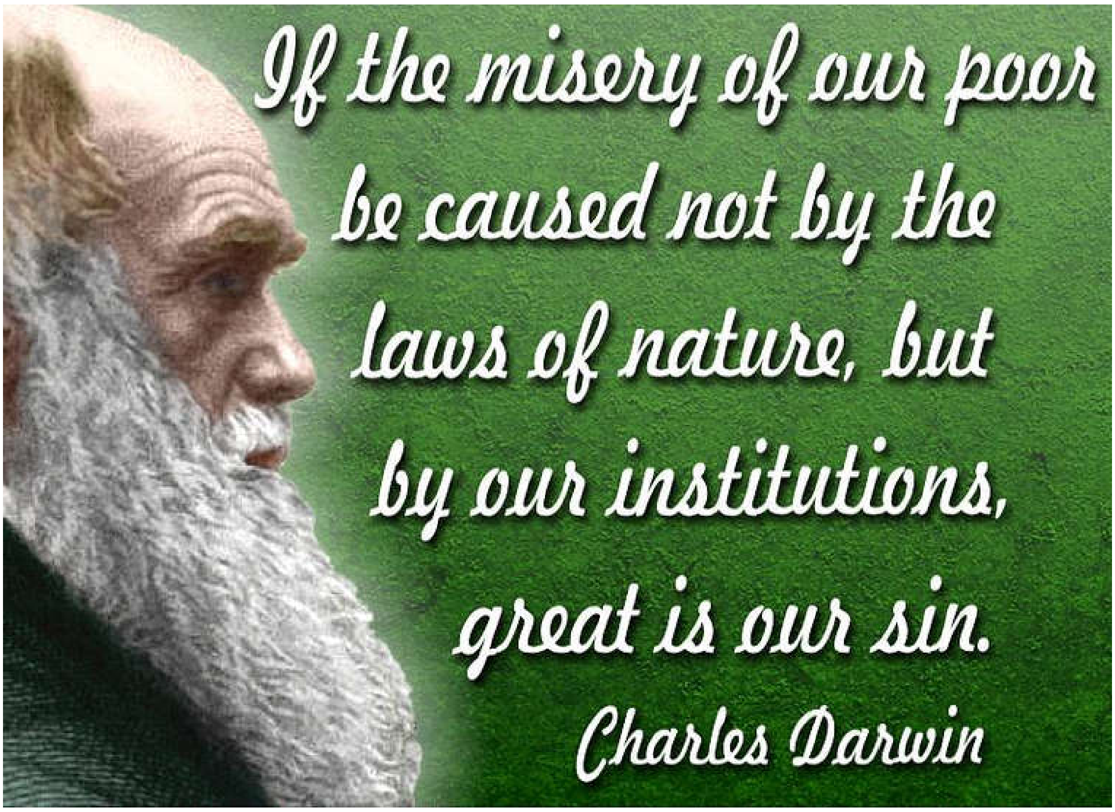
In chaotic dynamical systems come in a great variety of types: discrete and continuous, 2-dimensional, 3-dimensional and higher, particle-based and fluid-flow-based, and so on. Mathematically, we may suppose all of these systems share SDIC (Sensitive dependence on initial conditions). But generally they will also display properties such as unpredictability, non-computability, Kolmogorov-random behaviour, and so on—at least when looked at in the right way, or at the right level of detail.

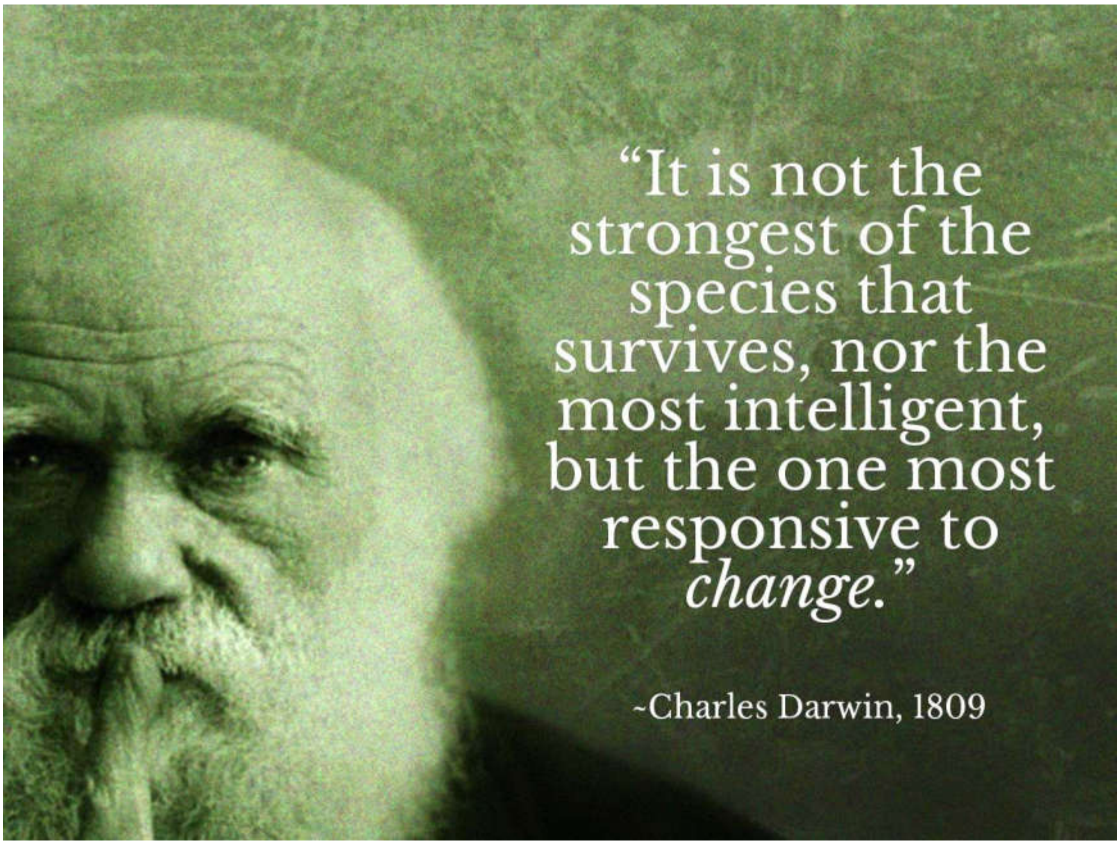
Also laws of Physics are different in different parts of the world. Near or at Singularities, such as near Blackhole, the known laws breakdown. We do have concepts of Planck length, Planck space, Planck time etc. The maximum temperature that we can theoretically have is the temperature where photons are emitted with wavelength of Planck length. At that high temperature more particles, and virtual particles are created. The energy starts getting converted to mass, and thus temperature can 't increase any more.

Country	Deployed warheads*	Other warheads†	Total 2014	Year of first nuclear test
USA	1920	5380	7300	1945
Russia	1600	6400	8000	1949
UK	160	65	225	1952
France	290	10	300	1960
China		250	250	1964
India		90–110	90–110	1974
Pakistan		100–120	100–120	1998
Israel		80	80	. .
North Korea		6–8	6–8	2006
Total	**3970**	**12 350**	**16 300**	

* 'Deployed' means warheads placed on missiles or located on bases with operational forces. All estimates are approximate and are as of January 2014.

† Warheads that are in reserve, awaiting dismantlement or that require some preparation (e.g. assembly or loading on launchers) before they become fully operationally available.


If the misery of our poor
be caused not by the
laws of nature, but
by our institutions,
great is our sin.
Charles Darwin


"It is not the
strongest of the
species that
survives, nor the
most intelligent,
but the one most
responsive to
change."
~Charles Darwin, 1809

The Rich and Poor divide is very huge in this world. Privileged are those who have the luxury to sue someone or other for slightest “discomfort”. In some cases “mental discomfort” is sighted as the cause for suing ….

Larksville woman sues county over son’s injuries at park

ERIC MARK / PUBLISHED: OCTOBER 24, 2014

A Larksville woman whose son was injured at Seven Tubs Nature Area in Bear Creek Township has sued Luzerne County, claiming negligence.

Melissa Moser, in a suit filed in Luzerne County Court on Thursday, claims that her 14-year-old son was badly hurt when a concrete barrier fell on him and trapped him during a family outing to the county-owned park on May 13. The boy was playing near a parking lot with his siblings when the traffic-control barrier tipped and fell on him, according to court papers.

We do not have the same rules or facilities for all in this world. See the images below and think who these poor men can sue? Can they sue anyone? Do they have money to sue anyone?

These poor men can’t afford hearse service. Nor there is any Public help or support. Can they sue anyone for “mental discomfort” and / or agony ? Society has pampered rich women with privileged laws and facilities. **Who cares for poor Man ‘s Feelings** ?

In contrast poor boys and Men are always left to fend themselves.

You should be horrified to see how much important the feelings of Rich Feminists are …

See http://www.breitbart.com/big-government/2015/09/25/u-n-womens-group-calls-for-web-censorship/

County in UK makes it a hate crime to upset women

If you talk to a woman in Nottinghamshire, East Midlands in the United Kingdom and she doesn't want to be spoken to by you, prepare to get a call from the police. In an effort to crack down on the alleged tidal wave of abuse and harassment...

WASHINGTONEXAMINER.COM

If you talk to a woman in Nottinghamshire, East Midlands in the United Kingdom and she doesn't want to be spoken to by you, prepare to get a call from the police.

(**How dare Men, talk to rich women ?**)

http://www.washingtonexaminer.com/county-in-uk-makes-it-a-hate-crime-to-upset-women/article/2596356#!

[Who saves and helps Savvy, Rich, Painted faced, Wearing high heels, Women with Manicured and Pedicured nails ? Dirty hands … and White Knights …]

I can only say that … "Poverty is very sad ! "


LAUGHING Colours
laughingcolours.com
A MAN DOES ALL THE
HARD WORK ONLY TO MAKE HIS
WOMAN LIKE A PRINCESS...!!

This book is dedicated to the following greats who died in Poverty, yet did their best in the subjects, they were passionate in. I couldn't achieve infinitesimal part of their passion even being so well to do!

1) Nikolai Ivanovich Lobachevsky (Kazan, Russia) 1823 - known primarily for his work on hyperbolic geometry, otherwise known as Lobachevskian geometry. William Kingdon Clifford called Lobachevsky the "Copernicus of Geometry" due to the revolutionary character of his work. He was dismissed from the university in 1846, ostensibly due to his deteriorating health: by the early 1850s, he was nearly blind and unable to walk. He died in poverty in 1856.

Nikolai was an atheist.

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2) Egon Schiele - Prolific artist Egon Schiele succumbed to the Spanish Influenza that took 20,000,000 lives in Europe in 1918. Schiele 's wife Edith (who was six months pregnant at the time) died three days before him in their tiny apartment in Vienna. They were broke and hungry, and Schiele spent as much time as he could drawing. He was only 28 years old and spent his last moments alone drawing his wife's body before his own untimely death.

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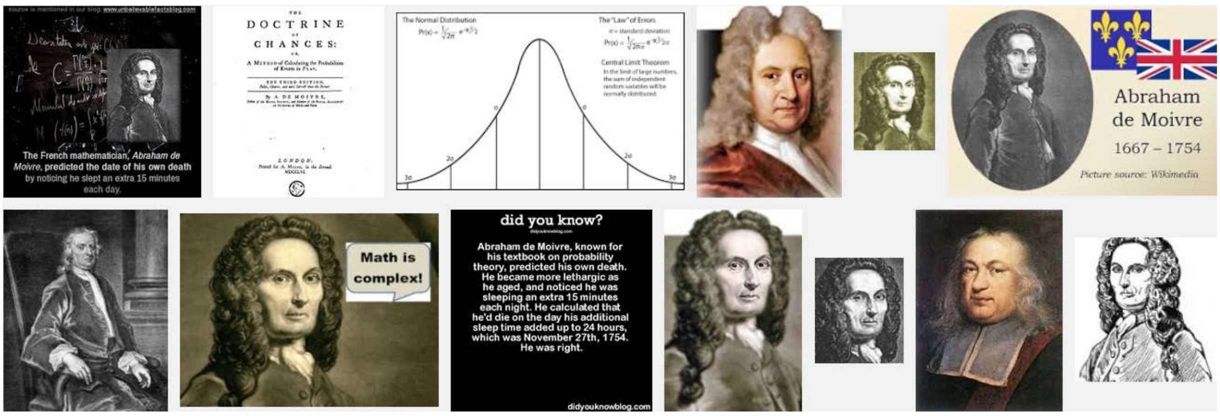

3) **Abraham de Moivre** 1667-1754 (87, natural causes) Despite being a gifted and renowned mathematician in France, de Moivre spent much of his life in poverty. He was a Calvinist, and when the Edict of Nantes was revoked in 1685 (a decision that is unequivocally considered to have damaged France), de Moivre left France for England. He remained virtually destitute, de Moivre was unable to secure employment and was often known to play chess for money in order to afford sustenance. Eventually succumbing to the ravages of poverty and old age, de Moivre predicted the day of his own death using a simple arithmetic progression in the number of hours he slept per day. The day he predicted 24 hours of sleep was the day he died.

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4) **Domenikos Theotokopoulos** AKA **El Greco** - Master of the Spanish Renaissance who studied under Titian, El Greco was known for his contorted figures in his paintings. Born in 1541, El Greco as he came to be known, studied in Rome before moving to Spain. What he wasn't known for was being a huge ladies man, or family man, as he followed various studios and painting masters across Europe. Some of his best known works were created for the Spanish royal family. El Greco was able to make a living as an artist for some time before he fell out of favor and became the subject of ridicule. He served as an inspiration for painters that brought forth the Expressionist and Cubist movements. Unfortunately after his work was scorned and laughed at he was unable to continue to make a living as a painter. It wasn't until 250 years after he died that the rest of the art world noticed his paintings. He was a big careerist and was described in letters in 1563 as a "maestro Domenigo" a "master" when he was just 22 years old. He died unrecognised and alone in Toledo, Spain on the 7th of April 1614.

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5) **Niels Abel** (1802-1829 Age - 26, pneumonia) Plagued by poverty and a lack of renown, Abel and his work went unrecognized during his lifetime. He spent time in Paris hoping to gain recognition and publish his work, but was unable to afford adequate means to sustain his health. In addition to being underfed, Abel contracted pneumonia. His pneumonia worsened on a trip to visit his fiancée for Christmas. He soon died, only two days before a letter arrived indicating that a friend had managed to find secure him a place as a professor in Paris. He never saw his work take root, nor did he ever secure a paying job as a mathematician, nor did he have opportunity to marry his fiancée.

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6) **Oscar Wilde** - His famous last words really set the tone for Oscar Wilde's end, "My wallpaper and I are fighting a duel to the death. Though Wilde was a celebrity of the age and his works sold well, he was known to have extravagant spending habits. One or other of us has got to go." After his imprisonment he had been given a very small yearly allowance from the estate of his deceased wife, and was not helped at all by his former lover Lord Alfred Douglas, who had at that time just inherited a large sum. Living essentially in poverty in Paris, he was known to wander, bumping into old friends and spending what little cash remained on alcohol. Reportedly, when a doctor attending to him during his last days asked to be paid for his services, Wilde joked that he would die as he had lived - beyond his means. He passed away in a hotel room in Paris completely bankrupt from paying legal fees for his arrest and imprisonment for the crime of homosexuality. If that wasn't bleak and cruel enough, it was during this period that his works were becoming extremely popular. Unnnfairrrrrrr.

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7) **Frank Ramsey** 1903-1930 (26, jaundice) Ramsey is known for his work in mathematics, specifically combinatorics and logic/foundations, but is also remembered as a gifted philosopher and economist. Ramsey suffered from lifelong liver problems, and was often unable to focus on work for more than a few hours a day. In spite of this, he gained renown as a promising young philosopher and mathematician, until a severe attack of jaundice hospitalized him in 1930. He died during an operation meant to alleviate the problem.

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8) **Claude Monet** – As the founder of French Impressionism, Monet's paintings usually dealt with landscape scenes in a moment. While his seminal work "Impression, Sunrise" is now studied and appreciated in art colleges around the world, it was widely derided by critics when it was first revealed. Monet received little but abuse from public and critics alike, who complained that the paintings were formless, unfinished, and ugly. He and his family endured abject poverty. By the 1880s, however, his paintings started selling.

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9) **Srinivasa Ramanujan** 1887-1920 (32, malnutrition/hepatic amebiosis) The story of Ramanujan is well known among mathematicians, if not in general. Described as a prodigy, savant, genius, etc., Ramanujan taught himself mathematics as a youth and began to devise results in analytical number theory and other areas of mathematics in isolation. He was quite poor and unable to afford school, and his exclusive devotion to mathematics precluded him from scholarship funding. He spent much of his life seriously ill, and spent a fair amount of time unable to secure any position as a scholar or mathematician. Eventually, he came to England to work with G.H. Hardy. Sadly, his long-term illness continued, and he succumbed to a combination of malnutrition and a parasitic liver infection.

10) **Vincent Van Gogh** – It is hard not to think of tragedy when considers the life of Vincent Van Gogh. If there was ever a fine line between madness and genius, Vincent Van Gogh crossed it quite early in his career. Without his time in insane asylums and self-inflicted ear mutilation, the world would have never had “The Starry Night” and “The Potato Eaters.” Despite his countless post-Impressionist chefs-d’oeuvres, Van Gogh only sold one painting in his lifetime. It sold for the equivalent of approximately $109 dollars. Although he is famous for his works such as “The Starry Night” this artist battled mental illness most of his life. Unfortunately he finally lost this battle and cut his ear off in 1888, committing suicide not long after that by shooting himself in the chest. His last words were, “The sadness will last forever.” He died broke and destitute.

See https://zookeepersblog.wordpress.com/vincent-van-gogh-who-prefered-to-paint-without-eating/

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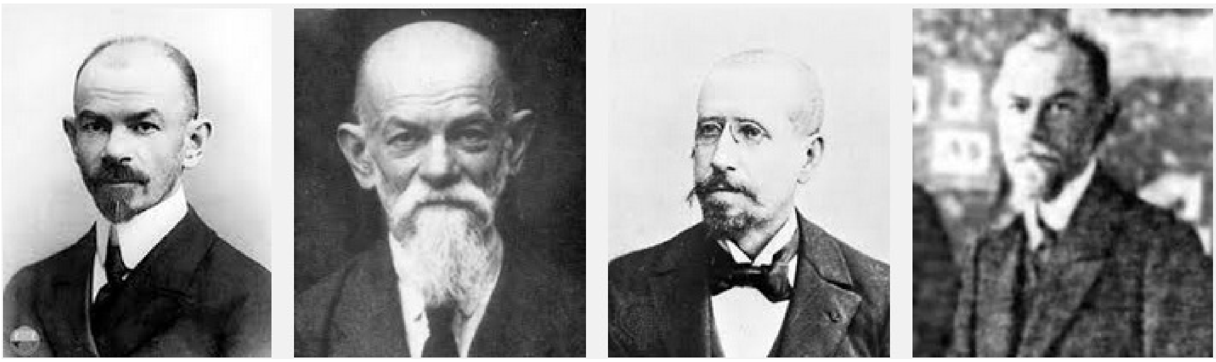

11) **Dmitri Egorov** 1869-1931 (61, starvation) Egorov made important contributions in the areas of analysis, differential geometry, and integral equations, including a fundamental result named for him in real analysis. Luzin was Egorov's first student, and was one member of a school that developed under Egorov to study real functions. Egorov became a leader and administrator in the Moscow Mathematical Society and at the Institute for Mechanics and Mathematics at Moscow State University. Egorov became a vocal opponent to the anti-religious persecution in the time following the Russian revolution, and was dismissed from the IMM. However, he remained active and well-respected in his position in the MMS, supported

by his peers in the organization. Outside influences began to manipulate the society, and within a year, Egorov was dismissed from his position and arrested. He went on a hunger strike in prison and died in the prison hospital (or, as some reports state, at a colleague's home).

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12) **Johannes Vermeer** – Vermeer was a 17th-century painter with eleven children, massive debt and a habit of working very slowly and painstakingly on his paintings. While Vermeer painted the “Girl with a Pearl Earring,” he certainly was not draped in them during his life. Instead of having the elite or nobility commission works, Vermeer’s genre of painting was catered to the provincial middle class. After the French invaded the Netherlands in 1672, the Dutch economy suffered terribly and Vermeer was left in hopeless debt. He suffered from a number of physical afflictions as well as mental illness. In 1675 Vermeer borrowed money in Amsterdam, using his mother-in-law as a surety. Soon after, the Dutch genre painter actually left his family in debt upon his death. After his death some of his paintings (he created about 40 in his lifetime) were sold with the names of other artists on them to make them more valuable. It took three centuries for Vermeer to be recognized as a master painter of the Dutch Golden Age for his use of light, tranquility and the unusual subject matter of peasants that populated his works. Though he did have patrons who paid him, he never made much and lived on the verge of poverty much of his life, eventually leaving his family in debt when he died at age 43.

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 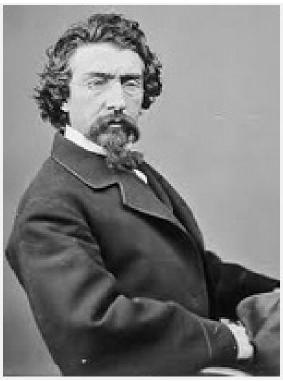  

13) Mathew Brady - The "Father of Photojournalism" is best known for his invaluable photographs of the American Civil War. Though he was a successful and well-known portrait

photographer before the war began (Abraham Lincoln's likeness on the $5 bill is modeled after Brady's portrait of him), he spent around $100,000 during the war on his photographs, which numbered in the thousands. The pictures brought the truth and grotesque horror of the war to the doorsteps of all Americans - a marked change from the propaganda and half-truths coming from print journalists at the time. Unfortunately, after the war no one wanted to be reminded of the horrors of it, and Brady was unable to sell his photographs or recoup his losses. Eventually Congress bought his collection for a mere $2,840, but Brady's life had already been ruined by poverty and alcoholism, and he died in relative obscurity in 1896.

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14) **Paul Gauguin** – Poverty became Gauguin's reality. Then his favorite daughter Aline died of pneumonia and Clovis, his son, died from a blood infection. Gauguin's escapades were far more exotic than his peers which eventually landed him in French Polynesia. There, he produced masterpieces like "Spirit of the Dead Watching," which largely inspired primitivism – an important art movement of the 19th century. After many years of poverty and sickness, Gauguin died from heart failure, alone and unaware of the mark his art would later make on the 20th century.

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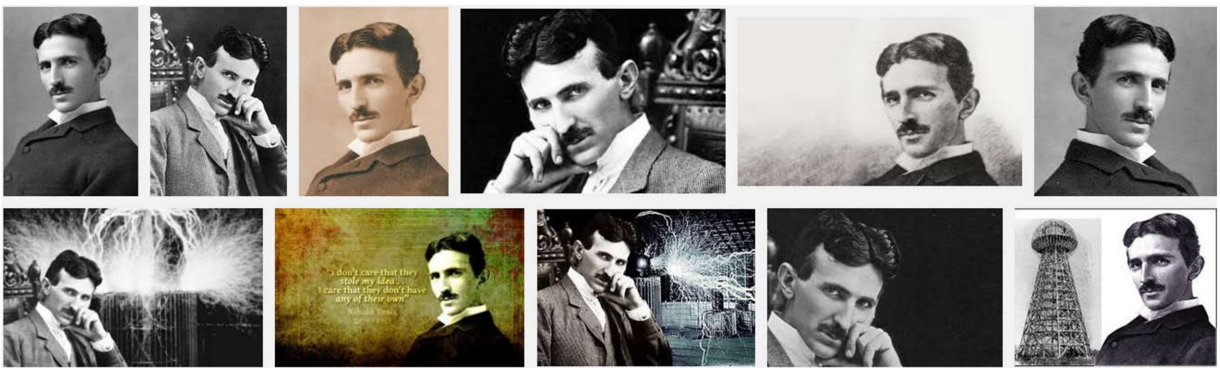

15) Nikola Tesla - Early in the 20th century, brilliant scientist Nikola Tesla was a world-famous inventor and regular headline news-maker. As for genius, we have Tesla to thank for alternating current, radio, wireless technology, neon lamps, and X-rays. Sadly, Tesla's life

was a series of run-ins with guys like Thomas Edison, who famously stiffed Tesla out of $50,000, and Guglielmo Marconi, who stole the credit for the invention of the radio by using 17 of Tesla's patents. Tesla died penniless in 1943 in the New Yorker Hotel, where he had lived for 10 years after being evicted from another hotel for not paying his bill.

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16) **Stephen Foster** - Though you may not be familiar with Stephen Foster's name, you undoubtedly know his songs. Foster is considered the "Father of American Music," penning the works "Camptown Races," "Swanee River," "Jeanie With the Light Brown Hair," "Beautiful Dreamer" and "Oh! Susanna" among many others, some of which function as current state songs. Foster's melodies were popular in his time (and remain so today, despite some controversy), and he wished to make a living as a professional songwriter. Unfortunately, the lack of copyright laws or a structure for the payment of royalties meant Foster made very little to nothing on performances and reprints of his work. Foster died at the age of 37 with 38 cents in his pocket.

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17) **Jean-Honore Fragonard** - Jean-Honore Fragonard was born in Grasse, Provencal in 1732 and became one of the most famous painters of the Rococo period. His family moved to France in 1738, where he was heavily influenced by the Baroque style. His art career started out promisingly enough, having attended the Ecole Royale des Eleves Protégés in Paris. Fragonard was then sent to Italy, where he spent time at the French academy in Rome. He had some success after returning to France, preferring to do private commissioned work. Some of his best known pieces were "Coresus and Callirhoe" and "The Swing". He was well-known for his sensual and erotic style, complimented by his sense of whimsy and fantasy.

Unfortunately, Fragonard was unable to adapt to the new style that eventually came into popularity over “Rococo” called “Neo-classical”. That ended his career and he died in relative obscurity and poverty in 1806.

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18) **Herman Melville** - The celebration of the Moby Dick author’s genius did not begin until well after he could enjoy — or profit from — the recognition. It took a solid 30 years after Herman Melville’s death before his epic whaling novel was recognized as a masterpiece of American literature. By then he had long since abandoned any hopes of living off his writing, instead working as a customs inspector for 19 years. When he died of a heart attack in 1891, he was broke and virtually unknown. The only paper to mention his passing referred to him as a "long forgotten" author.

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19) **James Barry** - James Barry born in Ireland in 1741 was a self-taught artist. He’s best known for his six part series of paintings, “The Progress of Human Culture”. He completed these for the Great Room of the Royal Society of Arts. He became a member of the Royal Academy in 1773 and taught as a Professor there from 1782 to 1799. Barry was one of the earliest of the “romantic” painters in Britain and although he died in poverty in 1806 he was thought to be the most important Irish Neoclassical artist.

20) **Joseph Gandy** - Reviews for a 2006 book on the life of Joseph Gandy referred to him as a "stifled genius" and "our greatest architectural artist." But history has mainly forgotten the genius that was Gandy, who lived and worked in Britain in the early 1800s. Despite being a major figure in Romantic culture and creating some of the best architectural drawings of all time, he was a commercial failure and was thrown into debtor's prison. He died in a windowless asylum that his family had him committed to, and the whereabouts of his grave are unknown.

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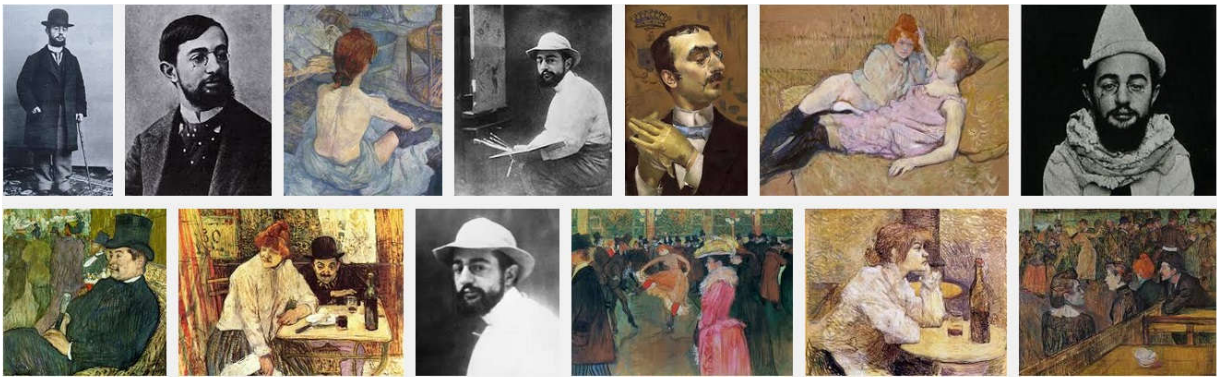

21) **Henri de Toulouse** - Lautrec was born in France in 1864. He was a close friend of Vincent Van Gogh, even using him as a subject for his painting. Toulouse-Lautrec is considered one of the great painters of the Post-Impressionist period. He favored painting the theatrical life of Paris in the 1800's, giving his audiences personal and provocative peeks inside the Moulin Rouge. Unfortunately, Toulouse-Lautrec suffered from a variety of health issues including pycnodysostosis (a disease that causes very short brittle bones). This may have been the culprit that caused his short stature. Depression caused Toulouse-Lautrec to begin drinking and he died in poverty in 1901 from complications of alcoholism as well as syphilis.

22) Richard Heck - 2010 Nobel Chemistry prizewinner died aged 84 in Manila. He was Penniless. Famous for his Heck reaction that he discovered in the late 1960s and then spent three decades refining, he won the Nobel for it along with two Japanese chemists working in a similar field.

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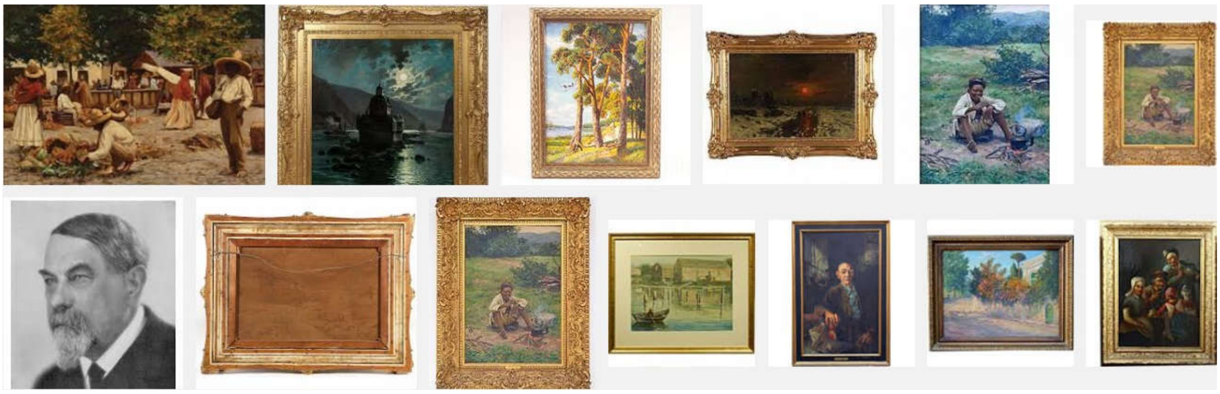

23) Gustave C. Langenberg Born in 1859 in Germany this painter became known as “The Painter on Horseback”. He painted many portraits including a portrait of Queen Wilhelmina, which hangs even today at the Royal Palace at The Hague. Langenberg fought in the Boer War as a member of the British Army. He painted many battle scenes of his time there. Afterward spending time in Mexico, Langenberg painted Mexican scenes including the Hill Indians and Mexican natives. Although he toured much of the world and spent time with Kings and Queens, he died alone and penniless in 1915.

24) **Rembrandt Harmenszoon van Rijn Rembrandt** was born in 1606 and he became one of the greatest painters of all time and certainly the most important in Dutch history. Historians credit him with bringing on the “Dutch Golden Age”. He was best known for his portraits. Rembrandt also painted many biblical scenes. He was credited with having great empathy into the human condition, which helped him to capture his subjects in a way no one else could seem to manage. Unfortunately his life was fraught with tragedy and after his wife died and his friends deserted him, he was pushed into bankruptcy and unable to find any more work. He died in obscurity and poverty in 1669.

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25) **Amedeo Modigliani** - Born in 1884, Modigliani was an Italian artist. He painted and sculpted, spending most of his career in France. He was known for his unique portraits and lush nudes. Modigliani’s family was very poor and tragedy followed him from an early age. He was a true bohemian, drinking absinthe, smoking hashish, and attending wild parties. Modigliani lived fast and hard and died of tubercular meningitis at the age of 35, leaving his nine-month pregnant wife behind. She was so distraught over his death she committed suicide the very next day jumping five stories to her death.

26) **Franz Schubert** - Like van Gogh, Schubert was exceptionally prolific in his short life as a classical composer (he died at the age of 31, just one year after the death of his contemporary, Beethoven). Also similarly to van Gogh, Schubert's works were of little interest to those of his age, and considered inferior to Bach and Beethoven. Because of his financial difficulties, Schubert often lead a rather bohemian and at time nomadic lifestyle, but it did not slow down his production. His music influenced later composers such as Brahms and Mendelssohn, and the complexity and beauty of his melodies are now thought to be on par with Mozart (you may recognize one little song of his called "Ave Maria"), solidifying his place in the canon of neglected geniuses who died in obscurity.

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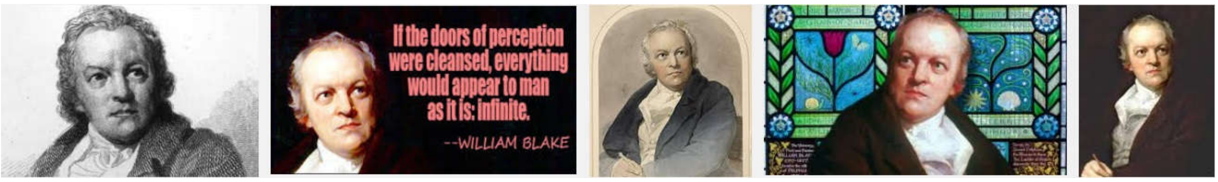

27) **William Blake** - William Blake was another artistic luminary working in obscurity in his day. Though he died poor and unknown, he did not have any debts. Blake was one of the first artists of the 18th century to rebel against Rationalism and move forward into the Romantic Age, and was unsurprisingly considered "mad" because of it. At the time of his death Wordsworth wrote of him, "There was no doubt that this poor man was mad, but there is something in the madness of this man which interests me more than the sanity of Lord Byron and Walter Scott." Blake was known not only for his paintings but also for his fantastic engravings that illustrated his poetry. Despite attempts at exhibitions of his works, no interest was attracted at the time, which did not deter (thankfully) Blake from continuing to produce. He was buried in an unmarked grave at Bunhill Fields in 1827.

28) **Edgar Allan Poe** - Without a doubt now one of the most recognizable names in literature, Edgar Allen Allan Poe was one of the first writers to attempt to make a living on just that, and unfortunately embodied the Romantic notion of life as a starving artist because of it. Facing a myriad of rejections early in his career, even after Poe was published (in 1839 with a volume of short stories, "Tales of the Grotesque and Arabesque") he initially received no money for his work. Despite the relative success of stories such as "The Gold Bug," Poe was unable to make enough money to support his family. Whether attempting to start his own magazines or simply working at journals that ultimately failed, Poe's revenue stream seem to have a life-long curse of bad luck. His beloved wife died in 1847, and two years later Poe was hospitalized and died in utter poverty under famously mysterious circumstances.

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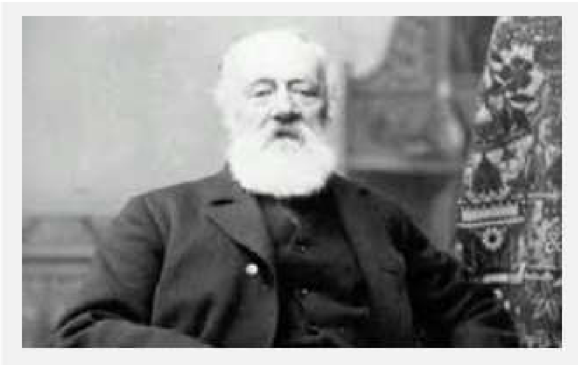
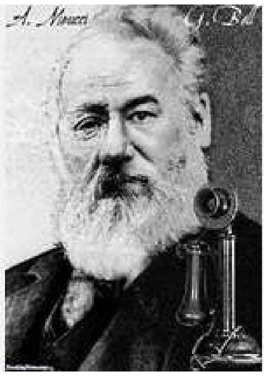
29) **Sammy Davis, Jr.** - The famous Rat Pack singer is reported to have made over $50 million in his lifetime, but died in 1990 $15 million in debt (much of it, like in the case of Joe Louis, was owed to the IRS). Though he made around $1 million a year at the height of his career, the notorious "swinging world" of the Rat Pack nearly bankrupted Davis. According to Matt Birkbeck's book "Deconstructing Sammy," Davis actually rejected surgery in 1989 on his throat that may have saved him, because of his dismal finances. He reasoned that without his voice he couldn't sing and therefore couldn't make any more money. Birkbeck spoke to NPR in 2008 to talk about Sammy's regrettable decline from superstardom to poverty.

30) **Antonio Meucci** - At least in the United States, Alexander Graham Bell has enjoyed far more acclaim than Antonio Meucci, whose name likely invokes a resounding "Who?" from most Americans. But in 2002, Congress gave Meucci his just credit for the invention of the telephone, or the "teletrofono" as he had called it. Bell simply called it "mine" when he stole the idea from Meucci 's papers, which he had sent to Bell's company in the hopes of securing financial backing. Meucci sued him but **died, penniless**, in 1889, never having been able to profit from his genius.

See

http://www.kellenmyers.org/deaths.html

http://blog.redbubble.com/2014/02/6-famous-artists-who-died-poor-and-alone/

http://www.realclearscience.com/blog/2015/02/mathematicians_die_in_horrible_ways.html

http://www.finearttips.com/2011/10/10-famous-artists-who-died-before-their-art-was-recognized/

http://www.therichest.com/rich-list/poorest-list/10-famous-artists-that-died-penniless/

Did you notice that these great passionate Men, **did not quit** from their work or Passion. They did not switch to some other means of " **making money** " even in abject Poverty! Men are in Love (war) with their Work, Creations, Problems, Research, Search of new Knowledge …

Kamikaze Pilots can only be Men. Passionate great men doesn't know "how to quit" or simply Can't quit.





It is quite expected that, the advice for quitting will come from women …

https://www.youtube.com/watch?v=6MBaFL7sCb8

https://www.youtube.com/watch?v=wfNX1cHk-fE

In case of calamity there are broadly “Two Ways” to survive. **Women prefer to runaway**, hide (change jobs / change family / change Protector). This is a very valid way, a very intelligent / safe way, to continue living. **Running away ensures Survival**.

But the Second Way, which most Men Prefer, is to fight it out! It is to “**Solve the Problem**” to survive! This is a very valid way, but bit foolish / unsafe way ! This ensures living. **After the problem is solved it ensures Survival**.

This book is for young students say around the age of 13 to Max 20 years. So to elaborate the above survival techniques, let us see some very simple or common example.

If there is a fire then all **women rush out to extinguish the fire**, risking whatever …. While Men are hardly seen, as every Man has taken recluse in some far away safe place …

Am I saying or seeing something wrong ?

Why are the Maths Department of every College, or Every IIT is full with Women ?

99% Women, and rarely 1% Men somehow making it ?

This book is dedicated to Hardworking Men who solve Problems …

Preface

We all know that in the species "Homo Sapiens ", males are bigger than females. The reasons are explained in standard 10, or 11 (high school) Biology texts. **This shapes or size, influences all of our culture**. Before we recall / understand the reasons once again, let us see some random examples of the influence

Random - 1

If there is a Road rage, then who all fight ? (generally ?). Imagine two cars driven by adult drivers. Each car has a woman of similar age as that of the Man. The cars " touch " or " some issue happens". Who all comes out and fights ? Who all are most probable to drive the cars ?




(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win)

Random - 2

Heavy metal music artists are all Men. Metallica, Black Sabbath, Motley Crue, Megadeth, Motorhead, AC/DC, Deep Purple, Slayer, Guns & Roses, Led Zeppelin, Aerosmith ….. the list can be in thousands. All these are grown-up Boys, known as Men.




(**Men strive for perfection. Men are eager to excel. Men work hard. Men want to win**.)






Random - 3

Apart from Marie Curie, only one more woman got Nobel Prize in Physics. (Maria Goeppert Mayer - 1963). So, … almost all are men.

(**Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women**.)

Random - 4

The best Tabla Players are all Men.

(**Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women**.)

Random - 5

History is all about, **which all Kings ruled**. Kings, their men, and Soldiers went for wars. History is all about wars, fights, and killings by men. Who won, and who controlled !

Boys start fighting from school days. Girls do not fight like this

(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Random - 6

The highest award in Mathematics, the " Fields Medal " is around since decades. Till date only one woman could get that. (Maryam Mirzakhani - 2014). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 7

Actor is a gender neutral word. Could the movie like " Top Gun " be made with Female actors ? The best pilots, astronauts, Fighters are all Men.

Random - 8

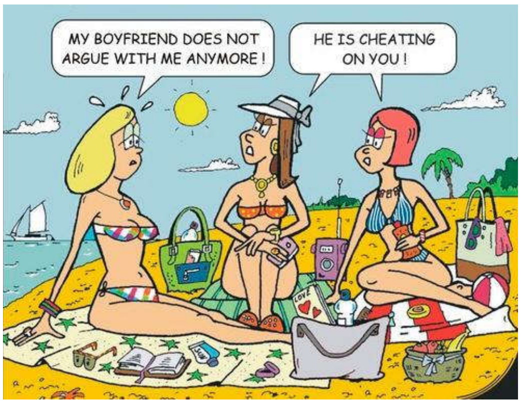
In my childhood had seen a movie named " The Tower in Inferno ". In the movie when the tall tower is in fire, **women were being saved first**, as only one lift was working….

Many decades later another movie is made. A box office hit. " The Titanic ". In this also …. **As the ship is sinking women are being saved**. **Men are disposable**. Men may get their turn later… (never) !!

Movies are not training programs. Movies do not teach people what to do, or not to do. **Movies only reflect the prevalent culture**. Men are disposable; is the culture in the society. Knowingly, unknowingly, the culture is depicted in Movies, Theaters, Stories, Poems, Rituals, etc. I or you can't write a story, or make a movie in which after a minor car accident the Male passengers keep seating in the back seat, while the both the women drivers come out of the car and start fighting very bitterly on the road. **There has been no story in this world, or no movie made, where** after an accident or calamity, Men are being helped for safety first, and women are told to wait.

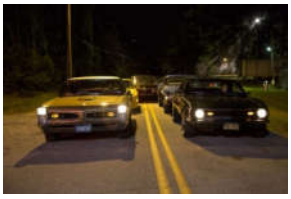
Random - 9

Artists generally follow the prevalent culture of the Society. In paintings, sculptures, stories, poems, movies, cartoon, Caricatures, knowingly / unknowingly, " the prevalent Reality " is depicted. **The opposite will not go well with people**. If deliberately " the opposite " is shown then it may only become a special art, considered as a special mockery.



Random - 10

Men go to "girl / woman's house" to marry / win, and bring her to his home. That is a sort of winning her. When a boy gets a " Girl-Friend ", generally he and his friends consider that as an achievement. The boy who " got / won " a girl-friend feels proud. His male friends feel, jealous, competitive and envious. Millions of stories have been written on these themes. Lakhs of movies show this. Boys / Men go for " bike race ", or say " Car Race ", where the winner " gets " the most beautiful girl of the college.




(Men want to excel. Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Prithviraj Chauhan ' **went** ` to " **pickup** " or " **abduct** " or " **win** " or " **bring** " his love. There was a Hindi movie (hit) song … " Pasand ho jaye, to ghar se utha laye ". It is not other way round. Girls do not go to Boy's house or man's house to marry. Nor the girls go in a gang to " pick-up " the boy / man and bring him to their home / place / den.

Random - 11

We have the word "ice cold". While, when it snows heavily, the cleaning of the roads is done by Men. Ice avalanche is cleared by Guns, by Men.


Can women do this please ?


Random - 12


There are many remote mines in this world which are connected by rails through Hilly regions. These railroads move through steep ups and downs. Optimum speed of the train has to be maintained !! The expert driver has to ensure that the brakes do not burn out, if driven too slow. The speed should be enough so that next climbing can be done. Sudden braking is not possible ! ; as the load of the wagons will derail the train, and that will mean huge loss and deaths. The **Drivers are Men who risk their lives** in every journey.

Fukushima Daiichi nuclear disaster happened on March 11, 2011. This was primarily by the tsunami following the Tōhoku earthquake (magnitude 9.0). Lots of radioactive materials were scattered in the environment thorough “vent” to reduce the internal pressure and the hydroponic explosions of the nuclear reactors.

Old Men, Pensioners, Seniors offered to cleanup the Nuclear damage as **'suicide corps'** See http://edition.cnn.com/2011/WORLD/asiapcf/05/31/japan.nuclear.suicide/

Old People Line Up To Clean Radiation in Japan

kelly
5/31/11 10:00pm · Filed to: JAPAN
119.0K 111

I deeply appreciate such gesture to "Save" the society. While I wish to draw your attention to a much deeper/important questions !!

Why old women did not Volunteer to clean the Nuclear site ?

Old women are not pregnant ! Women get menopause sometime in their early 40s. Why is it so common in the Society to "Save" older women as well, and "spare" or "deprive" old men ? ***Why old men are treated so badly*** ? **Why are Men eager to fight every war** ?

[Climbing Everest or any Mountain Peak, or say crossing Atlantic solo, or reaching the North Pole / South Pole; Almost ALL are Men isn't it .... Researching into technology, inventing and discovering new frontiers of Science is also a war! In every case it is Almost ALL Men]

Very Sad, bad habit of Million years, is driving the world for so much of "Good" and "BAD" !

The reader / student should not assume that I have not read enough Philosophy; where it is taught that GOOD or BAD are only individual's mental interpretations. I am mature enough to say the above words as .... ' Million years of Good Habit of **"Fighting to Win and Survive"** has led Men to all sorts of difficulties, accidents, discomforts, loss .... '

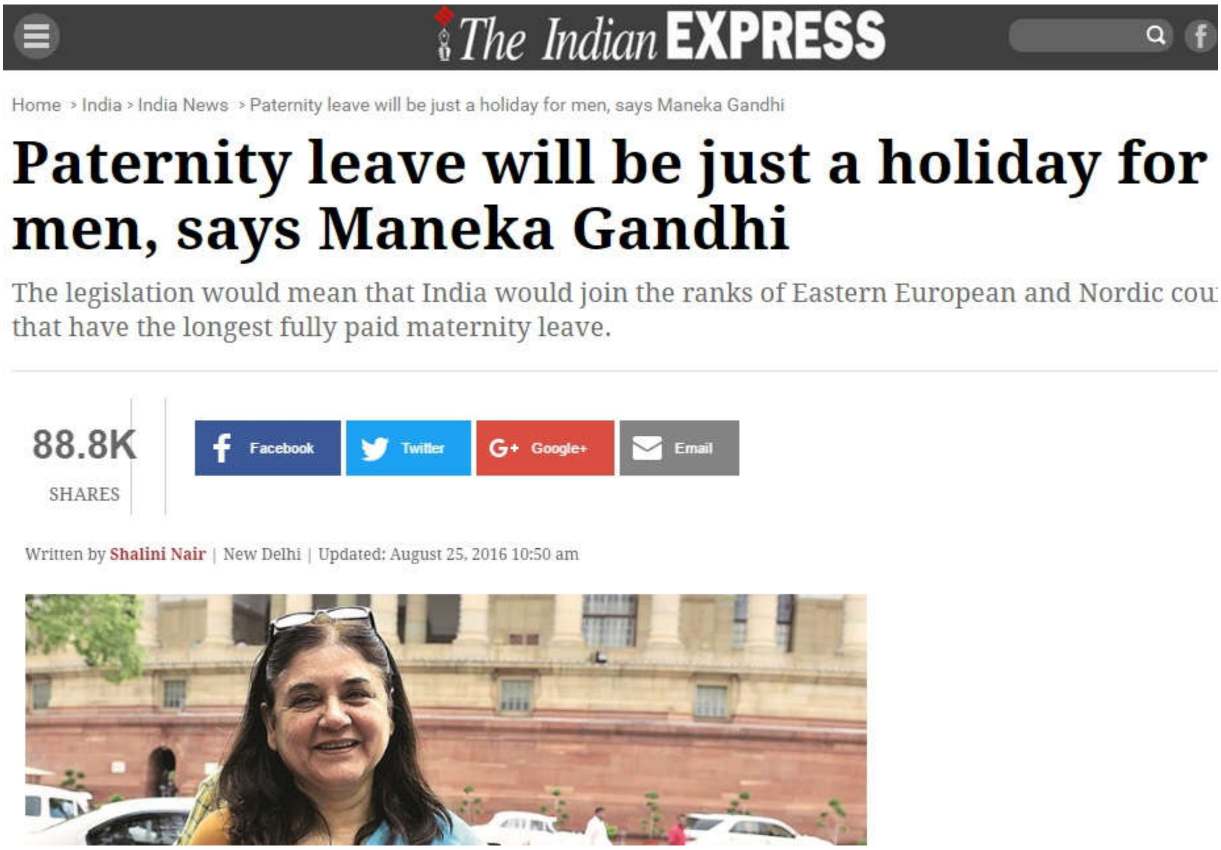
Most women are just Thankless to Men, and their efforts. Women just use Men like parasite or Leeches. They see all the facilities' and benefits as their right !

(Unfortunately most men submit themselves to be used / exploited like this ! MGTOW s are one of the exceptions.)

In all countries the Laws / Traditions / Customs / Society norms etc have been systematically twisted in favor of women to ensure that Women get "everything". While Nothing is available for Men !

For example Money, Job, Certificate, Facilities etc are given to Widow and (may be Mom) of the deceased MAN; who died 'fighting ' **!** The Law or norm **is not for the father of the Soldier.** **[Think … who is dying ? Who is surviving ? Who is getting the benefits ? who is being deprived ?]**

(These images are a few amongst Millions of images which are available. All make the same point)



The Indian EXPRESS

Home > India > India News > Paternity leave will be just a holiday for men, says Maneka Gandhi

Paternity leave will be just a holiday for men, says Maneka Gandhi

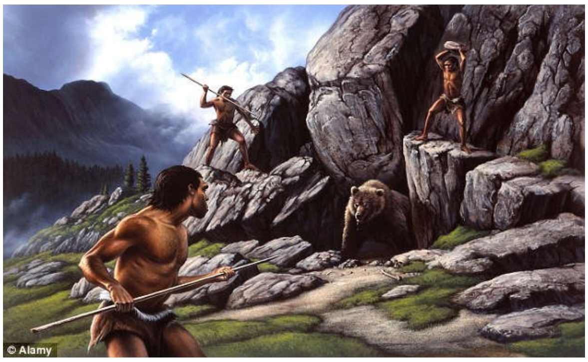
The legislation would mean that India would join the ranks of Eastern European and Nordic cou that have the longest fully paid maternity leave.

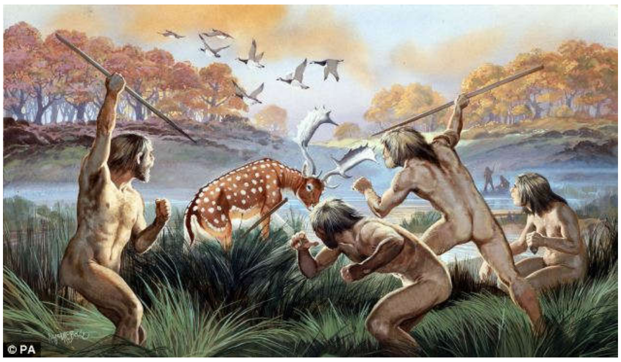
88.8K SHARES

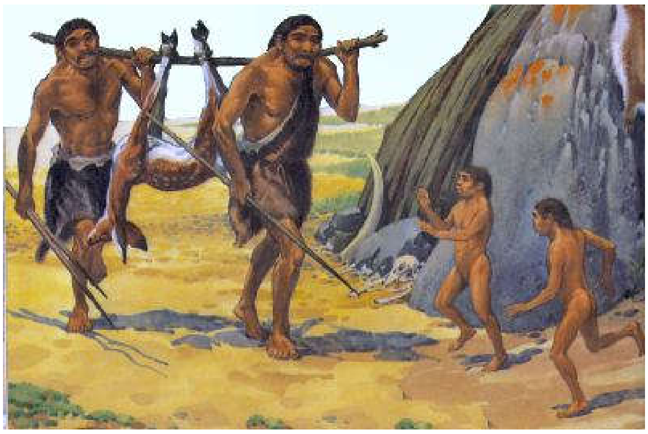
Facebook Twitter Google+ Email

Written by Shalini Nair | New Delhi | Updated: August 25, 2016 10:50 am

Men are only for working ! (<u>sorry, hunting !</u>) always … that's what most people think !







Every woman has a womb. The women (rather their Wombs) were protected / kept safe, so that children are born. That was the survival method to continue the species…

Let us name the best of the Mathematicians …

Leonhard Euler, Isaac Newton, Carl Gauss, Fermat, Henri Poincaré, Lagrange, David Hilbert, G.W. Leibniz ...

(See http://fabpedigree.com/james/mathmen.htm)

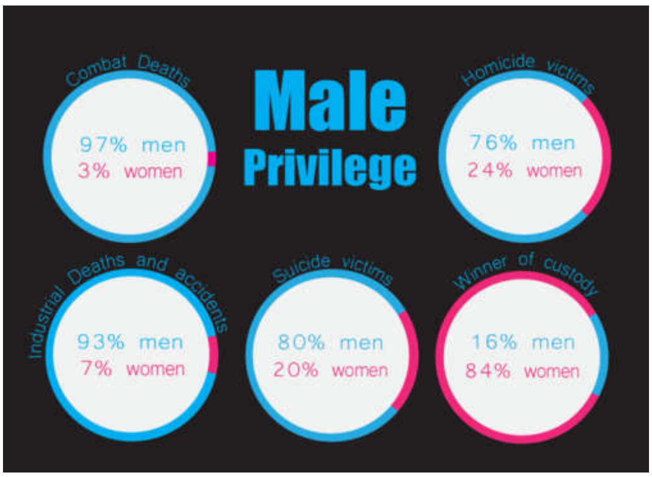
Why all these great names are of Men ? Why women could not contribute, in the cozy safe home ?

A newly married couple goes out in car … and if there is a flat tire (known as puncture in India) then who opens the wheels ? who replaces from the stepney ?

Womb being protected ? Why women don't help ?

How much is the Society or Men paying for wombs ? This penance is till which age ?

No woman works for “ Male Suicide “ issues. Even-though, the rate of suicide in men are many times higher, than that of women. Women are never bothered about Men. Some women work only for “women issues “.



“It's so cool! If you use your imagination, you can blame men for *everything!*”

http://www.telegraph.co.uk/men/the-filter/11965029/Middle-aged-male-suicide-rate-rises-by-40-per-cent-since-2008.html

http://scroll.in/article/669061/married-men-are-most-likely-to-commit-suicide-in-india

Meet the Woman Who Shot Her Son with the Same Gun She Used to Kill Her Husband 20 Years Earlier

By Afarin Majidi - August 3, 2015

1. Katherine Knight – Kills Husband and Eats Him.

This lady, Katherine Knight stabbed his poor husband 37 times with a butcher's knife then skinning him and hanged his body with a meat hook in their lounge room. Katherine, the first Australian woman to be sentenced to a natural life term without parole. She had a history of violence in relationships. She mashed the dentures of one of her ex-husbands and slashed the throat of another husband's eight-week-old puppy before his eyes. A heated relationship with John Charles

3. Stacey Castor, poisoned husband with antifreeze and then framed her daughter.

Stacy Castor staged a scene to make her dead husband appeared to have committed suicide but getting the cops suspicious then investigated her past only to found out that her former husband was dead from a 'heart atack' . suspicios, the cops enquire an autopsy of the former husband and found ethylene glycol substance same like the second husband's autopsy.

10. The woman who cheated on her husband after he had donated his own kidney to her.

4. Omaima Aree Nelson, Killed, Chopped Up, Cooked and Ate Husband.

Model Omaima Aree Nelson tried to grind her husband up in the garbage disposal. But she just couldn't get rid of all of 6-foot-4, 230 lbs. of him so she boiled, breaded, deep-fried and ate body parts. (Link).

5. Angry Wife Cuts Off Husband's Balls While He is Sleeping.

An angry Chinese woman in Xiaoxian, Jiangxi province, China, sliced off her husband's testicles while he was sleeping, in order to protect her marriage.

8. 76-year-old lady who is suspected of murdering four of her five husbands.

Jeff Carstensen was spooked when he learned his grandmother planned to buy him a $100,000 life insurance policy — and name herself the beneficiary. As he and many others who came into Betty Neumar's orbit have learned, bad things tend to happen to the people around her.

Human beings are in general not comfortable with New ideas or New Paradigms or say new doctrines. New ideas take time to shape up !

(I am aware of Hundredth monkey effect ... scientists were conducting a study of macaque monkeys on the Japanese island of Koshima in 1952. These scientists observed that some of these monkeys learned to wash sweet potatoes, and gradually this new behavior spread through the younger generation of monkeys—in the usual fashion, through observation and repetition. Watson then concluded that the researchers observed that once a critical number of monkeys was reached, i.e., the hundredth monkey, this previously learned behavior instantly spread across the water to monkeys on nearby islands.

https://en.wikipedia.org/wiki/Hundredth_monkey_effect)

http://www.dailymail.co.uk/sciencetech/article-3317316/Monkeys-food-hygiene-Macaques-clean-potatoes-grain-eating-fewer-parasites.html

Robindranath Thakur, the first Nobel Laureate of Asia, was follower / believer of Bromho. His father Debendranath Thakur,(As son of Dwarkanath Tagore, a close friend of Ram Mohan Roy) philosopher and religious reformer, active in the Brahmo Samaj ("Society of Brahmā," also translated as "Society of God"), which aimed to reform the Hindu religion and way of life. He was one of the founders in 1848 of the Brahmo religion, which today is synonymous with Brahmoism.

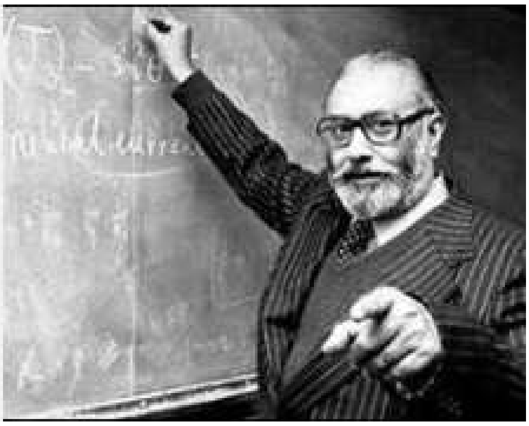
When Robindronath wanted to open a school in Calcutta, **many people did not want to send their children to a "Bromho Teacher "**. So In 1901 Tagore moved to Santiniketan to found an ashram.

Chatimtala Kaanch Ghor the Bramho Mandir, at Santiniketan

[English People could not pronounce Thakur. They used to distort it as Tagore .... Over time the family name is called as Tagore by most non-Bengalis]

Abdus Salam the only Physics Nobel Laureate of Pakistan was an Ahmadiyya; by faith. Ahmadiyya religion is not accepted in Pakistan. [The theological amendment in the constitution of Pakistan does not allow members of the Ahmadiyya faith to call themselves Muslims.] Abdus Salam had to shift to Trieste, Italy. Salam was buried in Bahishti Maqbara, a cemetery established by the Ahmadiyya Community at Rabwah, Punjab, Pakistan, next to his parents' graves. The epitaph on his tomb initially read "**First Muslim Nobel Laureate**". The Pakistani government removed "Muslim" and left only his name on the headstone. The word "Muslim" was initially obscured on the orders of a local magistrate before moving to the national level.





http://blogs.tribune.com.pk/story/19695/we-are-sorry-dr-abdus-salam/

http://blogs.tribune.com.pk/story/31969/dr-abdus-salam-and-all-the-wrong-choices-pakistan-made/

http://www.nobelprize.org/nobel_prizes/physics/laureates/1979/salam-bio.html

In some cases accepting the Truth takes very long time....

Pope John Paul II apologised on behalf of the Catholic Church for the mistreatment of Galileo in the 17th century. The dispute between the Church and Galileo has long stood as one of history's great emblems of conflict between reason and dogma, science and faith. At the time of his condemnation, Galileo had won fame and the patronage of leading Italian powers like the Medicis and Barberinis for discoveries he had made with the astronomical telescope he had built. But when his observations led him to proof of the Copernican theory of the solar system, in which the sun and not the earth is the center, and which the Church regarded as heresy, Galileo was summoned to Rome by the Inquisition. **Forced to Recant**. Galileo took back his statement, but still lived under house arrest for the rest of his life. It took 359 years and the leadership of Pope John Paul II (left) to recognize the wrong. On October 31, 1992, he formally apologized for the "Galileo Case" in the first of many famous apologies during his papacy.

https://www.youtube.com/watch?v=JUAsLcFPeNw

History of Gravity …

Galileo to Einstein https://www.youtube.com/watch?v=2H_zvoENNXo

https://www.youtube.com/watch?v=QGQq2aB3cWE

https://www.youtube.com/watch?v=mPxwgyJtJXI

The New York Times

World

WORLD | U.S. | N.Y. / REGION | BUSINESS | TECHNOLOGY | SCIENCE | HEALTH | SPORTS | OPINION

AFRICA AMERICAS ASIA PACIFIC EUROPE MIDDLE EAST

After 350 Years, Vatican Says Galileo Was Right: It Moves

By ALAN COWELL,
Published: October 31, 1992

ROME, Oct. 30— More than 350 years after the Roman Catholic Church condemned Galileo, Pope John Paul II is poised to rectify one of the Church's most infamous wrongs -- the persecution of the Italian astronomer and physicist for proving the Earth moves around the Sun.

With a formal statement at the Pontifical Academy of Sciences on Saturday, Vatican officials said the Pope will formally close a 13-year investigation into the Church's condemnation of Galileo in 1633. The condemnation, which forced the astronomer and physicist to recant his discoveries, led to Galileo's house arrest for eight years before his death in 1642 at the age of 77.

The dispute between the Church and Galileo has long stood as one of history's great emblems of conflict between reason and dogma, science and faith. The Vatican's formal acknowledgement of an error, moreover, is a rarity in an institution built over centuries on the belief that the Church is the final arbiter in matters of faith.

http://www.nytimes.com/1992/10/31/world/after-350-years-vatican-says-galileo-was-right-it-moves.html

For new ideas …. See …

http://www.slate.com/articles/news_and_politics/foreigners/2009/06/the_herbivores_dilemma.html

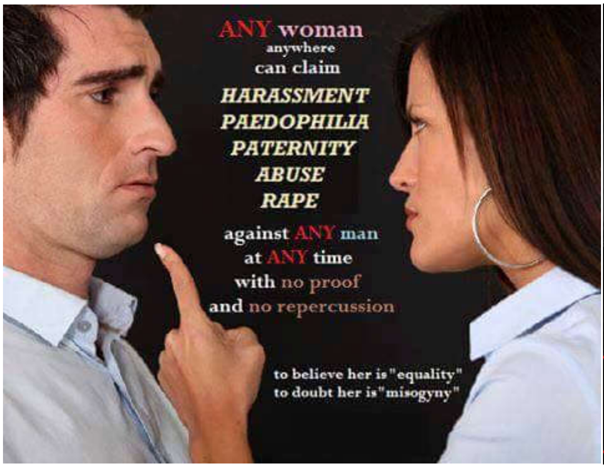
http://www.wisedup.org/antiphysical-men-giving-sex-relationships/

https://pairedlife.com/dating/Dating-10-Things-Men-Dont-Do-Anymore

Random – 13 (will you be comfortable with new ideas ?)

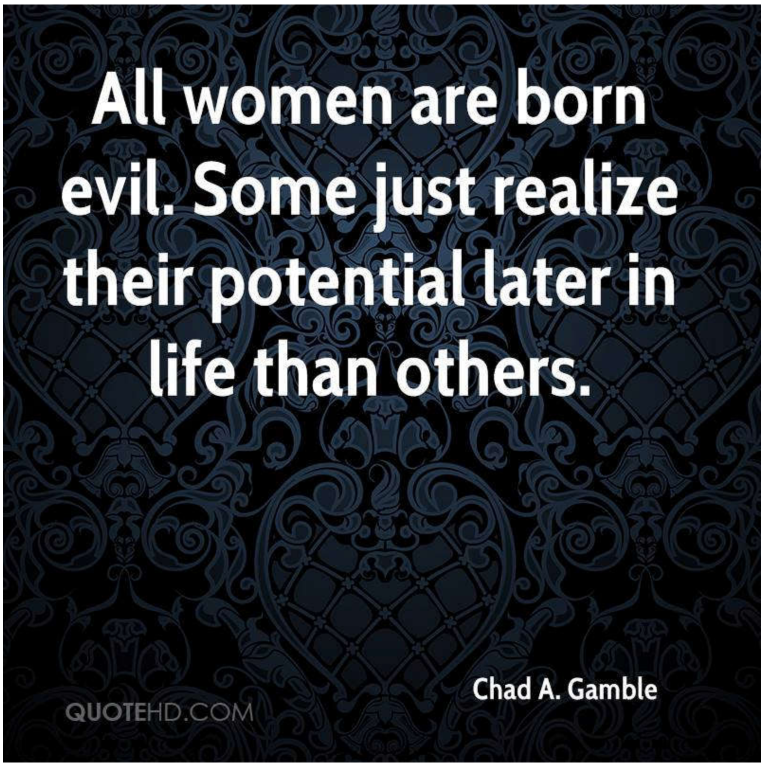
Almost all of us are very biased. Instead of I asking some questions; see the following images

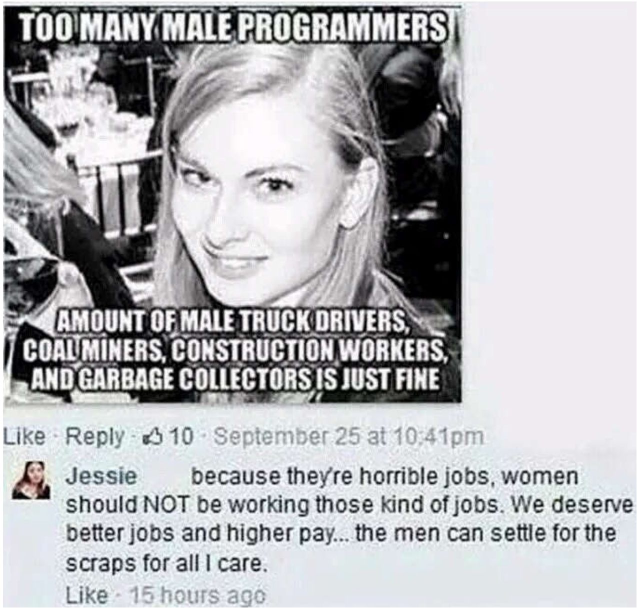
http://www.independent.co.uk/life-style/love-sex/women-are-genetically-programmed-to-have-affairs-evolution-university-texas-scientists-suggest-a7203501.html



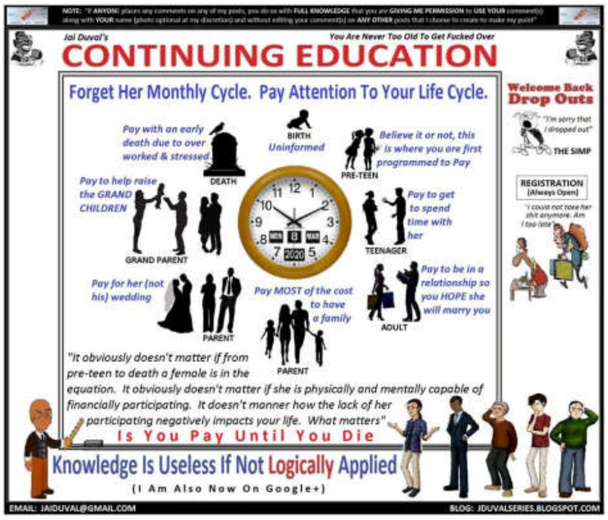
In all cultures the onus of Proving himself not guilty, **lies on the Man**; while it is enough for the woman just to accuse, and cry. **Tears are taken as proof of Crime** !

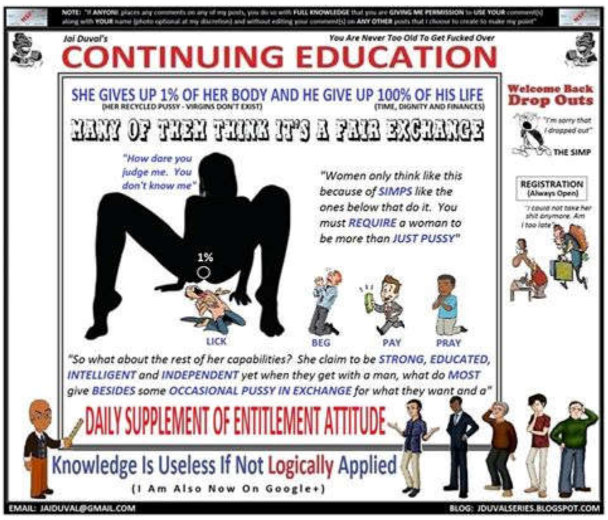

GARBAGE COLLECTOR
ALMOST 100% MEN
NO ONE HOWLS FOR EQUALITY
SEWER WORKER
ALMOST 100% MEN
NO ONE HOWLS FOR EQUALITY
MINER
ALMOST 100% MEN
NO ONE HOWLS FOR EQUALITY
OFFICE WORKER IN
AIR-CONDITIONED ROOM
OMG! EQUALITY, EMANCIPATION,
WOMEN ON THE LABOUR
MARKET, DISCRIMINATION!!!


All women are born evil. Some just realize their potential later in life than others.
Chad A. Gamble
QUOTEHD.COM


TOO MANY MALE PROGRAMMERS
AMOUNT OF MALE TRUCK DRIVERS, COAL MINERS, CONSTRUCTION WORKERS, AND GARBAGE COLLECTORS IS JUST FINE
Like · Reply · 10 · September 25 at 10:41pm
Jessie because they're horrible jobs, women should NOT be working those kind of jobs. We deserve better jobs and higher pay... the men can settle for the scraps for all I care.
Like · 15 hours ago

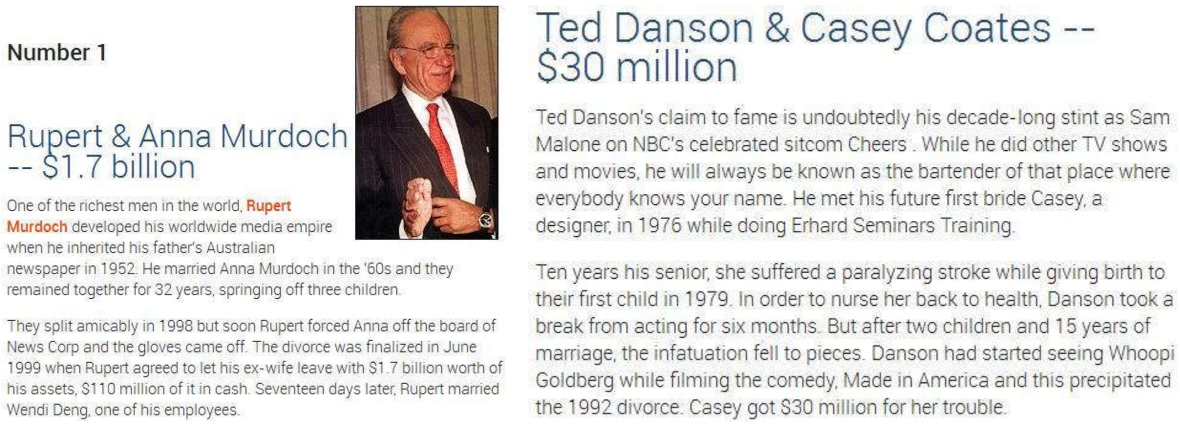

Proof that girls are evil
First we state that girls require time and money.
GIRLS = TIME × MONEY
And as we all know "time is money"
TIME = MONEY
Therefore:
GIRLS = MONEY × MONEY = (MONEY)²
And because "money is the root of all evil":
MONEY = √EVIL
Therefore:
GIRLS = (√EVIL)²
We are forced to conclude that:
GIRLS = EVIL


CONTINUING EDUCATION
Forget Her Monthly Cycle. Pay Attention To Your Life Cycle.
Welcome Back Drop Outs
Is You Pay Until You Die
Knowledge Is Useless If Not Logically Applied


CONTINUING EDUCATION
SHE GIVES UP 1% OF HER BODY AND HE GIVE UP 100% OF HIS LIFE
MANY OF THEM THINK IT'S A FAIR EXCHANGE
Welcome Back Drop Outs
LICK
BEG
PAY
PRAY
DAILY SUPPLEMENT OF ENTITLEMENT ATTITUDE
Knowledge Is Useless If Not Logically Applied

Random - 14

Rich people; often are very hard working. Successful business men, establish their business (empire), amass lot of wealth, with lot of difficulty. Lots of sacrifice, lots of hard work, gets into this. **Rich people's wives had <u>no contribution</u> in this wealth creation**. Women are smart, and successful upto the extent to choose the right/rich man to marry. So generally what happens in case of Divorces ? Search the net on " most costly divorces " and you will know. The women;(who had no contribution at all, in setting up the business / empire), often gets in Billions, or several Millions in divorce settlements. [Just because the wife has womb]



Number 1

Rupert & Anna Murdoch -- $1.7 billion

One of the richest men in the world, Rupert Murdoch developed his worldwide media empire when he inherited his father's Australian newspaper in 1952. He married Anna Murdoch in the '60s and they remained together for 32 years, springing off three children.

They split amicably in 1998 but soon Rupert forced Anna off the board of News Corp and the gloves came off. The divorce was finalized in June 1999 when Rupert agreed to let his ex-wife leave with $1.7 billion worth of his assets, $110 million of it in cash. Seventeen days later, Rupert married Wendi Deng, one of his employees.

Ted Danson & Casey Coates -- $30 million

Ted Danson's claim to fame is undoubtedly his decade-long stint as Sam Malone on NBC's celebrated sitcom Cheers . While he did other TV shows and movies, he will always be known as the bartender of that place where everybody knows your name. He met his future first bride Casey, a designer, in 1976 while doing Erhard Seminars Training.

Ten years his senior, she suffered a paralyzing stroke while giving birth to their first child in 1979. In order to nurse her back to health, Danson took a break from acting for six months. But after two children and 15 years of marriage, the infatuation fell to pieces. Danson had started seeing Whoopi Goldberg while filming the comedy, Made in America and this precipitated the 1992 divorce. Casey got $30 million for her trouble.

See https://zookeepersblog.wordpress.com/misandry-and-men-issues-a-short-summary-at-single-place/

See http://skmclasses.kinja.com/save-the-male-1761788732

<u>It was Boys and Men, who brought the girls / women home. The Laws are biased, completely favoring women. The men are paying for their own mistakes</u>.

See https://zookeepersblog.wordpress.com/biased-laws/

(Man brings the Woman home. When she leaves, takes away her share of big fortune!)

http://www.uplifting-love.com/2013/08/80-percent-of-divorces-are-filed-by.html

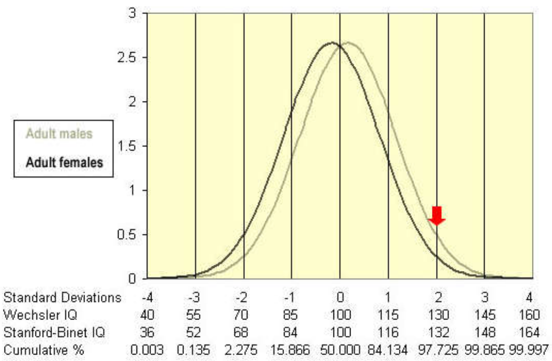
Mileva Marić wife of Albert Einstein; was the only woman among Albert Einstein's fellow students at the Zurich Polytechnic. They got married in 1903, they had two sons, Hans Albert and Eduard. They separated in 1914, with Marić taking the boys and returning to Zurich from Berlin. They divorced in 1919. Albert Einstein was confident or rather sure of winning the Nobel Prize. He had agreed to pay the prize money (after he gets it), to Mileva, for the separation and Divorce. After Einstein received the Nobel Prize in 1921, he transferred the money to Marić.

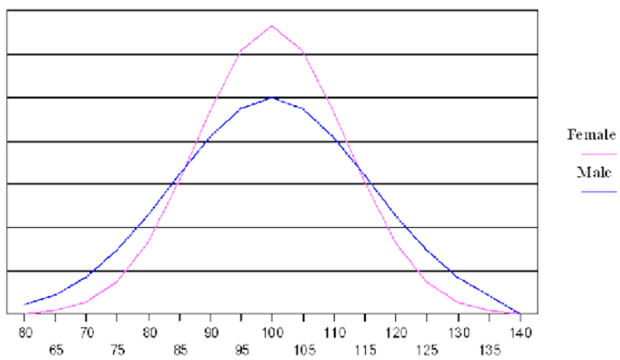
Just see how bad it has been ... Nobel Prize Money for Separation and Divorce!

-

Random - 15

A standardized test of Intelligence will never be possible. It never happened before, nor ever will happen in future. No IQ test results will be acceptable by all. In the net there are thousands of charts which show that the intelligence scores of girls / women are lesser. Debates of Trillion words, does not improve performance of Girls.





I am not wasting a single second debating or discussing with anyone, on this. **I am simply accepting ALL the results**. IQ is only one of the variables which is required for success in life. Thousands of books have been written on " Networking Skills ", EQ (Emotional Quotient), Drive, Dedication, Focus, " Tenacity towards the end goal " … etc. **In each criteria, and in all together, women (in general) do far worse than men**. Bangalore is known as " ….. capital of India ". [Fill in the blanks]. The blanks are generally filled as " Software Capital ", " IT Capital ", " Startup Capital ", etc. I am member in several startup eco-systems / groups.

I have attended hundreds of meetings, regarding " technology startups ", or " idea startups ". These meetings have very few women. (Generally in most meetings there are no women at all !). Starting up new companies are all **" Men's Game " / " Men's business "**. Only in Divorce settlements women will take their goodies, due to Biased laws. **There is no dedication, towards wealth creation, by women**. Women want easy money.

Max Roscoe

is an aspiring philosopher king, living the dream, travelling the world, hoarding FRNs and ignoring Americunts. He is a European at heart, lover of Latinas, and currently residing in the USA.

July 8, 2016

Culture

Women Who Sell Their Bodies For Money Don't Want To Be Called Prostitutes

Random - 16

Many men, as fathers, very unfortunately treat their daughters as " Princess ". Every " non-performing " woman / wife was " princess daughter " of some loving father. Pampering the girls, in name of " equal opportunity ", or " women empowerment ", have led to nothing.

See http://skmclasses.kinja.com/progressively-daughters-become-monsters-1764484338

See http://skmclasses.kinja.com/vivacious-vixens-1764483974

There can be thousands of more such random examples, where " Bigger Shape / size " of males have influenced our culture, our Society. Let us recall the reasons, that we already learned in standard 10 - 11, Biology text Books. In humans, women have a long gestation period, and also spends many years (almost a decade) to grow, nourish, and stabilize the child. (Million years of habit) Due to survival instinct Males want to inseminate. Boys and Men fight for the " facility (of womb + care) " the girl / woman may provide. Bigger size for males, has a winning advantage. Whoever wins, gets the " woman / womb / facility ". The male who is of " Bigger Size ", has an advantage to win…. Leading to Natural selection over millions of years. In general " Bigger Males "; the " fighting instinct " in men; have led to wars, and solving tough problems (Mathematics, Physics, Technology, startups of new businesses, Wealth creation, Unreasonable attempts to make things [such as planes / Flying Machines], Hard work ….)

So let us see the IIT-JEE results of girls. Statistics of several years show that there are around 17, (or less than 20) girls in top 1000 ranks, at all India level. Some people will yet not understand the performance, till it is said that … year after year we have around 980 boys in top 1000 ranks. Generally we see only 4 to 5 girls in top 500. In last 50 years not once any girl topped in IIT-JEE advanced. Forget about Single digit ranks, double digit ranks by girls have been extremely rare. It is all about " good boys ", " hard working ", " focused ", "Bel-esprit " boys.

In 2015, Only 2.6% of total candidates who qualified are girls (upto around 12,000 rank). while 20% of the Boys, amongst all candidates qualified. The Total number of students who appeared for the exam were around 1.4 million for IIT-JEE main. Subsequently 1.2 lakh (around 120 thousands) appeared for IIT-JEE advanced.

IIT-JEE results and analysis, of many years is given at https://zookeepersblog.wordpress.com/iit-jee-iseet-main-and-advanced-results/

In Bangalore it is rare to see a girl with rank better than 1000 in IIT-JEE advanced. We hardly see 6-7 boys with rank better than 1000. Hardly 2-3 boys get a rank better than 500.

See http://skmclasses.weebly.com/everybody-knows-so-you-should-also-know.html

So what " some women " are doing ?

Thousands of people are exposing the heinous crimes that Motherly Women are doing, or Female Teachers are committing. See https://www.facebook.com/WomenCriminals/

Some Random Examples must be known by all

It is extremely unfortunate that the " woman empowerment " has created. This is the kind of society and women we have now. I and many other sensible Men hate such women. Be away from such women, be aware of reality.

Mother Admits On Facebook to Sleeping with 15 Yr Old Son, They Have a Baby Together - Alwayzturntup

Sometimes it hard to believe w From Alwayzturntup

ALWAYZTURNTUP.ME

'Sex with my son is incredible - we're in love and we want a baby'

Ben Ford, who ditched his wife when he met his mother Kim West after 30 years, claims what the couple are doing 'isn't incest'

MIRROR.CO.UK

Woman sent to jail for the rest of her life after raping her four grandchildren is described as the 'most evil person' the judge has ever seen

Edwina Louis rape...

See More

Former Shelbyville ISD teacher who had sex with underage student gets 3 years in prison

After a two day break over the weekend, A Shelby County jury was back in the courtroom looking to conclude the trial of a former Shelbyville ISD teacher who had...

KLTV.COM | BY CALEB BEAMES

Woman sent to jail for raping her four grandchildren

A Ohio grandmother has been sentenced to four consecutive life terms after being found guilty of the rape of her own grandchildren. Edwina Louis, 53, will spend the rest of her life behind bars.

DAILYMAIL.CO.UK

http://www.thenativecanadian.com/.../eastern-ontario-teacher-...

The N.C. Chronicles.: Eastern Ontario teacher charged with 36 sexual offences

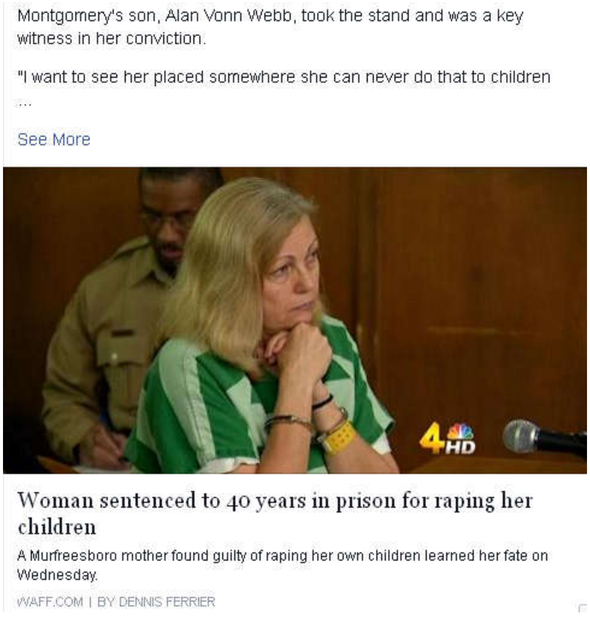
anti feminism, Child abuse, children's rights, Feminist hypocrisy,

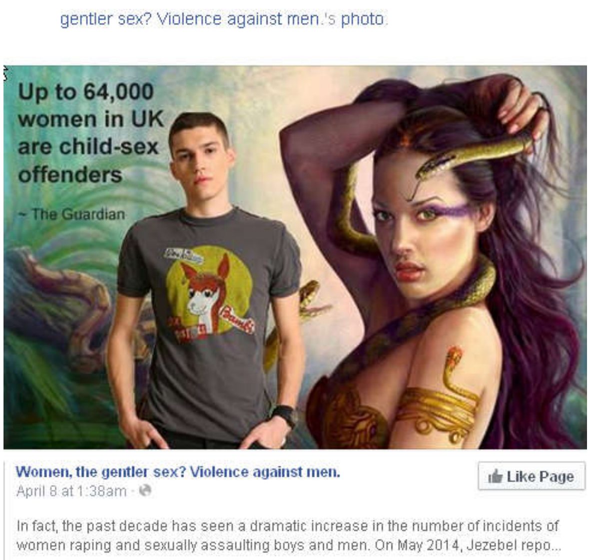
THENATIVECANADIAN.COM | BY BLACKWOLF

Hyd woman kills newborn boy as she wanted daughter - Times of India

Having failed to bear a daughter for the third time, a shopkeeper's wife slit the throat of her 24day-old son with a shaving blade and left him to die in a street on Tuesday night.Purnima's first child was a stillborn boy, followed by another boy born five years ago.

TIMESOFINDIA.INDIATIMES.COM





In Facebook, and internet + whatsapp etc we have unending number of posts describing frustration of men / husbands on naughty unreasonable women. **Most women are very illogical, Punic, perfidious, treacherous, naughty, gamey bitches**.

We also see zillions of Jokes which basically describe how unreasonable women / girls are. How stupid they are, making life of Boys / Men / Husband a hell.

While each of these girls was someones daughter. Millions of foolish Dads are into Fathers rights movement, who want their daughter back for pampering.

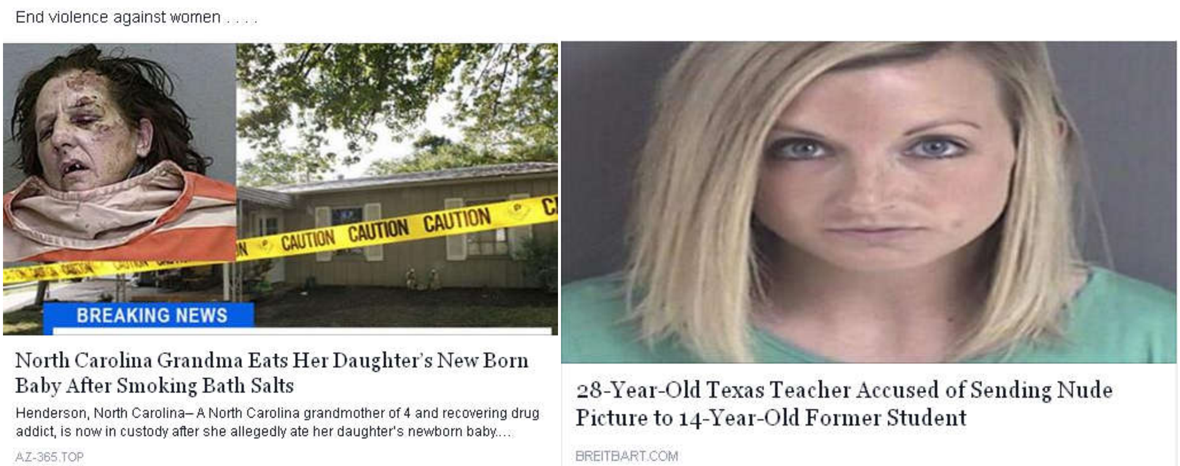
Most girls are being cockered, coddled, cosseted, mollycoddled, featherbedded, spoilt into brats.

Foolish fathers are breeding Monsters who are filing false rape cases. Enacting Biased Laws. Filing False domestic violence cases. Filing false sexual assault cases. Asking for alimony, and taking custody of the Daughter, not allowing the " monster " to meet dad. The cycle goes on and on and on.

Foolish men keep pampering future demons who make other Men's life a hell. (Now read this again from beginning). Every day we see the same posts of frustration.

https://nicewemen.wordpress.com/

Each women as described below was someone's Pampered Princess …



http://latest.com/.../attractive-girl-gang-lured-men-alleywa.../

Attractive Girl Gang Lured Men Into Alleyways Where Female Body Builder Would Attack Them

A Mexican street gang made up entirely of women has been accused of using their feminine wiles to lure men into alleyways and then beating them up and...

LATEST.COM

http://www.wfmj.com/.../youngstown-woman-convicted-of-raping-...

Youngstown woman convicted of raping a 1 year old is back in jail

A Youngstown woman who went to prison for raping a 1-year-old boy fifteen years ago is in trouble with the law again.

WFMJ.COM

Women are raping boys and young men

Rape advocacy has been maligned and twisted into a political agenda controlled by radicalized activists. Tim Patten takes a razor keen and well supported look into the manufactured rape culture and...

AVOICEFORMEN.COM | BY TIM PATTEN

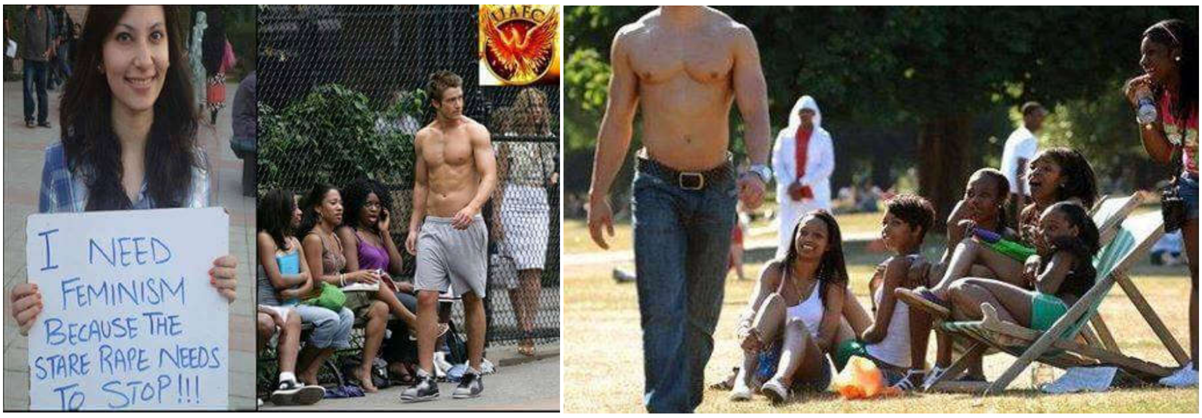
End violence against women . . . .

Bronx Woman Convicted of Poisoning and Drowning Her Children

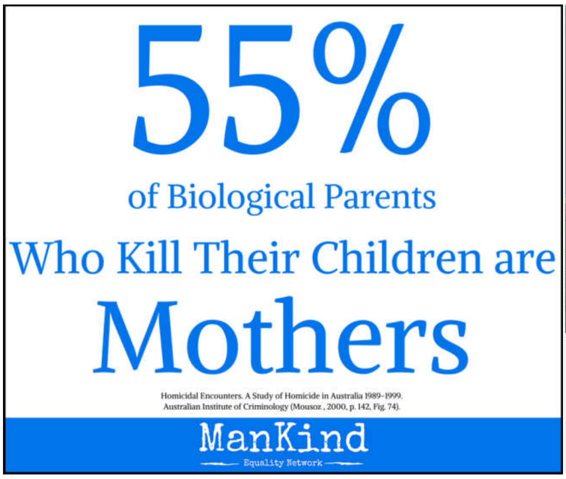
Lisette Bamenga researched methods on the Internet before she killed her son and daughter in 2012.

NYTIMES.COM | BY MARC SANTORA

Monster women have very easy and cozy life. Easy to demand anything and get law in favor !



If the lawmakers submit to these strange demands of say … " Stare Rape ! "; then we can easily see what kind of havoc that will create.


55%
of Biological Parents
Who Kill Their Children are
Mothers
ManKind
Equality Network


FIRST AT 4PM
JOSHUA TREE
EYEWITNESS NEWS
Woman charged with killing baby also had previous infant die
Woman charged with killing baby also had previous infant die
ABC7.COM | BY ROB MCMILLAN


Female Sex Predators: A Crime Epidemic shared a link.
Yesterday at 12:40am
WTVA.com | Woman pleads guilty to having sex with a dog
PITTSBORO, Miss. (WTVA) -- A Calhoun County woman has pleaded guilty to charges she had unnatural intercourse with a dog. Sheriff Greg Poll
WTVA.COM
Like
Comment
Share
Mhra Leander Pallat, Eric Antonio Alvarado and 31 others
Top Comments


Oklahoma Teacher Receives 15-Year Prison Sentence For Sex With 15-Year-Old Boy
A former Oklahoma middle school teacher has pleaded guilty to 6 counts of rape, child enticement...
THREEPERCENTERNATION.COM

In several countries or rather in several regions of the world, family system has collapsed, due to bad nature and naughty acts of women. Particularly in Britain, and America, almost 50% people are alone, lonely, separated, divorced or failed marriages. In 2013, 48% children were born out of wedlock. **It was projected that by 2016, more than 51% children will be born, to unmarried mothers**. In these developed countries " paternity fraud " by women, are close to 20%. You can see several articles in the net, and in wikipedia etc. This means 1 out of 5 children are calling a wrong man as dad. The lonely, alone " mothers " are frustrated. They see the children as burden. Love in the Society in general is lost, long time ago. The types of " Mothers " and " Women " we have now ....


This is the type of women we have in this world. These kind of women were also someones daughter
Mother Stabs Her Baby 90 Times With Scissors After He Bit Her While Breastfeeding Him!
Eight-month-old Xiao Bao was discovered by his uncle in a pool of blood Needed 100 stitches after the incident; he is now recovering in hospital Reports say his...
MOMMABUZZ.COM

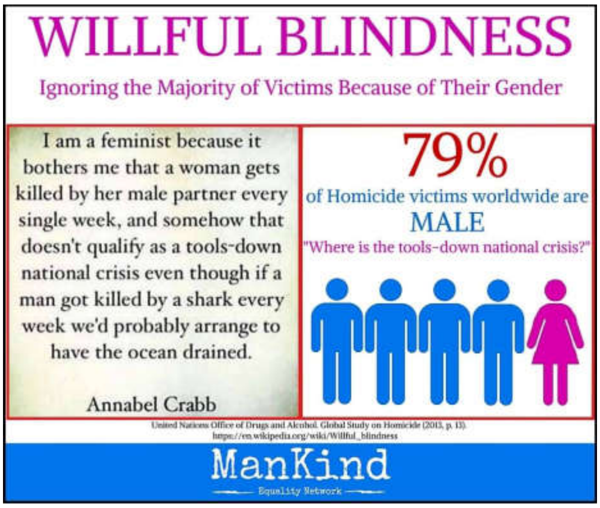

c'est sacré!
MY PUSSY MY RULES
NO GOD IN MY VAGINA


NATIONAL GO TOPLESS DAY
AUG 23 @ NOON - 3 PM
NYC - CENTRAL PARK
GOING TOPLESS IS LEGAL IN NY !!!
GoTopless.org


NUDITY ISN'T SEXUAL
WOMEN'S


TOPLESS : EGALITE POUR TOUS OU POUR PERSONNE !
LES SEINS DES FEMMES NE SONT PAS INDECENTS
SORRY TO BOTHER YOU, BUT WE ARE CHANGING THE WORLD!


FREEDOM FOR WOMEN
NO ISLAMISM



Muslim Woman Caught RAPING Her Own Son - Gives Disgusting Excuse to Judge | John Hawkins' Right Wing News

RIGHTWINGNEWS.COM

By now if you have assumed that Indian women are not doing any crime then please become friends with MRA Guri https://www.facebook.com/profile.php?id=100004138754180

He has dedicated his life to expose Indian Criminals

Delhi Woman Who Tried To Rape An Auto Driver, While Her Friend Filmed The Act, Has Been Arrested

Men are raped too!

MENSXP.COM | BY NIKITA MUKHERJEE

Muslim mother, 43, jailed for sex offences against girl, nine

Raheelah Dar, 43, from Middlesbrough, has been jailed for seven years for carrying out a string of sex offences against a nine-year-old girl.

DAILYMAIL.CO.UK

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Mother who had been forced into an arranged marriage is jailed for filming herself having sex with her 14-year-old son and sending the clips to relatives in Pakistan

- **Vile mother filmed having sex with her teenage son in sick porn video**
- **Clips sent to cousin in Pakistan who allegedly asked her to make film**
- **She also sent her relative indecent images of her three-year-old daughter**

By ALEX MATTHEWS FOR MAILONLINE
PUBLISHED: 12:44 GMT, 1 August 2016 | UPDATED: 11:23 GMT, 2 August 2016

Wife Stabs Husband And Runs Away After He Stops Her From Gambling

The husband said his wife had become a habitual gambler who was also addicted to liquor.

INDIATIMES.COM

Teacher learns fate for 6 months of sex with boy

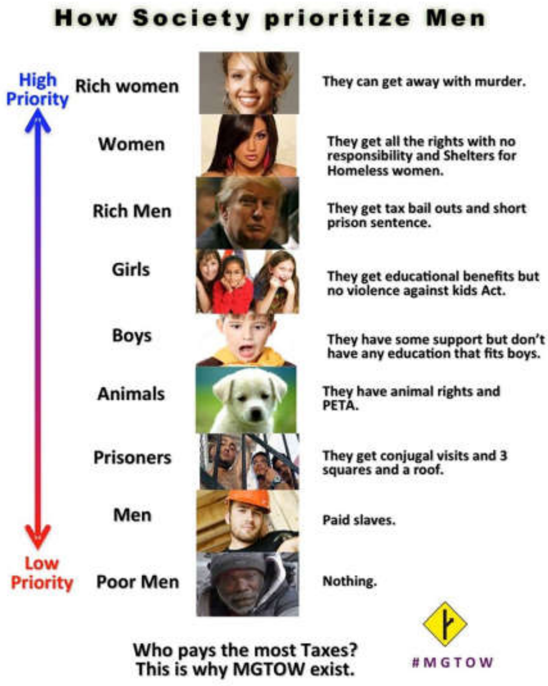
(CBS8) — SAN DIEGO (CNS) – A Crawford High School teacher and coach who carried on a six-month sexual relationship with a 15-year-old male student was sentenced Friday to a two-year prison term. Toni Nicole Sutton, 38, pleaded guilty…

WND.COM

Mom jailed for 40 years after body of daughter, 9, found in fridge

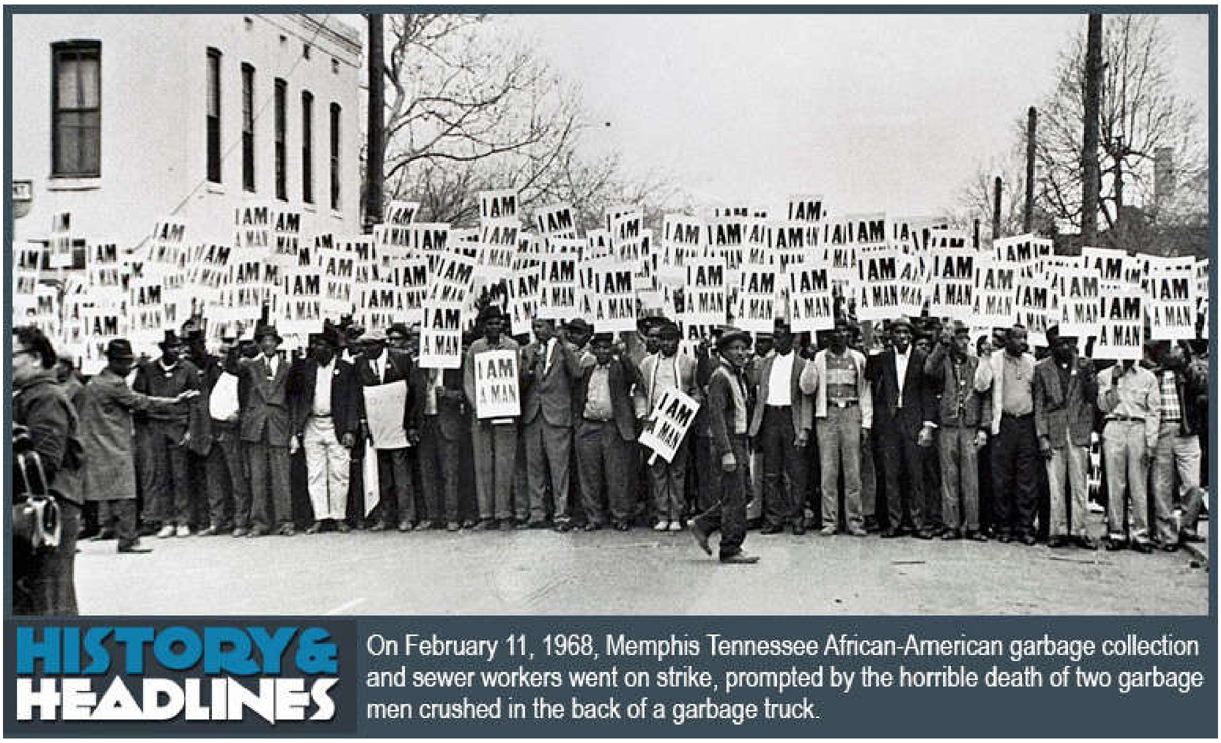
Amber Keyes, 37, was sentenced in the death of Ayahna Comb in Houston on Friday. Ayahna, who had cerebral palsy, had been in the fridge for six months…

DAILYMAIL.CO.UK



Professor Subhashish Chattopadhyay

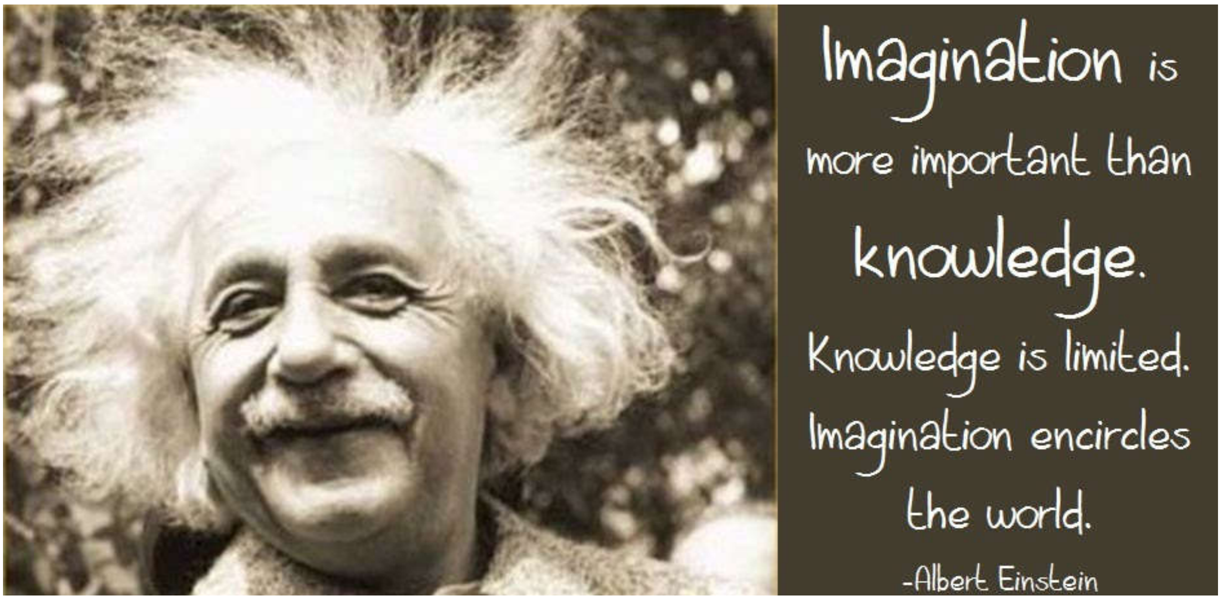
Read http://www.warrenfarrell.org/TheBook/index.html



Read http://www.pdfarchive.info/pdf/S/Sm/Smith_Helen_-_Men_on_strike.pdf

Read http://judgybitch.com/2013/09/17/what-would-happen-if-no-men-showed-up-for-work-today/

Preface for Science



Many Scientists have made, very good TV programs; to teach Science. Carl Sagan, Desmond Morris, Jacques Cousteau, Neil deGrass Tyson, James Burke, Jacob Bronowski, Bill Nye, Andrew Pontzen, Sean Carroll, Michio Kaku, Brian Cox, Brian Greene, Freeman Dyson, Dr. Don Lincoln ... the list is long. BBC, Discovery Channel, Nova, Nature, Science Planet .... the list of good Channels is big.

Even though these programs are being delivered free, (add education programs of Govt. of India, which are also very good); not sure how many are correctly learning.

https://www.youtube.com/watch?v=4sLGCeeA1UI&list=PLaMjJl9Tuw7HoCo8wzZNwMC7jjo3nrEvx

As I randomly talk to lots of students ... I find ...

The Science understanding of Urban, Rich children, in general; is abysmal.

The Science fiction movies, showing Aliens; or winning war with Aliens are more popular and influential. Doraemon making "time machine" so easily, and doing "time travel" so often intrigues children more. (for General Knowledge see http://skmclasses.weebly.com)

India is an uniquely peculiar country; has 1.3 Billion people, obsessed with thousands of stupid things. Superstitious Religious Rituals, Hundreds of festivals, 'What to do' and 'what not to do' [on a full moon day, on a No Moon day, on 11 th day of Lunar month], before and after an eclipse, what to eat and what not to eat, what to wear and what not to wear, Caste, Gotra, “methods and steps” for Puja or Prayer, hundreds of ways to control or restrict or influence others etc... ; keeps people busy.

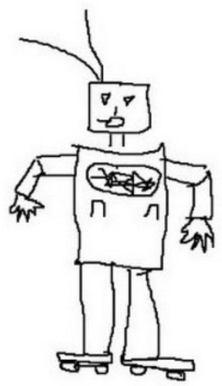
Students have major influence and learning's from these superstitious life style, and fiction / 'stupid movies' rather than from good Science TV shows.

[if you ask any Science Question to any student, first reaction is "Ye to course mein nahi hai"!]

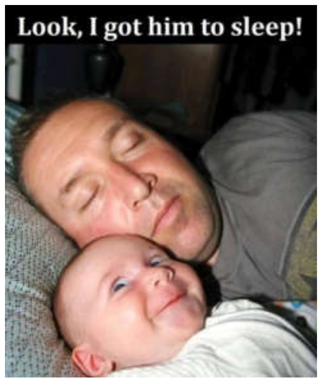
Another most important obsession of Indians is to become Engineers; well somehow .... 14 Lakh (1.4 million) students appear for IIT JEE exam. (Not about IITs or NITs etc) Almost all are stark idiots; study "Engineering" in some college or other .... the story goes on.

In general students / people in India do not know or understand the following ...

One of the most important drawbacks of Human beings is **Anthropophilia**. We love to imagine that ... God, Aliens, Robots etc, are similar to us. Tell a small child to draw a Robot, and almost 100% cases you see a Humanoid being drawn. It is not about the child being intelligent or smart. It is a fundamental 'mental block' that we harbor in general. [when I was a kid, and if someone had told me to draw a Robot, I would have surely drawn a Humanoid]

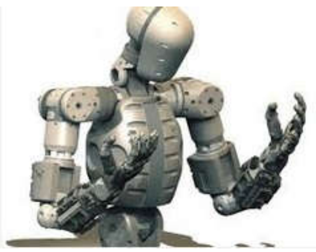


(if I tell you to draw a "Chemical Robot" then ?)



We feel comfortable with Humanoid Robots only

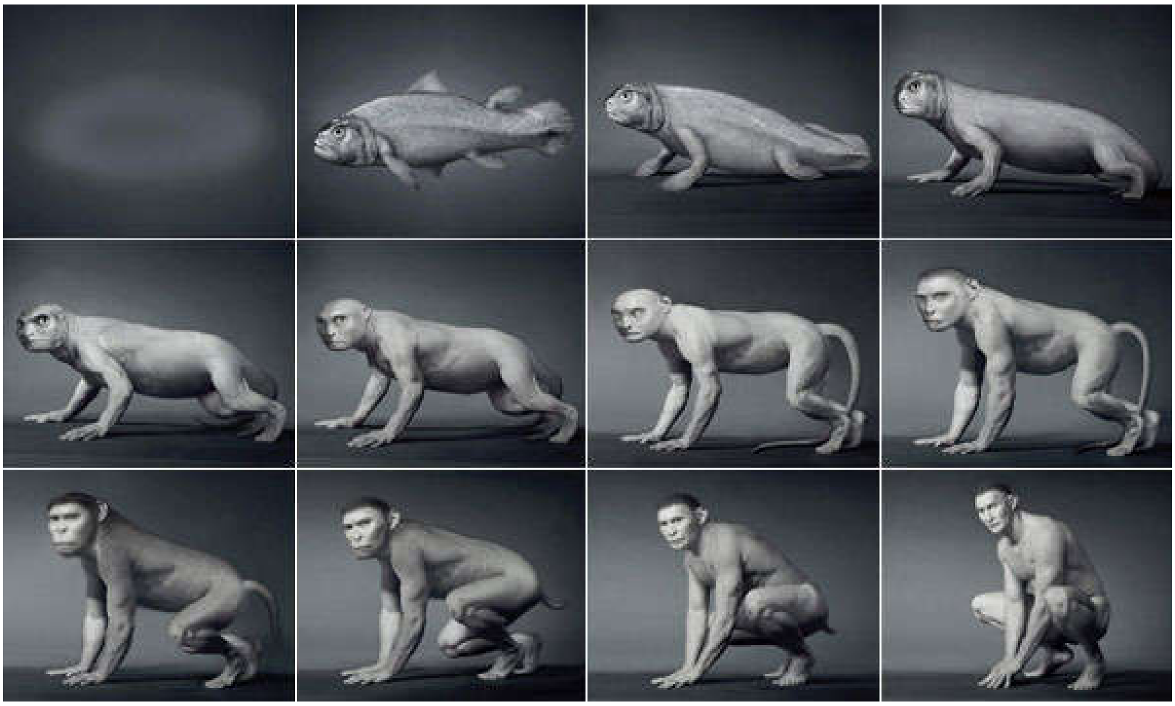





It takes lot of Training and maturity to understand that all machines are Robots. A car is a Robot. A crane is a Robot. Mars Rover is a Robot. Robots can be of any size and shape, serving a particular purpose.

Similarly Aliens do not have to look like us. We have five fingers in our hands, and five toes in our legs because Monkeys have the same. We all evolved step by step from some primitive fish, which had five bones / cartilages in its fins. The fish from which we all evolved had 2

pairs of fins. The pair of fins which was nearer to the head became hands, and the pair at the rear became legs.



Now imagine an Alien evolving from a fish, which had 3 pairs of fins ! or say 17 pairs ! then that may lead to ....

Some children will be quick to identify that Aliens may not evolve from fish, can be different pathways ... in that case they will look very different from us isn't it !

As I write all these in 2016, I say .... " Soon we will find various life-forms in Mars, Moons of Jupiter, Jupiter, and Asteroids ! "

Back to Anthropophilia ... It is very difficult to get rid of this. Christiaan Huygens the great Dutch Scientist 'logically concluded' from observations as follows ...

Jupiter has Atmosphere, so it will rain in Jupiter, so Jupiter must have seas and Oceans, so the "life forms" in Jupiter must have boats, the boats need rope, and rope must be made from trees / fiber, so "they" should have hemp plants ...

Huygens was the first to make a submarine which could go down in water, by a few meters. In those days, around 1650 there was no plane, rocket or space travel. So do you see Huygens could not imagine Aliens in Jupiter flying in Planes or Rockets. While movies now show Aliens in Rockets!

[**Students must know about various limitations of Human beings**. Professor Daniel Kahnemen (2002 Nobel Laureate) has long list of Human Limitations in his book.

see https://vk.com/doc23267904_175119602

I collected some limitations, and wrote an article. See http://skmclasses.kinja.com/bias-we-all-are-biased-1761664826

Scientists have advised a list of "must learn" for students, to appreciate / understand Science better.

See Read http://edge.org/responses/what-scientific-concept-would-improve-everybodys-cognitive-toolkit

It is mandatory for students; to know all the points given in the above links; whom I personally teach]

Chimps and Humans have 96 Percent common genes; Research and Gene Study Finds. **But Humans and Chimps can't communicate, or discuss**. Orangutans are our nearest relatives. We humans are 97% the same as orangutans, gene study shows. But we can't converse with any other species. A little bit of sign language of say 100 "words" or a Dog understanding "instructions" of his master is **not** what is being referred here. **Earth has several Million species, while observations as of now, does not show "communication" across two separate species.** Let us not bring in Symbiotic relationship into this. It is about intelligent communication, discussions, debate, learning from each other etc. Can Humans communicate with insects or birds chirping ?

Imagine a World where Lions were communicating with insects, or say Otters communicating with birds ! The ecosystem as we know, has all these staying together ... so close ! All like a family !! http://www.telegraph.co.uk/science/2016/09/11/dolphins-recorded-having-a-conversation-for-first-time/

Simard discovered that **different tree species are in contact with one another.**

Some **birds** which fly very long distances; do that by sensing Magnetic fields. The eyes of the bird is sensing these feeble magnetic field of Earth by **Quantum entangled Particles**. As the light photons reach and “react” with various Chemicals, the entangled particles are released. These particles “enable” the birds brain to detect Magnetic fields. Does one bird communicate or Guide another with similar mechanisms ?

Trees, it turns out, have a completely different way of communicating: they use scent. It was found that acacias start pumping toxic substances into their leaves to rid themselves of the large herbivores, when being eaten. Beeches, spruce, and oaks all register pain as soon as some creature starts nibbling on them. When a caterpillar takes a hearty bite out of a leaf; the tissue around the site of the damage changes. In addition, the leaf tissue sends out electrical signals, just as human tissue does when it is hurt. However, the signal is not transmitted in milliseconds, as human signals are; instead, the plant signal travels at the slow speed of a third of an inch per minute. Accordingly, it takes an hour or so before defensive compounds reach the leaves to spoil the pest’s meal. Trees live their lives in the really slow

lane, even when they are in danger. If the roots find themselves in trouble, this information is broadcast throughout the tree, which can trigger the leaves to release scent compounds. And not just any old scent compounds, but compounds that are specifically formulated for the task at hand. [Discussing more of this later in the book]

Now do we see the limitations about our obsession, with "communicating" with Aliens ?

The nearest stars are several light years away. Even if we improve the technology to travel 1000 times faster than the fastest rockets it will take thousands of years to travel to nearest "Earth like" planets. **I personally rule out any more discussions on travelling and meeting and communicating with Aliens**.

The life forms (which we will soon find) in Mars, Moons of Jupiter, Jupiter etc **have to be analyzed for DNA**. Will these life-forms have DNA ? Will these Aliens have molecules similar to what we see in organisms here in Earth ? These are important questions in Xenobiology, Astrobiology etc. **We have to wait for data**.

Science is study of data, experimental verification, logical conclusions.

We have made XNA. We have made various kinds of Artificial life, including Arsenic, Selenium based pathways. But extremeophiles also have the same kind of DNA or molecules that we see in all organisms. Same kinds of mRNA etc. Why didn't life grow and evolve multiple times ? We don't know as of now. Or did life evolve / grow multiple times in the same way ? Intelligent human beings will keep researching, and we will know the answers.

The only Sanskrit word in Standard 11-12 Science CBSE text books is Tincal (which is the word for Borax). The books (rightly) are full with German names. Students are unaware the Potassium was derived from an Arabic word Potash, ashes of (roots) of plant.

(not talking about last 50 or 100 years) **Not a single chemical element were purified / synthesized or discovered in India, by any Indian**. Indium (In = #49): Indicum (Latin) means indigo. The pigment indigo was named after indicon (Greek) in allusion for its coming from India. On August 18th, 1868 by French astronomer Jules Janssen. While in Guntur, India, Janssen observed a solar eclipse through a prism, whereupon he noticed a bright yellow spectral line (at 587.49 nanometers) emanating from the chromosphere of the Sun. This led to discovery of Helium. In 1937, Discovery of Astatine was reported by the chemist Rajendralal De. Working in Dacca in British India (now Dhaka in Bangladesh), he chose the name "**dakin**" for element 85, which he claimed to have isolated as the thorium series equivalent of radium F (polonium-210) in the radium series. The properties he reported for **dakin** do not correspond to those of astatine; moreover, astatine is not found in the thorium series, and the true identity of **dakin** is not known.

[not considering the ancient elements which were known to others also ... Supher, Zinc, Mercury and http://www.thehindu.com/sci-tech/science/indian-role-in-producing-superheavy-element-117/article5986191.ece]

As a culture Indians preferred Ayurveda. Identify the trees, smash the leaves, take the bark and / or the roots, make a paste, in some cases add honey etc … and this paste or potion cures everything. If we do not have a medicine for some disease, or if the medicine is not effective, then the argument is … "we did not search the trees in the jungle enough !". The belief being solution / medicine for every disease is out there in the jungle!

This culture is grossly opposite to get into the details, identify the molecules, find the reaction pathways. Modern techniques is not seen as good. In fact opposite … **older things are considered better**. The claim often is "some grandfather's grandfather was a great Ayurvedic Doctor, since several generations they are using some paste, and they now the best.

With this kind of a culture Indians cannot and did not find **pharmacophores**.

[see http://www.eurekaselect.com/81348/article

http://www.ucdenver.edu/academics/colleges/medicalschool/departments/Pharmacology/Pages/history.aspx

http://adaptogens.org/adaptogen/history]

An extremely superstitious culture, avoiding to get-into any details, easy way of "chalta hai" had its Dark effect. Indians are averages and poor, because hardly there was any value-add !

Most people in India; think in the following way …

Let us see contribution of some Mathematicians and Scientists; who did great work but students generally don't know about them.

Eugene Wigner - After his sojourn in Berlin, Wigner returned to Budapest to work in his father's tannery. Somehow and somewhere from there, he returned to Berlin joining the Kaiser Wilhelm Institute working first under Karl Weissenberg and later under Richard Becker. There he explored quantum mechanics of Erwin Schrödinger and group theory (founded by

the genius Evariste Galois who was obsessed with polynomials equations and their solutions). At the age of 25, in 1927, in Germany somewhere he introduced the group theory into quantum mechanics. He published it formally in 1931 at the age of 29:

"**Group Theory and Its Application to the Quantum Mechanics of Atomic Spectra**."

He soon thereafter introduced symmetries (rotations, translations, and CPT- charge parity and time reversal symmetry) into quantum mechanics. He formulated and proved a theorem which became the cornerstone of the mathematical formulations of quantum mechanics. Eugene Wigner was so impressed with the usefulness of abstract mathematics in nuclear physics and quantum mechanics that he went on to write a landmark article in 1960 titled:

"The Unreasonable Effectiveness of Mathematics in the Natural Sciences".

In 1930, Princeton University recruited both Jeno Pal Wigner and Janos Von Neumann at 7 times the salary they were drawing in Europe. Both these geniuses anglicized their first names to "Eugene" and "John" respectively and soon thereafter became naturalized citizens of the United States.

-

Janos Bolyai (Transylvania, Hapsburg Empire) 1822 - one of the founders of non-Euclidean geometry — a geometry that differs from Euclidean geometry in its definition of parallel lines. The discovery of a consistent alternative geometry that might correspond to the structure of the universe helped to free mathematicians to study abstract concepts irrespective of any possible connection with the physical world.

Nikolai Ivanovich Lobachevsky (Kazan, Russia) 1823 - known primarily for his work on hyperbolic geometry, otherwise known as Lobachevskian geometry. William Kingdon Clifford called Lobachevsky the "Copernicus of Geometry" due to the revolutionary character of his work. He was dismissed from the university in 1846, ostensibly due to his deteriorating health: by the early 1850s, he was nearly blind and unable to walk. He died in poverty in 1856.

Nikolai was an atheist.

-

Bernhard Riemann (Breselenz, Jameln, Kingdom of Hanover) 1853: student of Gauss - Influential German mathematician who made lasting and revolutionary contributions to analysis, number theory, and differential geometry. In the field of real analysis, he is mostly known for the first rigorous formulation of the integral, the Riemann integral, and his work on Fourier series. His contributions to complex analysis include most notably the introduction of Riemann surfaces, breaking new ground in a natural, geometric treatment of complex analysis. His famous 1859 paper on the prime-counting function, containing the original statement of the Riemann hypothesis, is regarded, although it is his only paper in the field, as one of the most influential papers in analytic number theory. Through his pioneering

contributions to differential geometry, Riemann laid the foundations of the mathematics of general relativity.

-

Felix Klein (Düsseldorf, Prussia) 1870s - German mathematician and mathematics educator, known for his work in group theory, complex analysis, non-Euclidean geometry, and on the connections between geometry and group theory. His 1872 Erlangen Program, classifying geometries by their underlying symmetry groups, was a hugely influential synthesis of much of the mathematics of the day.

-

Marcel Grossman (Budapest) 1910s tutored Einstein on differential geometry and tensor calculus - mathematician and a friend and classmate of Albert Einstein. Grossmann was a member of an old Swiss family from Zurich. His father managed a textile factory. He became a Professor of Mathematics at the Federal Polytechnic Institute in Zurich, today the ETH Zurich, specializing in descriptive geometry.

-

Gregario Ricci-Curbastro (Italy) 1880s - Italian mathematician born in Lugo di Romagna. He is most famous as the inventor of tensor calculus, but also published important works in other fields. With his former student Tullio Levi-Civita, he wrote his most famous single publication, a pioneering work on the calculus of tensors, signing it as Gregorio Ricci. This appears to be the only time that Ricci-Curbastro used the shortened form of his name in a publication, and continues to cause confusion. Ricci-Curbastro also published important works in other fields, including a book on higher algebra and infinitesimal analysis, and papers on the theory of real numbers, an area in which he extended the research begun by Richard Dedekind.

-

Ernst Mach (Moravia, Austrian Empire) 1900s who totally abhorred Newton's idea of absolute space and time - Austrian physicist and philosopher, noted for his contributions to physics such as study of shock waves. Quotient of one's speed to that of sound is named the Mach number in his honor. As a philosopher of science, he was a major influence on logical positivism, American pragmatism and through his criticism of Newton, a forerunner of Einstein's relativity.

-

Hendrik Lorentz (Netherlands) 1900s - Dutch physicist who shared the 1902 Nobel Prize in Physics with Pieter Zeeman for the discovery and theoretical explanation of the Zeeman effect. He also derived the transformation equations which formed the basis of the special relativity theory of Albert Einstein. According to the biography published by the Nobel Foundation, "It may well be said that Lorentz was regarded by all theoretical physicists as the

world's leading spirit, who completed what was left unfinished by his predecessors and prepared the ground for the fruitful reception of the new ideas based on the quantum theory." For this he received many honours and distinctions during his life, including—from 1925 to his death in 1928—the role of Chairman of the exclusive International Committee on Intellectual Cooperation.

-

Willem De Sitter (Netherlands) 1920s - Dutch mathematician, physicist, and astronomer. De Sitter made major contributions to the field of physical cosmology. He co-authored a paper with Albert Einstein in 1932 in which they discussed the implications of cosmological data for the curvature of the universe. He also came up with the concept of the de Sitter space and de Sitter universe, a solution for Einstein's general relativity in which there is no matter and a positive cosmological constant. This results in an exponentially expanding, empty universe. De Sitter was also famous for his research on the planet Jupiter.

-

Alexander Friedmann (St. Petersburg, Russian Empire) 1920s - was a Russian and Soviet physicist and mathematician. He is best known for his pioneering theory that the universe was expanding, governed by a set of equations he developed now known as the Friedmann equations.

-

Georges Lemaitre (Belgium) 1920s - was a Belgian priest, astronomer and professor of physics at the Catholic University of Leuven. He proposed the theory of the expansion of the universe, widely misattributed to Edwin Hubble. He was the first to derive what is now known as Hubble's law and made the first estimation of what is now called the Hubble constant, which he published in 1927, two years before Hubble's article. Lemaître also proposed what became known as the Big Bang theory of the origin of the universe; which he called his "hypothesis of the primeval atom" or the "Cosmic Egg".

-

One of the greatest help we apes got; **was with the discovery or invention of mass spectrometry**.

The men who invented this device were (at least Two; as claimed by the Western English speaking world).

1. Englishman Francis William Aston in 1919

2. Canadian American Arthur Jeffrey Dempster in 1918.

Just imagine as Europe was involved in one of their bloodiest slaughter and carnage, these men were quietly working in their labs devising an instrument that could sort out atoms and ions based on their charge to mass ratio.

(I wish to emphasize yet again that even though atoms are a fact, we using the term atomic theory till date.)

By 1919, Aston had achieved 2 feats:

1. He showed that atoms of a single element could have different isotopes thereby establishing as fact that even non radioactive elements have isotopes.

2. He had invented the first mass spectroscope.

The Canadian Dempster had greatly improved on it, greatly increasing its accuracy in identifying compounds by mass of elements in a sample. This was a gigantic step to our understanding of nature.

-

David Goldberg - David Edward Goldberg (born September 26, 1953) is an American computer scientist, civil engineer, and professor at the department of Industrial and Enterprise Systems Engineering (IESE) at the University of Illinois at Urbana-Champaign and is most noted for his work in the field of genetic algorithms. He is the director of the Illinois Genetic Algorithms Laboratory (IlliGAL) and the chief scientist of Nextumi Inc. He is the author of Genetic Algorithms in Search, Optimization and Machine Learning, one of the most cited books in computer science.

In computer science and operations research, a genetic algorithm (GA) is a metaheuristic inspired by the process of natural selection that belongs to the larger class of evolutionary algorithms (EA). Genetic algorithms are commonly used to generate high-quality solutions to optimization and search problems by relying on bio-inspired operators such as mutation, crossover and selection.

-

Lotfi Zadeh - The term fuzzy logic was introduced with the 1965 proposal of fuzzy set theory by Lotfi Zadeh. Fuzzy logic had however been studied since the 1920s, as infinite-valued logic—notably by Łukasiewicz and Tarski. Fuzzy logic is a form of many-valued logic in which the truth values of variables may be any real number between 0 and 1, considered to be "fuzzy". By contrast, in Boolean logic, the truth values of variables may only be 0 or 1, often called "crisp" values. Fuzzy logic has been applied to many fields, from control theory to artificial intelligence.

-

Warren McCulloch and Walter Pitts - (1943) created a computational model for neural networks based on mathematics and algorithms called threshold logic. This model paved the way for neural network research to split into two distinct approaches. One approach focused on biological processes in the brain and the other focused on the application of neural networks to artificial intelligence.

In the late 1940s psychologist **Donald Hebb** created a hypothesis of learning based on the mechanism of neural plasticity that is now known as **Hebbian learning**. Hebbian learning is considered to be a 'typical' unsupervised learning rule and its later variants were early models for long term potentiation. Researchers started applying these ideas to computational models in 1948 with Turing's B-type machines.

Farley and Wesley A. Clark (1954) first used computational machines, then called "calculators," to simulate a Hebbian network at MIT. Other neural network computational machines were created by Rochester, Holland, Habit, and Duda (1956).

Frank Rosenblatt (1958) created the perceptron, an algorithm for pattern recognition based on a two-layer computer learning network using simple addition and subtraction. With mathematical notation, Rosenblatt also described circuitry not in the basic perceptron, such as the exclusive-or circuit, a circuit which could not be processed by neural networks until after the backpropagation algorithm was created by Paul Werbos (1975).

Neural network research stagnated after the publication of machine learning research by Marvin Minsky and **Seymour Papert** (1969), who discovered two key issues with the computational machines that processed neural networks. The first was that basic perceptrons were incapable of processing the exclusive-or circuit. The second significant issue was that computers didn't have enough processing power to effectively handle the long run time required by large neural networks. Neural network research slowed until computers achieved greater processing power.

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Interval arithmetic, interval mathematics, interval analysis, or interval computation, is a method developed by mathematicians since the 1950s and 1960s, as an approach to putting bounds on rounding errors and measurement errors in mathematical computation and thus developing numerical methods that yield reliable results. Very simply put, it represents each value as a range of possibilities. For example, instead of estimating the height of someone using standard arithmetic as 2.0 meters, using interval arithmetic we might be certain that that person is somewhere between 1.97 and 2.03 meters. In mathematics, a (real) interval is a set of real numbers with the property that any number that lies between two numbers in the set is also included in the set. For example, the set of all numbers x satisfying $0 \le x \le 1$ is an interval which contains 0 and 1, as well as all numbers between them.

This concept is suitable for a variety of purposes. The most common use is to keep track of and handle rounding errors directly during the calculation and of uncertainties in the knowledge of the exact values of physical and technical parameters. The latter often arise

from measurement errors and tolerances for components or due to limits on computational accuracy. Interval arithmetic also helps find reliable and guaranteed solutions to equations and optimization problems.

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Nassim Nicholas Taleb and Benoit Mandelbrot -

Nassim is a Lebanese-American essayist, scholar, statistician, former trader, and risk analyst, whose work focuses on problems of randomness, probability, and uncertainty. His 2007 book The Black Swan was described in a review by the Sunday Times as one of the twelve most influential books since World War II. He advocates what he calls a "black swan robust" society, meaning a society that can withstand difficult-to-predict events.

Benoit Mandelbrot was a Polish-born, French and American mathematician with broad interests in the practical sciences, especially regarding what he labeled as "the art of roughness" of physical phenomena and "the uncontrolled element in life." He referred to himself as a "fractalist". He is recognized for his contribution to the field of fractal geometry, which included coining the word "fractal'", as well as developing a theory of "roughness and self-similarity" in nature. He spent most of his career in both the United States and France, having dual French and American citizenship. In 1958, he began a 35-year career at IBM, where he became an IBM Fellow, and periodically took leaves of absence to teach at Harvard University. Because of his access to IBM's computers, Mandelbrot was one of the first to use computer graphics to create and display fractal geometric images, leading to his discovering the Mandelbrot set in 1979. He showed how visual complexity can be created from simple rules. He said that things typically considered to be "rough", a "mess" or "chaotic", like clouds or shorelines, actually had a "degree of order." His math and geometry-centered research career included contributions to such fields as statistical physics, meteorology, hydrology, geomorphology, anatomy, taxonomy, neurology, linguistics, information technology, computer graphics, economics, geology, medicine, cosmology, engineering, chaos theory, econophysics, metallurgy, taxonomy and the social sciences.

Nassim, Benoit Mandelbrot and many others showed that application of Fractals / Mandrelbot is better to predict several practical outcomes, in contrast to Gaussian distribution analysis.

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Charles Darwin told his friend that, he guesses; Life may have started in a shallow hot pond. Darwin was many hundred years ahead of his times.

The Murchison meteorite that fell near Murchison, Victoria, Australia in 1969 was found to contain over 90 different amino acids, nineteen of which are found in Earth life. Comets and other icy outer-solar-system bodies are thought to contain large amounts of complex carbon compounds (such as tholins) formed by these processes, darkening surfaces of these bodies.

The early Earth was bombarded heavily by comets, possibly providing a large supply of complex organic molecules along with the water and other volatiles they contributed.

The University of Waterloo and University of Colorado conducted simulations in 2005 that indicated that the early atmosphere of Earth could have contained up to 40 percent hydrogen—implying a much more hospitable environment for the formation of prebiotic organic molecules. The escape of hydrogen from Earth's atmosphere into space may have occurred at only one percent of the rate previously believed based on revised estimates of the upper atmosphere's temperature.

Researchers at the Rensselaer Polytechnic Institute in New York reported the possibility of oxygen available around 4.3 billion years ago. Their study reported in 2011 on the assessment of Hadean zircons from the earth's interior (magma) indicated the presence of oxygen traces similar to modern-day lavas.

700 Million years after Earth's origin, (around 3.8 Billion years ago), the Rocks have signatures of Microbe Life. Just 540 million year ago diversity of life happened (Cambrian Explosion). So for almost 3 Billion years Earth had only Microbes. The day was around 22 hours then, as Earth was rotating quicker.

Studies have been made of the amino acid composition of the products of "old" areas in "old" genes, defined as those that are found to be common to organisms from several widely separated species, assumed to share only the last universal ancestor (LUA) of all extant species. These studies found that the products of these areas are enriched in those amino acids that are also most readily produced in the Miller–Urey experiment. This suggests that the original genetic code was based on a smaller number of amino acids – only those available in prebiotic nature – than the current one.

Cyanobacteria are able to survive extreme conditions. They live in Antarctica as well as in mountain springs. One species was isolated even from polar bear hairs.

Cyanobacteria get their name from the bluish pigment phycocyanin, which they use to capture light for photosynthesis as they also contain chlorophyll. Their name comes from the Greek word for blue, cyanos. Cyanobacteria have been living on the Earth for more than 3 billion years. They alter genetically and develop various evolutionary lines. They have survived here for a uniquely long time. These are microscopic, they are rich in chemical diversity. the chloroplast in plants is a symbiotic cyanobacterium, taken up by a green algal ancestor of the plants sometime in the Precambrian. These bacteria are often found growing on greenhouse glass, or around sinks and drains. The Red Sea gets its name from occasional blooms of a reddish species of Oscillatoria, and African flamingos get their pink color from eating Spirulina.

The scientific community has gained a clearer understanding of the evolution of cyanobacteria of the Synechococcus group. It is one of the largest groups of cyanobacteria, widespread from the poles to the equator, in the sea as well as on land. Petr Dvorák, a phycologist from the Faculty of Science, has compared their genes and constructed, with the

help of molecular biology, the first complex phylogenetic tree of this group, an interpretation of its evolution.

It shows that; the beginning of life, coincides with a hypothetical event that occurred 4 billion to 3.85 billion years ago, known as the Late Heavy Bombardment, in which asteroids pummeled Earth and the solar system's other inner planets. These impacts may have provided the energy to jumpstart the chemistry of life.

Studies suggest that asteroid impacts may break down formamide — a molecule thought to be present in early Earth's atmosphere — into genetic building blocks of DNA and its cousin RNA, called nucleobases.

Chemist Svatopluk Civiš, of the Academy of Sciences of the Czech Republic, and his colleagues used a high-powered laser to break down ionized formamide gas, or plasma, to mimic an asteroid strike on early Earth. The reaction produced scalding temperatures of up to 4,230 degrees Celsius, sending out a shock wave and spewing intense ultraviolet and X-ray radiation. The chemical fireworks produced four of the nucleobases that collectively make up DNA and RNA: adenine, guanine, cytosine and uracil.

The Amino acids joinup to make various Proteins. These lead to microbes. Stromatolites produced Oxygen, and increased the Oxygen content in the atmosphere over Billion years. The Oxygen also made Iron oxide out of Iron dissolved in Water, which deposited as layers of Iron ore.

See about Trilobites at https://research.amnh.org/paleontology/trilobite-website/twenty-trilobite-fast-facts

http://www.fossilmuseum.net/Tree_of_Life/Stromatolites.htm

http://jrscience.wcp.muohio.edu/fieldcourses01/PapersMarineEcologyArticles/Stromatolites-Thelongestl.html

Dvorák and his colleagues utilised also a genome sequence of a new genus of cyanobacteria found in a peatbog in Slovakia. It was named Neosynechococcus. Algology (from algae) is a branch of biology studying algae and cyanobacteria. It deals with the systematisation, phylogenesis, and ecology of these organisms. It also includes physiology, biochemistry, and genetics.

See https://www.youtube.com/watch?v=SOGwoFkPtT8

The Miller–Urey experiment was a chemical experiment that simulated the conditions thought at the time to be present on the early Earth, and tested the chemical origin of life under those conditions. Earth favoured chemical reactions that synthesized more complex organic compounds from simpler inorganic precursors. Considered to be the classic experiment investigating abiogenesis, it was conducted in 1952 by Stanley Miller, with assistance from Harold Urey, at the University of Chicago and later the University of California, San Diego. Scientists examining sealed vials preserved from the original experiments (of Stanley Miller)

were able to show that there were actually well over 20 different amino acids produced in Miller's original experiments.

See https://www.youtube.com/watch?v=57merteLsBc

In 1961, Joan Oró found that the nucleotide base adenine could be made from hydrogen cyanide (HCN) and ammonia in a water solution. His experiment produced a large amount of adenine, the molecules of which were formed from 5 molecules of HCN. Also, many amino acids are formed from HCN and ammonia under these conditions. Experiments conducted later showed that the other RNA and DNA nucleobases could be obtained through simulated prebiotic chemistry with a reducing atmosphere.

See https://www.youtube.com/watch?v=xyhZcEY5PCQ

Next Study Evolution

- http://evolution.berkeley.edu/evolibrary/article/side_0_0/origsoflife_05

https://www.youtube.com/watch?v=QqG01ihQjoo

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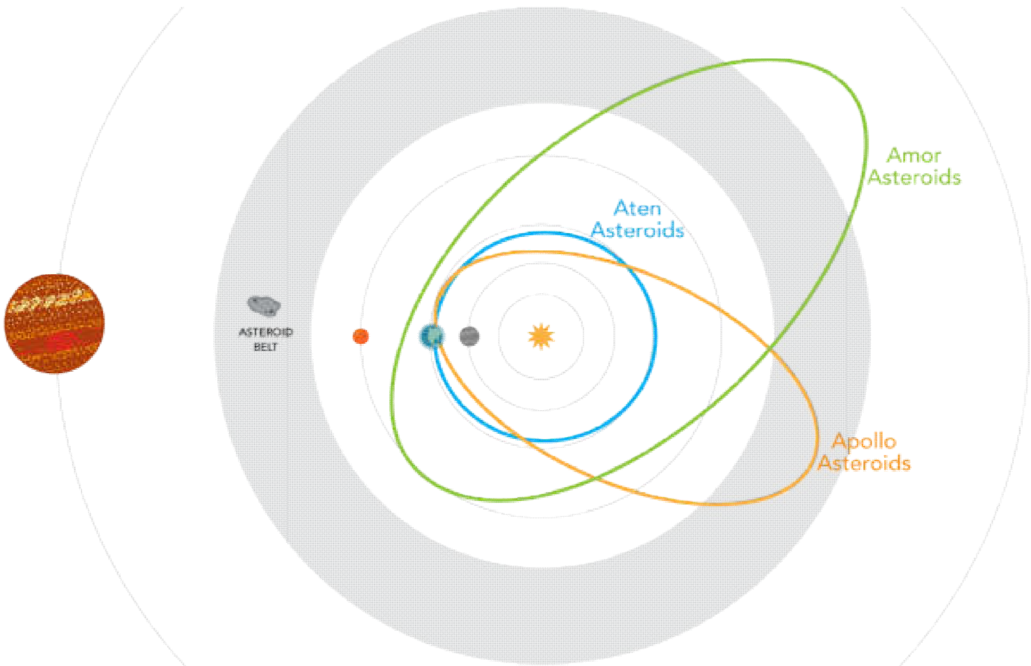
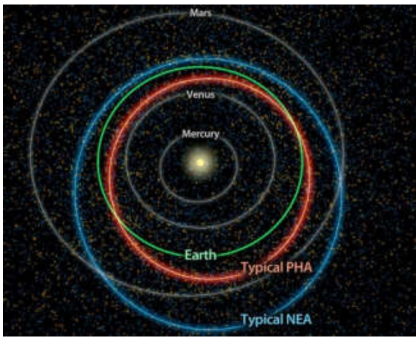
There are many near Earth Asteroids; that are being constantly monitored, since 1990s. This is to avoid any major impact that may wipeout life from Earth. International cooperation exists, to plan for destroying the Asteroid which is directed towards Earth. Near-Earth asteroids are in a different class than main belt asteroids, as they are much closer energetically to Earth.



There are three main orbits of near-Earth asteroids: Amor, Aten, and Apollo.

Most intersect with the Earth's orbit at some point during their trip around the sun, making this the prime time to analyze them with a telescope, or even rendezvous with them on a prospecting mission with our Arkyd spacecraft.

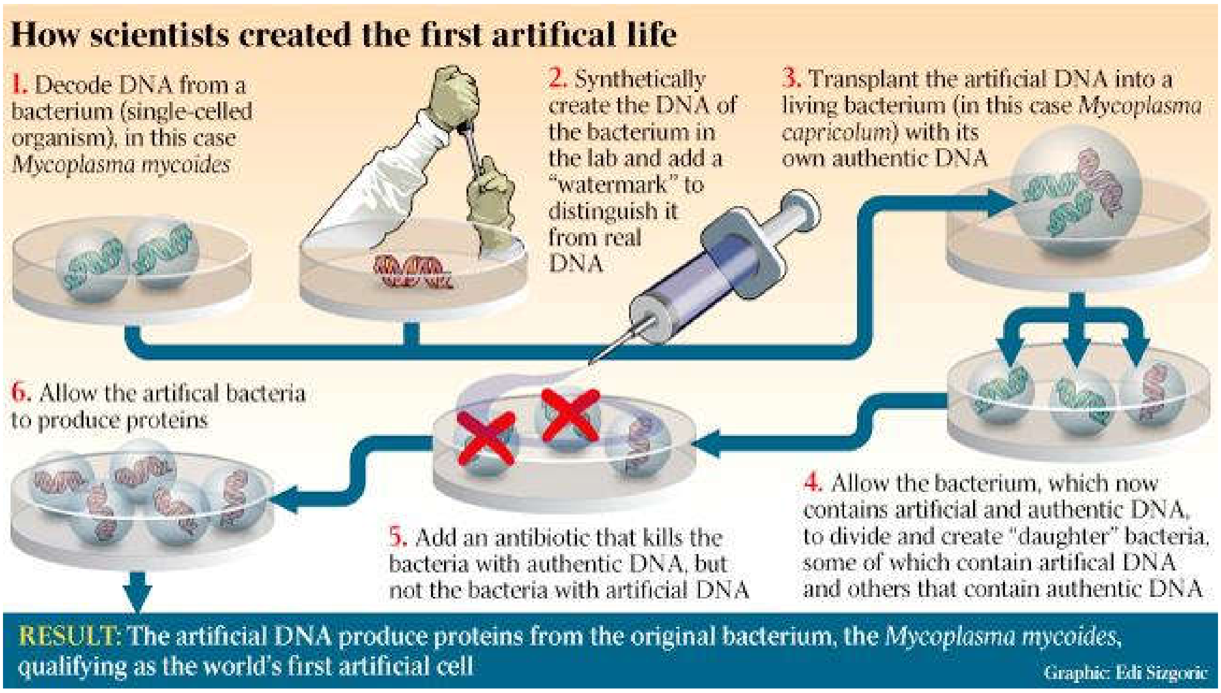
See http://www.planetaryresources.com/2015/10/studying-close-approaches-of-near-earth-asteroids/

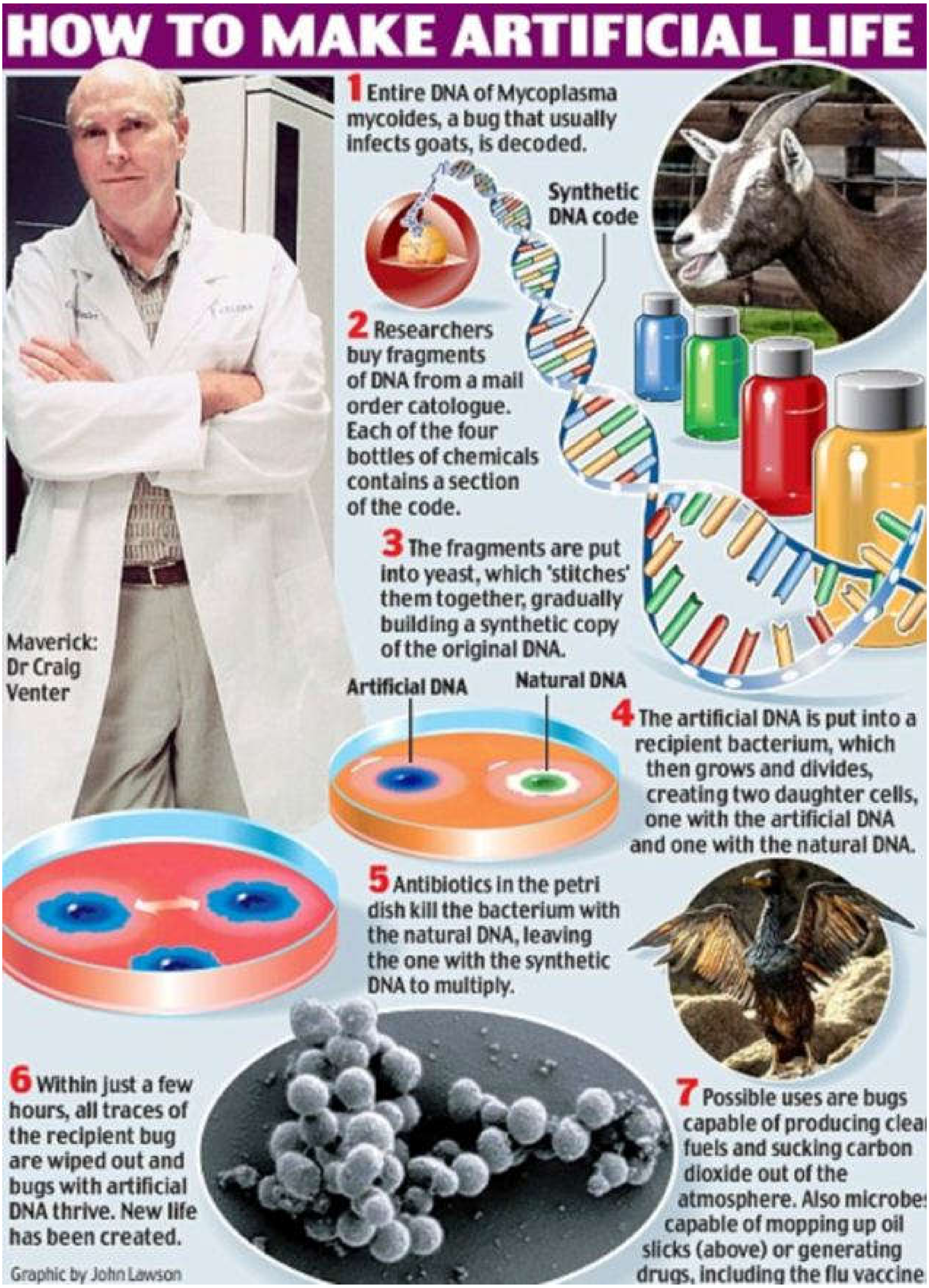


47,000 of the probable Asteroids have been listed.

http://www.dailymail.co.uk/sciencetech/article-2145699/New-Nasa-sky-scan-reveals-47-000-hazardous-near-Earth-asteroids-330ft-wide--BIGGER.html

Craig Venter and his team of Nobel Laureates, and other very smart Scientists, have been working on Artificial or Synthetic life for long.




HOW TO MAKE ARTIFICIAL LIFE
1 Entire DNA of Mycoplasma mycoides, a bug that usually infects goats, is decoded.
Synthetic DNA code
2 Researchers buy fragments of DNA from a mail order catologue. Each of the four bottles of chemicals contains a section of the code.
3 The fragments are put into yeast, which 'stitches' them together, gradually building a synthetic copy of the original DNA.
Maverick: Dr Craig Venter
Artificial DNA
Natural DNA
4 The artificial DNA is put into a recipient bacterium, which then grows and divides, creating two daughter cells, one with the artificial DNA and one with the natural DNA.
5 Antibiotics in the petri dish kill the bacterium with the natural DNA, leaving the one with the synthetic DNA to multiply.
6 Within just a few hours, all traces of the recipient bug are wiped out and bugs with artificial DNA thrive. New life has been created.
7 Possible uses are bugs capable of producing clea fuels and sucking carbon dioxide out of the atmosphere. Also microbe capable of mopping up oil slicks (above) or generating drugs, including the flu vaccine
Graphic by John Lawson

See https://www.youtube.com/watch?v=ayfF1v7rifw

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Gordon Allport and S. Odbert - The OCEAN model of "Big Five personality traits", rather modern Psychology was started by these two Men. The Big Five personality traits, also known as the five factor model (FFM), is a model based on common language descriptors of personality (lexical hypothesis). These descriptors are grouped together using a statistical technique called factor analysis (i.e. this model is not based on experiments). This widely examined theory suggests five broad dimensions used by some psychologists to describe the human personality and psyche. The five factors have been defined as openness to experience, conscientiousness, extraversion, agreeableness, and neuroticism, often listed under the acronyms OCEAN or CANOE. Beneath each proposed global factor, a number of correlated and more specific primary factors are claimed. For example, extraversion is said to include such related qualities as gregariousness, assertiveness, excitement seeking, warmth, activity, and positive emotions.

In 1884, Sir Francis Galton was the first person who is known to have investigated the hypothesis that it is possible to derive a comprehensive taxonomy of human personality traits by sampling language: the lexical hypothesis. In 1936, Gordon Allport and S. Odbert put Sir Francis Galton's hypothesis into practice by extracting 4,504 adjectives which they believed were descriptive of observable and relatively permanent traits from the dictionaries at that time. In 1940, Raymond Cattell retained the adjectives, and eliminated synonyms to reduce the total to 171. He constructed a self-report instrument for the clusters of personality traits he found from the adjectives, which he called the Sixteen Personality Factor Questionnaire. Based on a subset of only 20 of the 36 dimensions that Cattell had originally discovered, Ernest Tupes and Raymond Christal claimed to have found just five broad factors which they labeled: "surgency", "agreeableness", "dependability", "emotional stability", and "culture". Warren Norman subsequently relabeled "dependability" as "conscientiousness".

After "**God, Puja & Prayer**", being the 1st ; the 2nd worst illusion, that hampers Science; is "**Gut feeling**". The Havoc or mayhem of "Gut feeling" is very prominently seen regarding Psychology, or People skills (of most people). Close to 99% people conduct interviews and take 'people decisions', without caring anything about Psychology.

Long back I wrote "***Millions of Interviews are being conducted every day, where the interviewer knows nothing about Psychology, while believes that her gut feeling is guiding for correct decisions***". [the reader will have to agree with this, if he heard about OCEAN model for the first time, here]

https://zookeepersblog.wordpress.com/interview-techniques-and-the-things-you-cannot-find/

https://zookeepersblog.wordpress.com/are-people-very-logical-and-rational-then-why-should-we-be-polite-with-all/

https://zookeepersblog.wordpress.com/correlated-adjectives-this-personality-trait-predicts-your-tendency-to-lie-and-cheat/

Psychology stands on the conclusions drawn after experiments. Some most important experiments being Milgram Experiment, Stanford Prison experiment, Hawthorne experiment, Bad Samaritan experiment, Attractiveness experiments, Evolutionary Psychology experiment, Decoy experiments, Equity theory of Motivation experiments, etc …

The experiments that I used to talk about while teaching **Senior Corporate Managers** are listed at

https://zookeepersblog.wordpress.com/psychology-experiments-and-summary-of-the-subject/

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Is Economics a Branch of Science ?

Not discussing about Economists here, as my personal opinion about, "works and contribution of Economists" is very poor. All of them argue and fancy in disagreeing with each and every thing told by someone. Economics has no consensus, no agreed rules, driven more by politics, and / or dynamic situations. No prediction by any Economist comes Correct or True; consistently. Media interviews thousands of these "strange foolish guys", and trys to "understand" an average. Randomly someone's prediction matches the actual outcome, and Predictions of 999 of the other morons deviate. These guys are always busy, analyzing and confirming that in past what had happened was "inevitable", while in the same breath, they accept that "no clue about the future". **None** had predicted the “**inevitable**” though. The stupidest of all the doomsters is Thomas Malthus. He has a "world record" of its kind, as ALL his predictions came wrong.

[The second record holder will be of course Sigmund Freud. All explanations given by Freud are wrong, and crap. Modern Psychologists, call Freud worst than a quack. See how Professor Bloom, from Yale laugh at Freud, (and I agree with Prof. Bloom), in the class... https://www.youtube.com/watch?v=P3FKHH2RzjI&list=PL6A08EB4EEFF3E91F

even Aristotle did better than these stupids. See something what Aristotle said is true, given below in this book]

Personally I have read several books in Economics, and several thousand (may be more than 10,000) scholarly articles. All will call me a fool, for every prediction; I make on Economy, or anything in Economics. As usual no one will agree with me, I know. I never try to talk about Economics, as you all saw, here, just now! I agreed with Millions of others, '**to Not to**' believe in anything an Economist says or predicts.

A very small "summary" of what these 'idiots' have done is at

https://zookeepersblog.wordpress.com/a-butcher-makes-kima-of-economics/

[My friends occasionally say … "**even Russia has Russian economists**" …]

Nassim Taleb has called for cancellation of the Nobel Prize in Economics, saying that the damage from economic theories can be devastating. (and I agree with him).

http://www.zerohedge.com/news/2016-11-06/economics-broken-and-there-no-internal-incentive-fix-it-5-reasons-smash-ivory-towers

In contrast to economics, Finance Law/Rules and Marketing Tricks/Techniques are supreme. Very correctly Millions call these subjects as "Financial Science" and "Marketing Science".

The learning's here are generally not attributed to a particular person. There are many Key concepts, which are correct; and accurate! These enable people to take right decisions, to make money, be profitable, to generate employment, to avoid and reduce loss, to sale, and keep businesses going.

For whatever we do, we have to deal with people, and earn money or make profit. So the basic understanding of Psychology, the Laws of Finance, and the 'Tricks and trades' of Marketing (Science) are must for all. Human beings in general, harbor many limitations; which Economists disregard. One of the first assumptions of Economics, "The Rational Human beings" is wrong.

See the list of Biases at http://skmclasses.kinja.com/bias-we-all-are-biased-1761664826

Some of the key concepts of Finance are NPV (Net Present Value), ROI (Return on Investment), Risk/Return Tradeoff, Diversification, ROCE (Return on Capital Employed), Discounted Cash flow, Time value of Money, Liquidity, Budgeting etc. The list is big. It takes many months of correct studies, to understand and master these. Those who apply these rules and learning's well; are paid well. People in general do not disagree to fight with what Finance Gurus says.

It is extremely important for every student to know that everyone is not working or running after profit, or ROI. The world is full with Philanthropic acts. There are Billions of Altruists. Too much of priority towards money, makes people cold, cruel, isolated, un-helping, and in-human …

See https://zookeepersblog.wordpress.com/do-you-know-who-was-dashrath-manjhi/

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Marketing Science is **Art**. Successful Marketing gurus are paid very well. I have not seen insults and fights, towards Marketing Gurus. People just do **not** hate them like Economists. There are some key concepts.

See https://zookeepersblog.wordpress.com/25-points-on-brand-and-marketing/

Personally I will always remain a toddler, regarding Tricks and details of Marketing.

When I was in Standard 9, my Aunt (Cousin sister of my Mom), started a very small chemical business. She was staying in a different city, and I "managed" the business affairs, in Jamshedpur. I had to meet lot of people at various offices, advertise, give sale pitches, sale, follow-up with people, get payments, and generate profit etc.

This gave me very interesting exposure to human behavior, organizations, processes, human nature and follies, greed etc. Much later I managed my own IIT JEE coaching / Business.

With this background, I am adding "**a Pinch of Salt**" in the **Ocean of Management**.

[meaning, I do not think, my words are going to teach or contribute anything]

Regarding advertisement, I have observed that people are in silos, or islands. Mostly unaware what is going on in other islands. People expect advertisement in their own silo, or island. So advertisement is required to be done in multiple mediums / channels. If I advertise in newspaper, (say about Govt. of India, official Olympiads), some people will say … "school did not tell anything". If I advertise in Google adwords, guys in Facebook will not know. Any amount of "Radio Messages" done, will not stop people saying … "the CSR (corporate Social responsibility) department did not send any mailer ! …

It is extremely costly to advertise in every island. Small businesses just cannot afford such expenditures. So advertisement always remains insufficient, as per my perception. Effectiveness of the advertisements, and success is always unknown. As per my perception, the young MBA's handling the budget randomly try various things, playing randomly with "others money". Randomly there is some result/response, that is termed / "show cased" as

success. Gurus handling crores of advertisement budget will have their own “correct” experience. 99.99% people / small businesses are not relevant in that.

[Google adwords in my experience or observation; is very costly, and not at all effective. **Adwords is absolute waste of money**. **Facebook** in contrast maintains lots of connections, the visitors repeat of their own, so much more persistent.]

As per my perception; Advertisement is not a communication, at all. It is an enabler, so that **if someone searches**, then can find the links / details quickly. Only those who search, if they get some details, of something; earlier than another; the former has higher chance being considered.

[Did you notice that top 50 or 100 Management Gurus, and / or “Best selling Management Books“ are not Indian]

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Science is closely related to Technology. I personally cannot distinguish.

3D Printing was started by Chuck Hull

As of 2016 (apart from Lakhs of Industrial Applications) Body-parts are being 3D printed

See https://www.youtube.com/watch?v=a1Ikv3yHs0w

And https://www.youtube.com/watch?v=_RO5DSIB1GE

Xenotransplantation

https://www.youtube.com/watch?v=6rKUBBjaa0g

https://www.youtube.com/watch?v=qFQo28AahAE

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Artificial Blood

Since 1990s various kinds of Artificial Blood has been made. I read many reports! Research to improve is always on.

https://www.youtube.com/watch?v=9I7oUuZBG4c

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Artificial Photosynthesis or Chlorophyll

https://www.youtube.com/watch?v=hU-T0ht2OdQ

https://www.youtube.com/watch?v=N8LHqoNber4

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Nanotechnology

https://www.youtube.com/watch?v=xlYIex2TF5g

https://www.youtube.com/watch?v=7hRjhxi2uL0

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Metamaterials

https://www.youtube.com/watch?v=taSfueSfmag

https://www.youtube.com/watch?v=26J5n_8_6TQ

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Molecular Motors

https://www.youtube.com/watch?v=WH5rwsu5tzI

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Quantum Computer

https://www.youtube.com/watch?v=0dXNmbiGPS4

https://www.youtube.com/watch?v=u9zx7QOKPno

For list of emerging Technologies see

https://en.wikipedia.org/wiki/List_of_emerging_technologies

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Bio-batteries: creating energy from bacteria (or Microbial Fuel Cell)

Research reported by Dr Tom Clarke's team at the University of East Anglia's Department of Biological Sciences has shown how thousands of tiny molecular wires embedded in the surface of a bacterium called Shewanella oneidensis can directly transmit an electric current to inorganic minerals such as iron and manganese oxides, or the surface of electrodes. The phenomenon, known as direct extracellular electron transfer (DEET), occurs because of the way that some bacteria living in environments lacking oxygen export electrons that are generated through their respiratory cycle. Examples include Shewanella, and some species of another bacterium known as Geobacter.

See http://eandt.theiet.org/magazine/2013/07/growing-power.cfm

Regarding Indian Scientists https://journosdiary.com/2016/09/10/iisc-india-bacteria-power-tiny-engine/

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Communication in trees

Trees, it turns out, have a completely different way of communicating: they use scent. Four decades ago, scientists noticed something on the African savannah. The giraffes there were feeding on umbrella thorn acacias, and the trees didn't like this one bit. It took the acacias mere minutes to start pumping toxic substances into their leaves to rid themselves of the large herbivores. The giraffes got the message and moved on to other trees in the vicinity. But did they move on to trees close by? No, for the time being, they walked right by a few trees and resumed their meal only when they had moved about 100 yards away.

The acacia trees that were being eaten gave off a warning gas (specifically, ethylene) that signaled to neighbouring trees of the same species that a crisis was at hand. Right away, all the forewarned trees also pumped toxins into their leaves to prepare themselves. The giraffes were wise to this game and therefore moved farther away to a part of the savannah where they could find trees that were oblivious to what was going on. Or else they moved upwind. For the scent messages were carried to nearby trees on the breeze, and if the animals walked upwind, they could find acacias close by that had no idea the giraffes were there.

This ability to produce different compounds is another feature that helps trees fend off attack for a while. When it comes to some species of insects, trees can accurately identify which bad guys they are up against. The saliva of each species is different, and the tree can match the saliva to the insect. Indeed, the match can be so precise that the tree can release pheromones that summon specific beneficial predators. The beneficial predators help the

tree by eagerly devouring the insects that are bothering them. For example, elms and pines call on small parasitic wasps that lay their eggs inside leaf-eating caterpillars. As the wasp larvae develop, they devour the larger caterpillars bit by bit from the inside out. Not a nice way to die. The result, however, is that the trees are saved from bothersome pests and can keep growing with no further damage. The fact that trees can recognize saliva is, incidentally, evidence for yet another skill they must have. For if they can identify saliva, they must also have a sense of taste.

A drawback of scent compounds is that they disperse quickly in the air. Often they can only be detected within a range of about 100 yards. Quick dispersal, however, also has advantages. As the transmission of signals inside the tree is very slow, a tree can cover long distances much more quickly through the air if it wants to warn distant parts of its own structure that danger lurks. A specialized distress call is not always necessary when a tree needs to mount a defence against insects. The animal world simply registers the tree's basic chemical alarm call. It then knows some kind of attack is taking place and predatory species should mobilize. Whoever is hungry for the kinds of critters that attack trees just can't stay away.

Trees can also mount their own defence. Oaks, for example, carry bitter, toxic tannins in their bark and leaves. These either kill chewing insects outright or at least affect the leaves' taste to such an extent that instead of being deliciously crunchy, they become biliously bitter. Willows produce the defensive compound salicylic acid, which works in much the same way. But not on us. Salicylic acid is a precursor of aspirin, and tea made from willow bark can relieve headaches and bring down fevers. Such defence mechanisms, of course, take time. Therefore, a combined approach is crucially important for arboreal early-warning systems.

Trees also warn each other using chemical signals sent through the fungal networks around their root tips. which operate no matter what the weather. Surprisingly, news bulletins are sent via the roots not only by means of chemical compounds but also by means of electrical impulses that travel at the speed of a third of an inch per second. In comparison with our bodies, it is, admittedly, extremely slow. However, there are species in the animal kingdom, such as jellyfish and worms, whose nervous systems conduct impulses at a similar speed. Once the latest news has been broadcast, all oaks in the area promptly pump tannins through their veins.

Tree roots extend a long way, more than twice the spread of the crown. So the root systems of neighbouring trees inevitably intersect and grow into one another—though there are always some exceptions. Even in a forest, there are loners, would-be hermits who want little to do with others. Can such antisocial trees block alarm calls simply by not participating? Luckily, they can't. For usually **there are fungi present that act as intermediaries to guarantee quick dissemination of news**. These fungi operate like fibre-optic Internet cables. Their thin filaments penetrate the ground, weaving through it in almost unbelievable density. One teaspoon of forest soil contains many miles of these 'hyphae'. Over centuries, a single fungus can cover many square miles and network an entire forest. The fungal connections transmit signals from one tree to the next, helping the trees exchange news about insects, drought,

and other dangers. Science has adopted a term first coined by the journal Nature for **Simard**'s discovery of the 'wood wide web' pervading our forests. What and how much information is exchanged are subjects we have only just begun to research. For instance, **Suzzane Simard** discovered that **different tree species are in contact with one another**, even when they regard each other as competitors. And the fungi are pursuing their own agendas and appear to be very much in favour of conciliation and equitable distribution of information and resources.

If trees are weakened, it could be that they lose their conversational skills along with their ability to defend themselves. Otherwise, it's difficult to explain why insect pests specifically seek out trees whose health is already compromised. It's conceivable that to do this, insects listen to trees' urgent chemical warnings, and then test trees that don't pass the message on by taking a bite out of their leaves or bark. A tree's silence could be because of a serious illness or, perhaps, the loss of its fungal network, which would leave the tree completely cut off from the latest news. The tree no longer registers approaching disaster, and the doors are open for the caterpillar and beetle buffet. The loners I just mentioned are similarly susceptible—they might look healthy, but they have no idea what is going on around them.

In the symbiotic community of the forest, not only trees but also shrubs and grasses—and possibly all plant species—exchange information this way. However, when we step into farm fields, the vegetation becomes very quiet. Thanks to selective breeding, our cultivated plants have, for the most part, lost their ability to communicate above or below ground—you could say they are deaf and dumb—and therefore they are easy prey for insect pests. That is one reason why modern agriculture uses so many pesticides. Perhaps farmers can learn from the forests and breed a little more wildness back into their grain and potatoes so that they'll be more talkative in the future...

To decide if trees are silent … researchers substitute grain seedlings because they are easier to handle. They started listening, and it didn't take them long to discover that their measuring apparatus was registering roots crackling quietly at a frequency of 220 hertz. Crackling roots? That doesn't necessarily mean anything. After all, even dead wood crackles when it's burned in a stove. But the noises discovered in the laboratory caused the researchers to sit up and pay attention. For the roots of seedlings not directly involved in the experiment reacted. Whenever the seedlings' roots were exposed to a crackling at 220 hertz, they oriented their tips in that direction. That means the grasses were registering this frequency, so it makes sense to say they 'heard' it.

It is well known that Music Played near trees help them grow faster. There are many commercial products claiming quicker growth in farms.

After reading all these some may imagine that this is what is happening in jungles ….

-

The list can go on forever. Students can read and learn more of their own…

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Even though Indian Rocket could send 104 Satellites to space in one go, Indian prefer to do the following ...


[In Febrary 2017 India launched 104 satellites]

Every Puja is remnant of "Caste System". Think … Who are performing the Pujas ? What is the Qualification of the Pujari ? What is his effectiveness ? How are the Pujaris chosen ?

Russian Dnepr rocket had sent **37 satellites** to Space, without Pujas !

I have met lot of people who think, that "Global Warming" is happening due to Cars, or because of burning Plastics …

In our atmosphere close to 1% is Argon, while only 0.04% in CO_2

Half of the world's oxygen is produced via phytoplankton photosynthesis. The other half is produced via photosynthesis on land by trees, shrubs, grasses, and other plants.

See http://news.nationalgeographic.com/news/2004/06/0607_040607_phytoplankton.html

See http://skmclasses.kinja.com/global-warming-is-not-due-to-human-activity-1761784651

My students and the readers of this book must know that; over the past 250 years, humans have added just one part of CO2 in 10,000 to the atmosphere. One volcanic cough can do this in a day. https://www.skepticalscience.com/print.php?r=50

http://time.com/3698572/science-maya-tolstoy-geophysical-research-letters-volcanoes-climate-change/

Temperature-Sea Levels-CO2-etc always have been fluctuating over ages - Global Warming

See https://archive.org/details/TemperatureSeaLevelsCO2EtcAlwaysHaveBeenFluctuatingOverAgesGlobalWarming

Know about the Giants of Science from Videos

https://archive.org/details/CasimirPolderDaviesUnruhBELLAspectGalileoMosleyChadwickFeynmanSchrodinger

https://www.youtube.com/watch?v=ecQazN9Z24w

Long back a Professor had advised me, to read all issues of Scientific American; say from 1920s, or as old as possible; to learn Physics. I did listen to him and read all old copies, that were available in the Library. Now in the net it is much easier for Students, to get the copies.

See https://archive.org/search.php?query="Scientific%20American"

In 1999 there was a Special Issue on Men

See https://archive.org/stream/ScientificAmericanspEd-Vol10No2-Men-1999#page/n1/mode/2up

Preface for Mathematics

Mathematics is about doing things intelligently. To highlight this let us take a simple example …

What is the 352 th term of the Following Fibonacci Sequence ?

1,1,2,3,5,8,…. ?

A naïve person will try to write the sequence upto 352 terms :-D

But an intelligent person will derive the n th term of the sequence as

$$f_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$ where f(0) is defined as 1 the first term

Even though every student of Standard 10 or 11 learns about Arithmetic Progression, Geometric Progression, AGP (ArithmeticoGeometric Progressions); their n th term, sum etc; but the most important idea of **counting without counting** does not come through.

Let me explain this with another surprising example !

Tell an **Engineer or a smart High school student** who knows Programming; to write a program, which will find Factorial of a Natural number.

Most probably you will get a pseudo code as follows (doesn't matter in which language the program is being written. Such as FORTRAN or C or Java etc)

Define F(0) = 1

Tell to enter n as a natural number less than 51

Get n

Check if n entered is Natural number or not. If not Natural number then tell to enter again.

if (n > 1)

[assuming n was entered as 50

{

For(n==1;n==50;n++) [depending on the grammar of the language]

F(n) = n X F(n-1) [idea of recursive functions]

Some halt condition such as n should not exceed 50

Display F(n);

}

Most people will be surprised as to why I am criticizing this !

Read on …

There is no logic in calculating the Factorials repeatedly. Factorial 50 or even Factorial 30 is very long calculation intensive. If it is only factorial of 50 numbers each should be calculated and kept in a lookup table. In an ATM while doing a transaction with a Debit Card or Credit Card all details are looked up in fraction of a second from several Million data. Looking up from a Table is very quick or easy. So even if we need Factorial of 500 numbers **we should peacefully calculate them separately** and keep them in a Lookup table.

The Engineers and the Students will never tell you about the “Most intelligent” Mathematics formula of Stirling ‘s Equation.

Stirling's formula

The Scottish mathematician **James Stirling** (1692-1770), educated at Glasgow and later expelled from his Oxford college for his Jacobite sympathies, was able to show that with $\mathbf{k = e}$ the sequence $(\mathbf{x_n})$

$$x_n = \frac{n!\,k^n}{n^{n+\frac{1}{2}}}$$

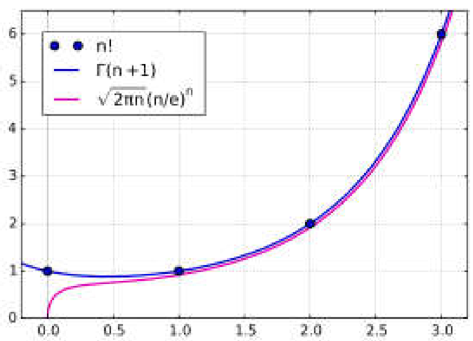
with converges to $\sqrt{(2\pi)}$.

This means that for large $\mathbf{n}$ we get the approximation

$$n! = \sqrt{(2\pi n)}\,(n/e)^n$$

This is usually called ***Stirling's formula*** – although in fact it may have been known earlier to the English mathematician **de Moivre**.

So you see again we can zoom into the n th term straight away. The formula gives more accurate results as we increase the number n



Next I discuss about Logic, Fallacies, Contradictions in Mathematics, Framework in Maths etc

Before **Friedrich Ludwig Gottlob Frege** Logic was only discussed in English (mostly).

https://en.wikipedia.org/wiki/Gottlob_Frege

Formal logic with Mathematical terms notations rules followed.

But came Cantor. He assumed infinity is possible; and just this assumption led to all kinds of Bigger infinity, lesser infinity, contradictory statements and absurdities etc. Ruckus (The act of making a noisy disturbance) got a new Definition in Mathematical? Work? of Cantor.

Stop bothering about Cantor and his “work?”; get a compact useful description of Mathematics at https://www.youtube.com/watch?v=OmJ-4B-mS-Y

So Mathematics is a very flexible Meta-Language for solving all sorts of problems. As per requirements of the situation / problem we model it appropriately and get our results.

Dr. Navsky has summarized various aspects of History of Mathematics very well in his blog.

Mathematics is totally deductive. (http://panarrans.blogspot.in/)

But deductive from where ?

It got to start from some place.

This starting point in mathematics are the most primitive assumptions called the axioms.

Its axiomatic origin was giving rise to lots of flaws and inconsistencies in mathematics that was giving men like David Hilbert nightmares.

Anything could be flawed, but not mathematics.

That was the idea then as still many of us are led to believe so even today.

David Hilbert was an optimist who thought that with rigor and determination, these little flaws would be ironed out and mathematics could be built up on solid axiomatic foundations.

Russell had tried to smoothen things out and yet while working on his Principia he had discovered a paradox in set theory that we had discussed in one of our bedtime stories.

Little did Hilbert know that his dreams were about to be shattered for good by a little know man then.

On September 7, 1930 Kurt Gödel had unleashed the first hint of his incompleteness theorems in a round table gathering of the Conference of Epistemology.

All of my bedtime stories that I have narrated, starting from Cantor and continuing up to Hilbert, are linked and form the basis for the thesis that Kurt Gödel chose to work upon in 1929.

If you recall, it was the 1928 Hilbert-Ackermann book “Principles of Mathematical Logic”, an introduction to the first-order logic where the famous question was posed:

“Are the axioms of a formal system sufficient to derive every statement that is true in all models of the system?”

Gödel took this question very seriously and decided to write a thesis on it under the guidance of Hans Hahn.

In all likelihood, all his contributions to mathematics had not given him as much recognition as the fact that he had Kurt Gödel as his student.

Hans Han also contributed indirectly to the founding of the Vienna Circle.

It did not take long for Gödel to make the breakthrough.

Within the year of 1929, Gödel defended his doctoral dissertation that he called the completeness theorem.

The year following the dissertation, on November 17, 1930 the Vienna Academy of Science published the incompleteness theorems under the title:

“On Formally Undecidable Propositions of Principia Mathematica and Related Systems.”

The language was German and the journal was Monatshefte für Mathematik.

-

Professor Wildberger gave a new Foundation to Mathematics by removing infinities.

https://www.youtube.com/watch?v=91c5Ti6Ddio&list=PLEqMsHbeR_kw-XQ-1oRnHaJHCs321Solz

-

The most important ideas that I will like to draw your attention to are …

Genetic Algorithms, Fuzzy Logic, Neural Networks and other self learning Networks, Artificial Intelligence related Algorithms, Interval Mathematics, Mandelbrot Mathematics, Chaos theory, Feigenbaum Analysis, Numerical Techniques.

Since 1986 we can practically solve every problem that we need to.

I read many books on mathematics. The Authors are showcasing the following ideas as important …

- idea of “ e “ and its uses

- idea of imaginary number and to use them to solve real life problems

- Statistics and Statistical Techniques of solving Problems

- Perfect numbers, Fibonacci numbers, Golden rectangles, Pascal's triangle, Euclid's algorithm, Logic, Proof, Sets, Calculus, Constructions, Curves, Topology, Dimension, Fractals, Chaos, The parallel postulate, Discrete geometry, Graphs, The four-colour problem, Probability, Bayes's theory, The birthday problem, Distributions, The normal curve, Connecting data, Genetics, Groups, Matrices, Codes, Advanced counting, Magic squares, Latin squares, The diet problem, The travelling salesperson, Game theory, Relativity, Fermat's last theorem, The Riemann hypothesis

The List also includes Primes numbers, Pi, Zero, Squares and Square Root etc. I personally do not see much use of Prime numbers.

From my side I will choose Genetic Algorithms, Fuzzy Logic, Neural networks, and Interval Mathematics.

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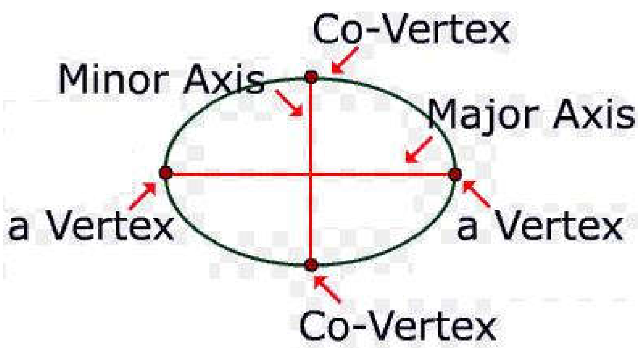
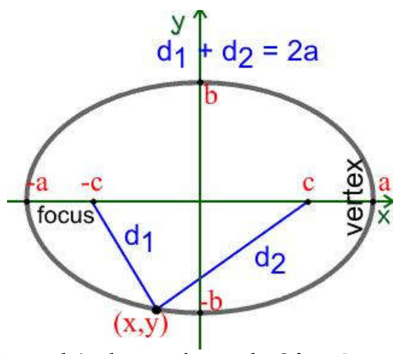
Radius of Curvature of an Ellipse

Let us learn a few basic facts about Ellipse



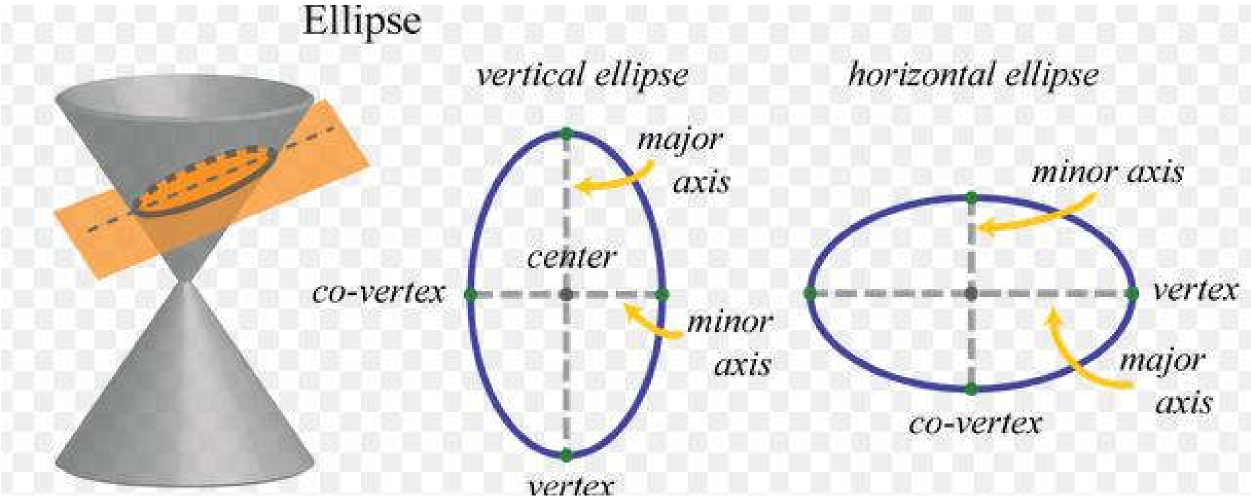


The major diameter is sometimes called the major axis. Let this have length 2*a. Let the minor diameter (minor axis) have length 2*b. We often say that a is the "semimajor axis" and that b is the "semiminor axis." Then the eccentricity of the ellipse is

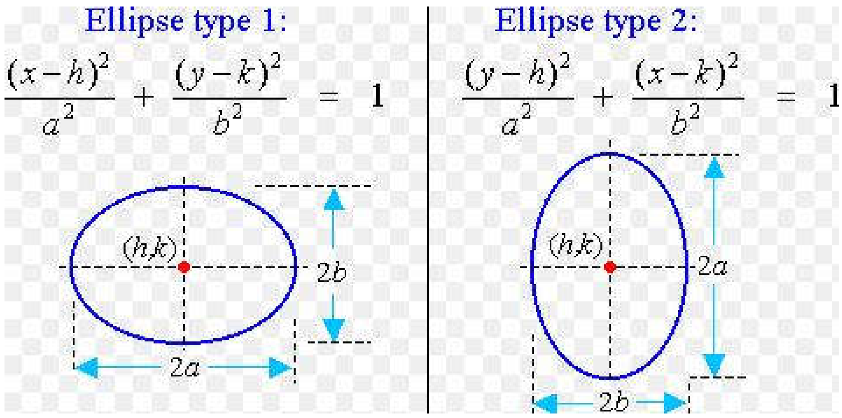
e = sqrt(a^2 - b^2) / a

This should be a number between 0 and 1. The distance from the center to the foci is c = a*e = sqrt(a^2 - b^2).

An Ellipse can be visualized as a Conic Section



While the equations of the Ellipse is given as shown below



In these (h, k) is the center of the Ellipse. For the ellipse a > b

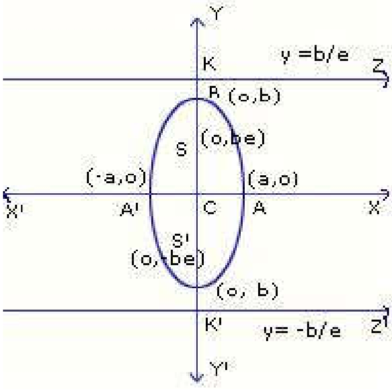
While if b > a then the calculations are shown below

$$a^2 = b^2(1-e^2)$$

$$\Rightarrow\ e^2 = 1-a^2/b^2$$

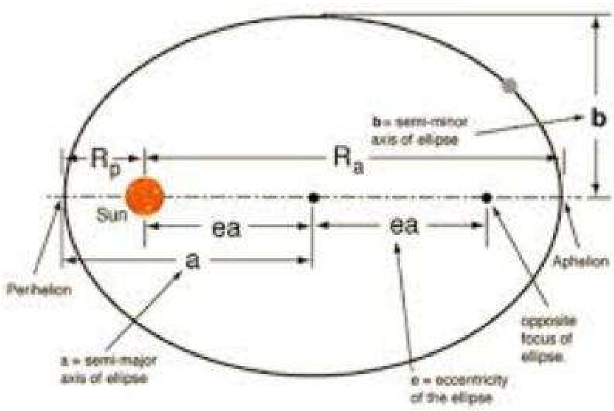
$$\Rightarrow\ e = \sqrt{1-\frac{a^2}{b^2}}$$

The shape of the ellipse is given below.



Now, this tells you where the foci are--they both lie on the major axis, at a distance of c from the center of the ellipse. But if you are trying to calculate the radius of curvature at the point y end (where the major axis intersects the ellipse), you can work directly from the formula for the ellipse:

```
x^2   y^2
--- + --- = 1          this assumes that the coordinate system
a^2   b^2               has the origin at the ellipse's center.
```



$R_a = a(1+e) \quad R_p = a(1-e)$

We need the radius of curvature at (x,y) = (a,0).

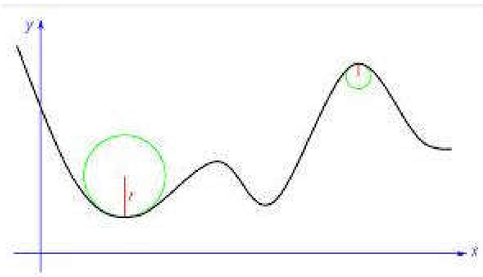
This is actually a question that is found using calculus:

$$\text{radius of curvature } R = \frac{[(x')^2 + (y')^2]^{(3/2)}}{x'y'' - y'x''}$$

Or it can be written as shown below

$$R = \left| \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}} \right|$$

$$\left(R = \frac{b^2}{a}\right)$$



where the x and y coordinates can be parameterized as

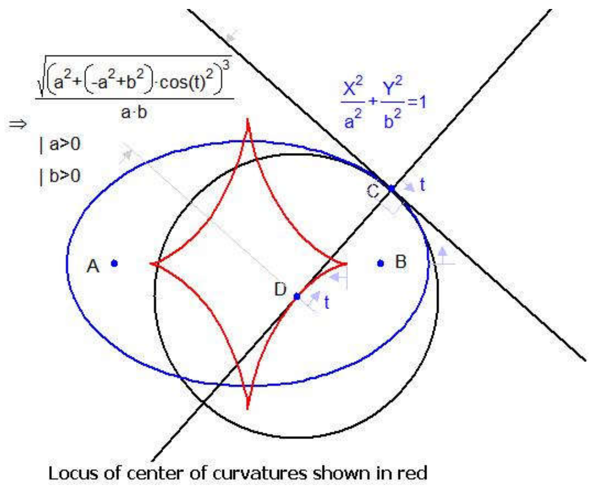
x(t) = a cos(t), y(t) = b sin(t)
x'(t) = -a sin(t), y'(t) = b cos(t)
x"(t) = -a cos(t), y"(t) = -b sin(t)
and plugging these into the expression for R gives us

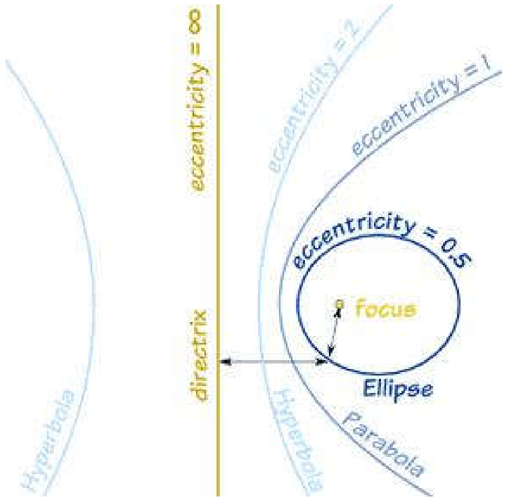
$$R = \frac{[a^2 \sin^2(t) + b^2 \cos^2(t)]^{(3/2)}}{ab\,[\sin^2(t) + \cos^2(t)]}$$

The point (x,y) = (a,0) occurs when t=0, so we plug t=0 into this expression to find the maximum possible radius of your cutting tool:

$$R(a,0) = \frac{[0 + (b^2)*1]^{(3/2)}}{a*b*1} = \frac{b^3}{a*b} = b^2/a$$

You can see that if b/a is small(i.e., the ellipse is very squashed), then the radius of curvature is b*(b/a), so that it is smaller than the semiminor axis b. And if b=a, then the ellipse is actually a circle, and it has radius of curvature equal to a, as required.





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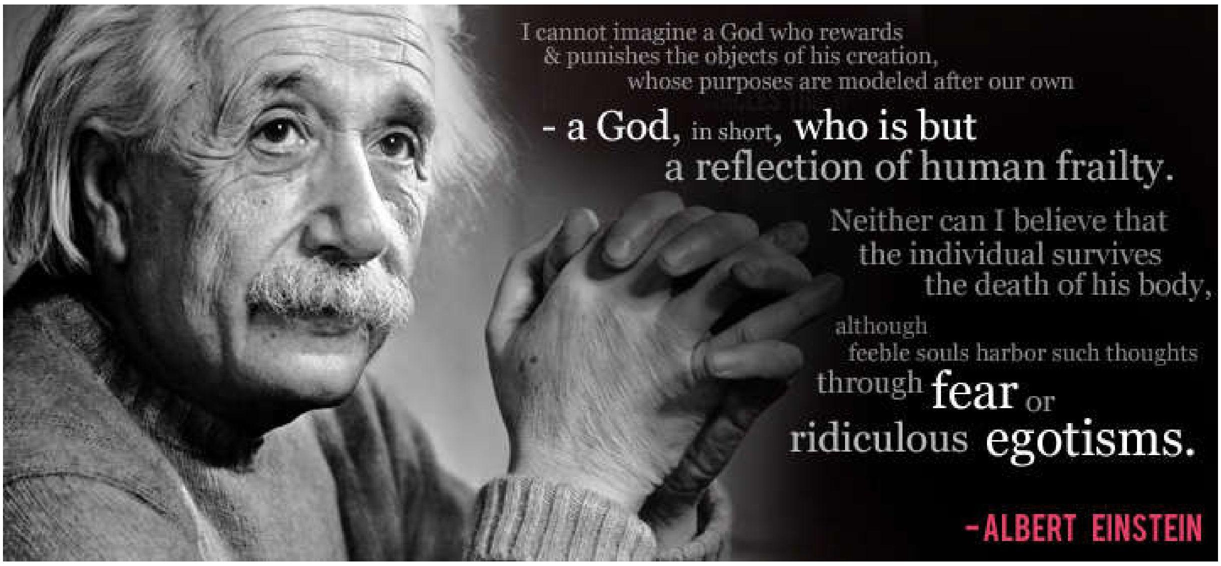
Motion in Variable Acceleration

Example – A body is Decelerating at Proportional to square of the distance …

https://archive.org/details/ABodyIsDeceleratingProportionalToXSquareWhatWillBeVelocoty

Physics is very closely related to Mathematics. So little bit on Physics …



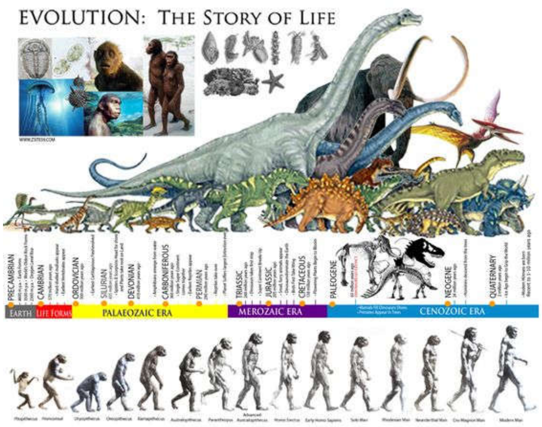
(Apart from Millions of smart people) **Several Nobel Laureates were Atheists**.

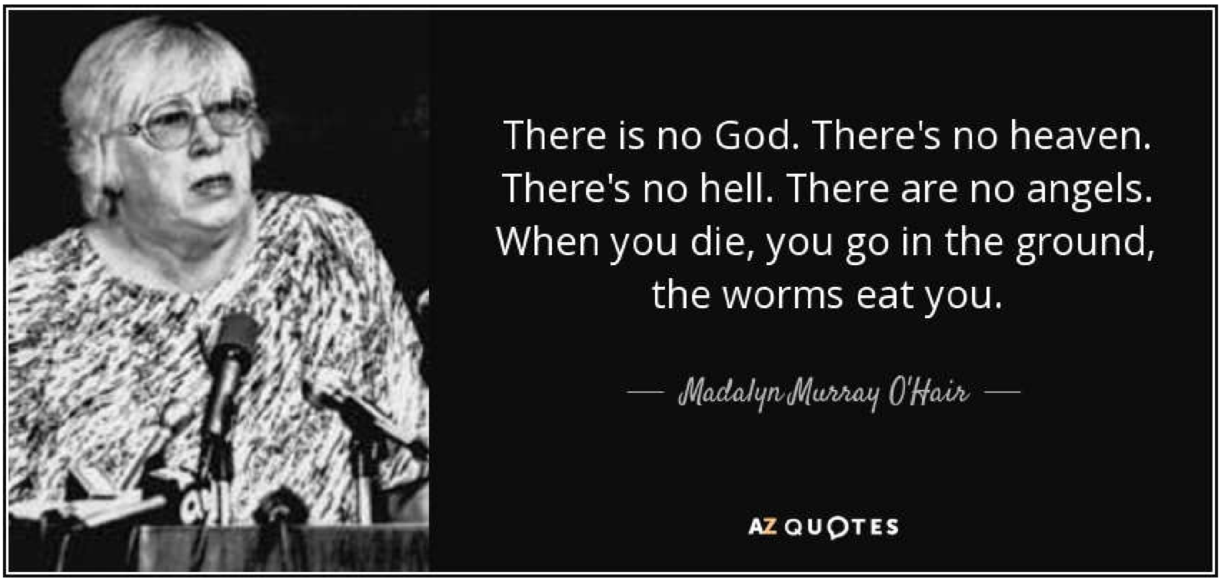
Some famous examples have been Albert Einstein (1921), Richard Feynman (1965), **Erwin Schrödinger** (1933), Paul Dirac (1933), **Lawrence M. Krauss** (2011), Niels Bohr (1922), Peter Higgs (2013), John Bardeen (**The only person receiving the Physics Nobel prize twice. 1956, 1972**), Frederick Sanger (**The only person receiving the Chemistry prize twice. 1958, 1980**), Marie Curie (1903, 1911), Frédéric Joliot-Curie and Irène Joliot-Curie (1935), Milton Friedman (1976), John Harsanyi (1994), Friedrich Hayek (1974), John Forbes Nash, Jr. (1994), **Amartya Sen** (1998), **Subrahmanyan Chandrasekhar** (1983), Enrico Fermi (1938), **C. V. Raman** (1930), Eugene Wigner (1963), Steven Weinberg (1979), Chen-Ning Yang (1957) etc

A bigger (incomplete) list can be seen at
https://en.wikipedia.org/wiki/List_of_nonreligious_Nobel_laureates

Important Scientists http://www.physicsoftheuniverse.com/scientists.html

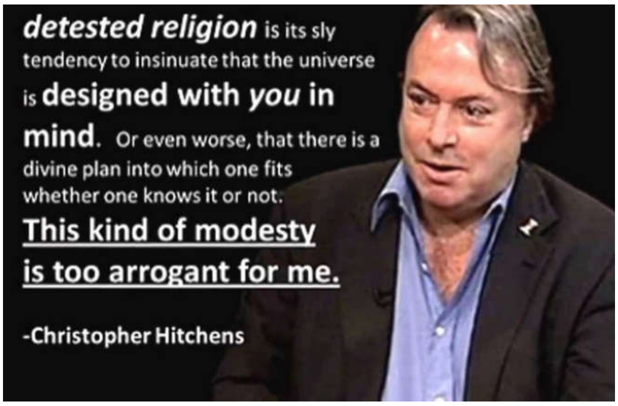
http://www.physicsoftheuniverse.com/facts.html

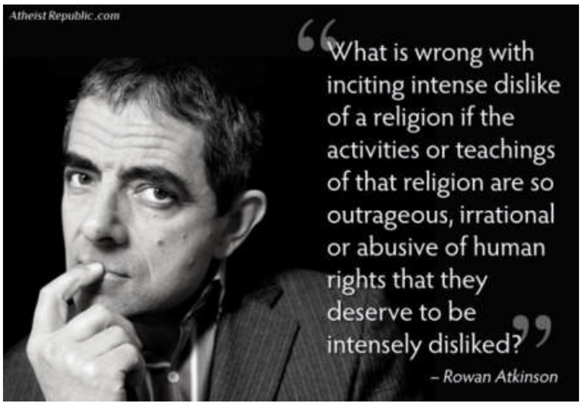




(When the body is burnt, oxides are the ash. The gases and water vapor spread in the air)

My **personal favorites** (among these Atheists) are **Richard Feynman**, **Peter Higgs**, **Lawrence Krauss**.





Richard Feynman openly laughed (Publicly and in class) about Gods, Fairies etc. see https://www.youtube.com/watch?v=j3mhkYbznBk

and https://www.youtube.com/results?search_query=Richard+Feynman

https://www.youtube.com/watch?v=JzWzLyGuPRY&list=PL_6G_2_0gFDqFjq4gZbmDvJT4bnvnNwr-

Approx 200 years ago; around 1800, Pierre-Simon Laplace developed a **new branch of Mathematics**, **Perturbation theory**. Perturbation theory was investigated by the classical scholars — Laplace, Poisson, Gauss — as a result of which the computations could be performed with a **very high accuracy**. The discovery of the planet Neptune in 1848 by Urbain Le Verrier, based on the deviations in motion of the planet Uranus (he sent the coordinates to Johann Gottfried Galle who successfully observed Neptune through his telescope), represented a triumph of perturbation theory.

Laplace was one the first persons who did **not** see or use "hand of God" (or role of God) to explain something. Newton's Gravitation equations for Two masses, were not enough to explain stability of multibody, rather multi planet and Sun system. **Perturbation Theory could accommodate cumulative effects of many small forces.**

While talking to Napoleon,(discussing the theory); Laplace said, (about God) "**that**" (God) hypothesis is **not** needed.

http://www.naturalhistorymag.com/universe/211420/the-perimeter-of-ignorance

https://en.wikipedia.org/wiki/Perturbation_theory

https://en.wikipedia.org/wiki/Pierre-Simon_Laplace

Peter Higgs was very unhappy about " Higgs Boson " being called "G..(I don't want to name this) Particle". **Stupid Journalists**, Media, and dumb people kept repeating that word, and **Peter requested to refrain from using this word.** Now for Madala Boson also the **Stupid Journalists**, Media, and dumb people are using that same G word.

Lawrence Krauss openly laughs and ridicules the Theists or any non-Atheists. The crap of Agnosticism does not work with me or Krauss.

Empty Space is not empty. Mass of Proton, Neutron is not sum of masses of Quarks

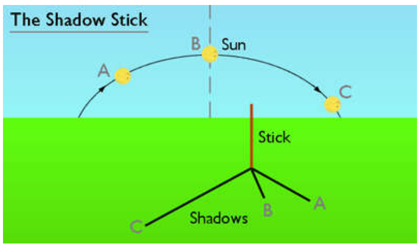
https://archive.org/details/EmptySpaceIsNotEmptyMassOfProtonNeutronIsNotSumOfMassesOfQuarks

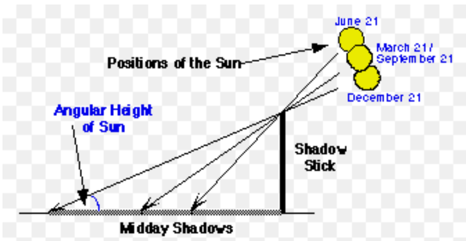
We are in Modern Times. I am lucky to learn the correct things quite early in my life, in a so " peaceful " society. When I was in standard 9, (in early 1980s), I was writing a book on Atheism. I was convinced to understand, learn, and imbibe the correct approach and knowledge.

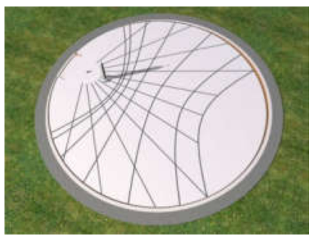
But that was not the case previously. Copernicus used to discuss and explain people widely and randomly, that Earth is rotating around the Sun, and it is not a Geocentric" universe. Nicolaus Copernicus had to waste lot of time arguing, fighting and convincing the stupids.

Measuring something, which is very slow; is very difficult. I have asked lot of "educated / engineer / Software or IT (senior position) Parents" that " How do we know that Earth is moving around the Sun in 365 days or say 365.242196 days " ? **Believe me I never got an answer**. The Modern iPad / smartphone **community in general does not know how 365.24 days was measured almost thousand years ago** !

A metal triangle was set at top of buildings (Mosques or churches) and the position of the shadow was marked at a particular time. Say 8 AM each day. The position of the shadow varied each day. It was seen that after 365 days the shadow matched the position but after sometime, not exactly at 8 AM but after a few hours (approx 6 hours) so at around 2 PM or slightly before.







See details of this at http://blog.world-mysteries.com/science/ancient-timekeepers-part-2-observing-the-sky/

http://blog.world-mysteries.com/science/ancient-timekeepers-part4-calendars/

See the video https://www.youtube.com/watch?v=IhqzW97_47w

https://thecuriousastronomer.wordpress.com/2012/10/

Much tougher questions are " How many different kind of years do we have ? "

Or " What is the difference between ' **Sidereal year** ' and ' **Tropical year** ' "

Meteors were coming from sky. These were called ' shooting stars '. Meteors often had Iron in them. Sidero is a combining form meaning "star," "constellation," used in the formation of compound words. Greeks used the word siderolite for Iron. Next the source of meteors; the sky itself was named the same. As year was measured using objects from sky; Sun and shadows; the year was named a " **Sidereal Year** "

To avoid embarrassing people; I don't ask ….

See the answers in https://www.youtube.com/watch?v=cGjP3vAZGa4

https://www.youtube.com/watch?v=qgsrVyW53DY

It took many centuries to introduce the leap year corrections. A century is a leap year only if divisible by 400 and not the rule of divisible by 4. Year 1900 was not a Leap year. But year 2000 was. I have met computer Science guys who are aware that Microsoft Database SQL-server do not accept some old dates, while Oracle database does not accept some specific dates of the past. But none whom I met knew the detailed or actual reasons.

See https://zookeepersblog.wordpress.com/everyone-must-know-about-the-calendar/

" How do you prove that day and night is happening due to rotation of Earth around it own axis in contrast to Sun is rotating around Earth " ?

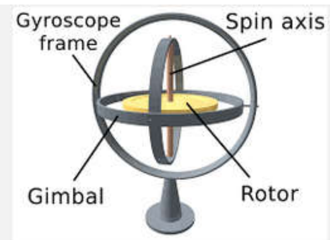
See http://www.visual-arts-cork.com/prehistoric-art-timeline.htm

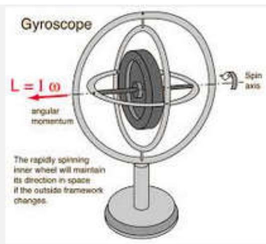
No student from Bangalore, whom I met, answered this. Though conservation of Angular Momentum is in course. (I am being polite) Hardly met any parent who knew the explanation. See https://www.youtube.com/watch?v=iqpV1236_Q0

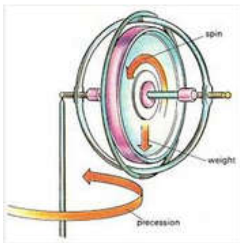
And https://www.youtube.com/results?q=Foucault%27s+pendulum

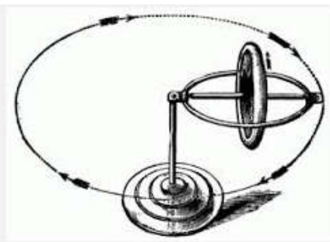
What about Gyroscopes ?

Approx 300 year back around 1750 the gyroscopes were made.









History of Gyroscope http://www.gyroscopes.org/history.asp

See about Gyroscopes in https://www.youtube.com/watch?v=cquvA_IpEsA

https://www.youtube.com/watch?v=awXTZt86gz0

https://www.youtube.com/watch?v=zbdrqpXb-fY

https://www.youtube.com/watch?v=N92FYHHT1qM

https://en.wikipedia.org/wiki/Earth%27s_orbit

https://www.youtube.com/watch?v=ZcWsjlGPPFQ

Must see
https://www.youtube.com/watch?v=SnMmBmzoVQc&list=PL68IJE2PG4AnVVMS7WvOYbJDmqf4umHG1

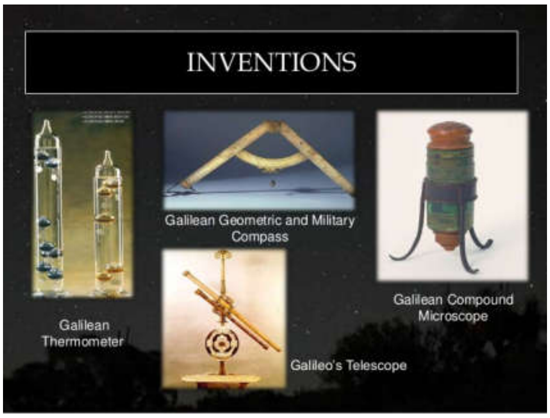
Must know …

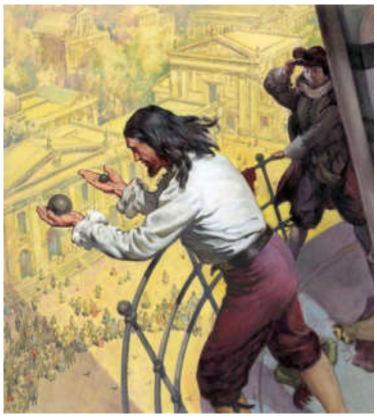
https://www.youtube.com/watch?v=zjV3PQ4f6IM&list=PLTve54sz-eh_P29Sbbv_j3bC97OFaArOd

Tyco Brahe took the boldest step to create the " Foundation of Science ". Experiments or "Double blind experimental observations" are the supreme. The **Theory follows the experimental verification**.

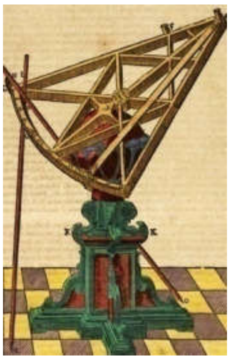
[There are some universities who award M.Sc in Psychology. A psychologist may **guess** something .... **But that is not reality or truth**. Till something is experimentally verified it remains as a Perception. Truth is known only after experiments. Because the subject Psychology; completely stands of experimental verification; so the Master in Science degree.]


Galileo Galilei
(1564-1642)



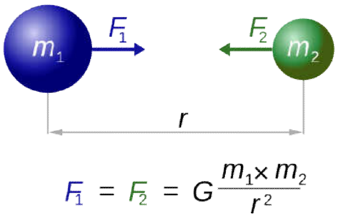


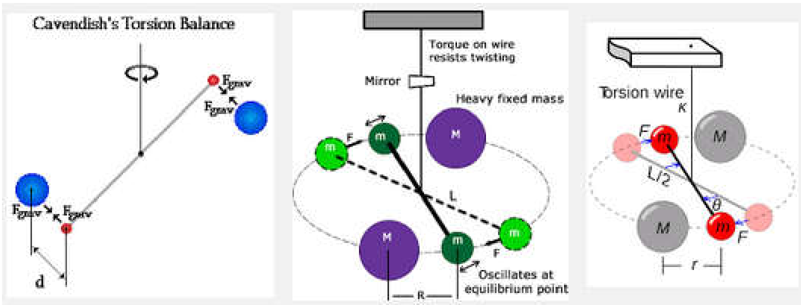
Galileo was the first person who wanted to experimentally verify the speed of light.



Tycho decided to observe the skies (around 1573). In those days sky was synonymous to God. He had the courage to go to the King to ask for donations to make an observatory. He said to the king that "he wants to observe the Gods and take conclusions ". Salute to Tycho's paradigm that even Gods can be observed and conclusions can be drawn.

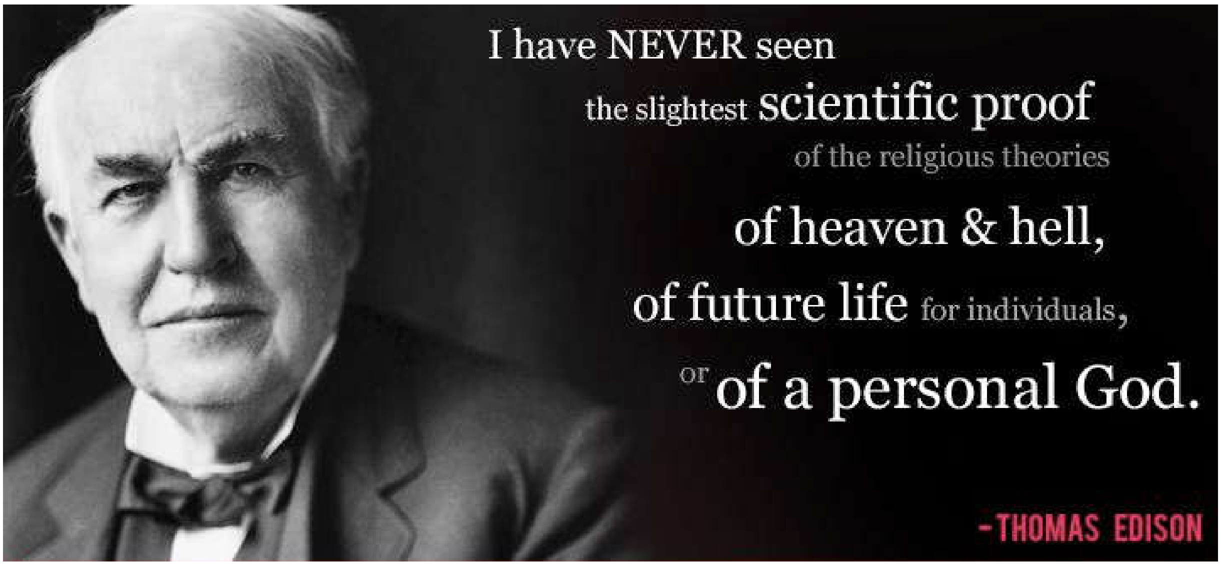
Amazing leap to start Science.





Since those days till now we observed and concluded about Kepler's Laws, Gravitation Laws, We concluded that there was no **Phlogiston** or Flogiston, Cavendish measuring value of G, measuring speed of light, X-Ray, Electromagnetism / Maxwell's equations, Radioactivity, No Aether was " observed " in Michelson Morley's experiments, Protons, Neutrons, General Theory of Relativity, Slowing of clocks at high speed, Bending of space, **Bending of light and gravitational lens**, YDSE, Quantum Mechanics, Ernst **Ruska** designed and built the first **electron microscope**, **Casimir Forces**, Virtual particles and more than 400 kinds of particles, Quarks, Unruh effect (an accelerating thermometer shows higher temperature), Negative Kelvin Temperature, Bose-Einstein condensates, Superconductivity, **Solution to EPR paradox by John Stewart Bell**, Violation of Parity in certain situations - Madam Wu, Yang and Lee, Quantum entanglement in Alain Aspect's Experiments, Black holes, mass of Neutrinos, Caesium Atomic Clocks, Dark Matter, Dark energy, Magnetic Monopole, Gravitational Waves, Nano Materials, Meta Materials, Quantum Computers .....
No God was observed, or no role of God was observed. There is no conspiracy theory going around in Science. Those who want to verify God have to die waiting

... Nothing ever will be reported regarding this illusion.



[Stupids had proposed the phlogiston theory. This was a superseded scientific theory that postulated that a fire-like element called phlogiston is contained within combustible bodies and released during combustion. The name comes from the Ancient Greek φλογιστόν phlogistón (burning up), from φλόξ phlóx (flame).]

In contrast see http://www.americanscientist.org/issues/pub/burn-magnet-burn

Some examples of stupidity to show / explain by contrasts; will be the right approach.

Aristotle used goat urine and Hippocrates recommended pigeon droppings. For what?

As a treatment for baldness. Men have never found baldness an appealing trait, in spite of stories that bald men are sexier. (Stories usually spread by bald men.) Virtually anything that can be done to a bald pate has been tried to stimulate hair growth. The ancient Egyptians were fond of rancid crocodile or hippo fat. If it smelled bad, surely it must do some good. It didn't. Cleopatra experimented with a goo made of ground horse teeth and deer marrow to spur Julius Caesar's dormant hair follicles into action. When this didn't work she traded him in for Mark Antony. During the Victorian era cold tea was brushed on the scalp, followed by citrus juice. In farming areas chickens were persuaded to leave deposits on a bald head and cows to lick it. Electric combs, suction caps and paint thinner have been tried. At a secluded farmhouse in Pennsylvania, Marcella Ferens rakes a glass instrument filled with a purple gas across the head to "sterilize the scalp." Then the subject holds a wire attached to some electrical machine while the operator holds a second wire as she massages the bald area with a secret formula. This forces the formula into the scalp. Some infomercials push shampoos with special emulsifiers to clean follicles as if baldness were due to plugged follicles. Others use jumbled language to promote spray paint to cover bald spots. The truth is that only Rogaine (minoxidil) rubbed on the scalp or Propecia (finasteride) taken orally have shown any effect in growing hair. Even with these the results are not impressive. The Bald Headed Men of America, headquartered appropriately in Morehead, North Carolina, was started when the founder was refused a job because he was bald. They take a different tack. If you want to waste your hormones growing hair....go ahead" Actually this is a wrong statement because it is high levels of dihydrotestosterone that can cause baldness. They are on firmer footing with their slogan. No rugs or drugs.

Aristotle used Goat Urine and Hippocrates recommended Pigeon droppings to cure baldness.

http://dazeinfo.com/2010/06/22/superstitions-across-different-countries-an-overview/

Australians bathed inside rotting whales to 'cure' rheumatism

The Australian National Maritime Museum has revealed that sufferers of rheumatism were once advised to sit inside the festering carcasses of whales in order to relieve their symptoms.

The museum has recently opened a new exhibit in Sydney, which seeks to uncover the diversity, origins and adaptation of whales, charting their development from land mammals to aquatic giants. The exhibition, entitled "Amazing Whales" also looks at the different relationships humans have had with the cetaceans, which includes their apparent medicinal qualities.

Those afflicted with rheumatism were advised to sit inside the belly of a dead whale for approximately 30 hours. If the patient could stay the course and withstand this bizarre practice, they were promised at least 12 months of relief from pain.

http://www.wired.co.uk/article/whale-bath

Weird Bizarre superstitions to cure disease

http://www.historyextra.com/feature/animals/10-historical-superstitions-we-carry-today

http://listverse.com/2013/01/21/10-crazy-cures-for-the-black-death/

Millions of People are making money out of superstitons of Fools

Rebirthing Therapy, Reiki, Energy-Deflecting Golfer Pendant, Maggot Debridement Therapy, Leech Therapy, Beer spas, Ozone Anti-Aging …… the list is very big.

http://webecoist.momtastic.com/2010/07/05/12-most-bizarre-modern-alternative-medical-treatments/

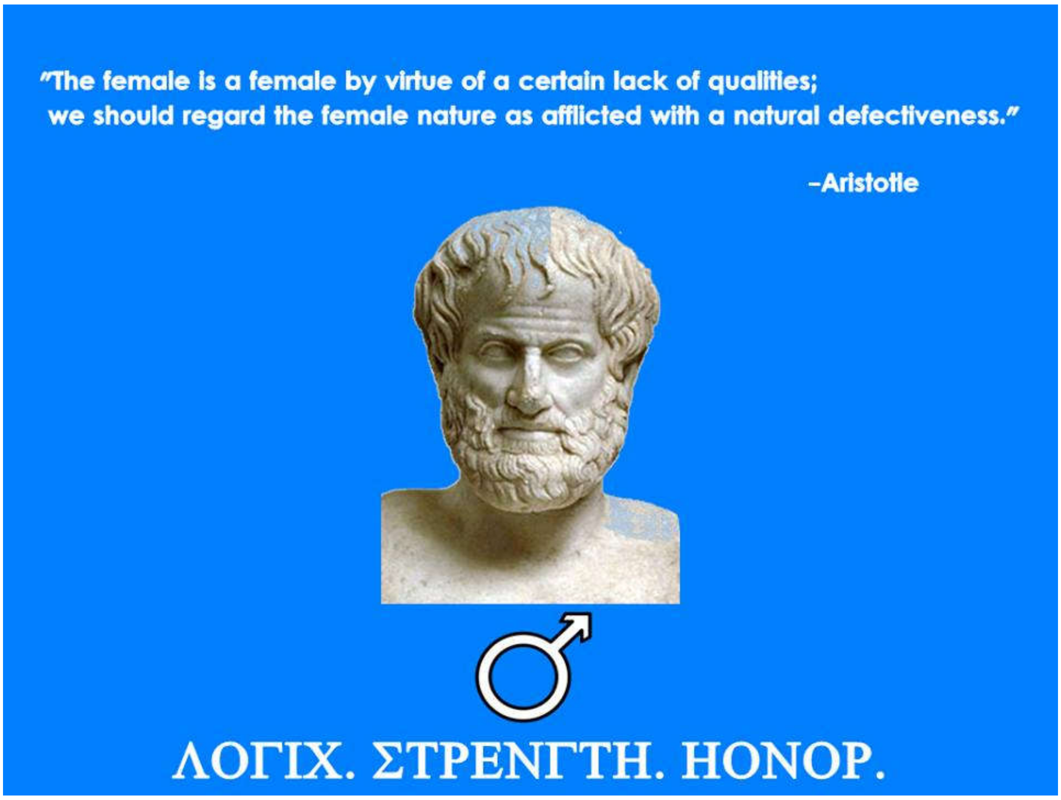
http://oddrandomthoughts.com/strange-and-bizarre-medicine-and-cures/

http://www.stylist.co.uk/life/13-strange-superstitions

So in simple words instead of taking opinions of Stupid Fools, or wasting any time arguing with them ….. Let study science correctly, without bias !

Aristotle is yet Famous, because Girls come to know about his name in school text books. Though not sure why !

Aristotle told at-least one statement correct !



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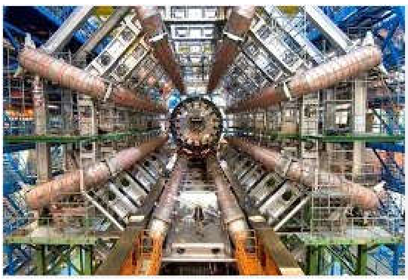
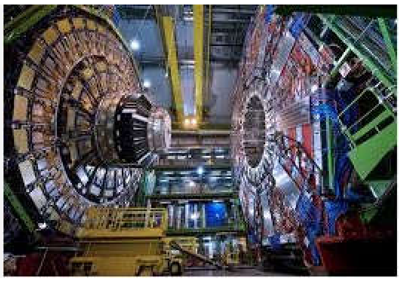
The monkeys in the previous page were all Female Monkeys

Aristotle was not correct (though not sure), Women are not missing anything …. No one is voting for Aristotle.

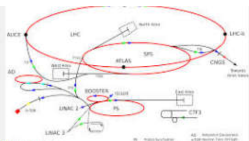
Not wrong as well (though not sure), very difficult to prove either way!

-










Most important physics experiments (**that a certain kind of Apes conducted**) can be seen at

See http://www.explainthatstuff.com/great-physics-experiments.html

http://physics-animations.com/Physics/English/top10.htm

https://en.wikipedia.org/wiki/List_of_experiments

https://www.quora.com/What-are-some-of-the-most-important-experiments-in-physics

Though my list will be as follows -

Michelson–Morley experiment proving there was no Aether, Measurement of e/m then e (charge of electron) and m (mass of electron), Fizeau's method of measuring the speed of light, Moseley 's experiment with X-Rays to discover Protons, Jagadish chandra Bose demonstrating controlled emission / transmission and receiving of Radio waves, Casimir experiments to show Casimir forces of virtual particles, Edington measuring bending of light, Flying atomic clocks in planes and confirming slowing down of time at high speeds, Victor Hess measured Radiation level variation at ground and high up in the atmosphere, Soviet physicist Sergey Vernov was the first to use radiosondes to perform cosmic ray readings with an instrument carried to high altitude by a balloon at heights up to 13.6 km, The proof of time dilation by Muon decay https://debunkingrelativity.com/muons-time-dilation/ , Measurement of Space-time curvature near Earth and thereby the stress–energy tensor (which is related to the distribution and the motion of matter in space) in and near Earth https://en.wikipedia.org/wiki/Gravity_Probe_B , Detecting Gravitational Waves.

[In 1909 Theodor Wulf developed an electrometer, a device to measure the rate of ion production inside a hermetically sealed container, and used it to show higher levels of radiation at the top of the Eiffel Tower than at its base. However, his paper published in Physikalische Zeitschrift was not widely accepted. In 1911 Domenico Pacini observed simultaneous variations of the rate of ionization over a lake, over the sea, and at a depth of 3 meters from the surface. Pacini concluded from the decrease of radioactivity underwater that a certain part of the ionization must be due to sources other than the radioactivity of the Earth. In 1912, Victor Hess carried three enhanced-accuracy Wulf electrometers to an altitude of 5300 meters in a free balloon flight. He found the ionization rate increased approximately fourfold over the rate at ground level. Hess ruled out the Sun as the radiation's source by making a balloon ascent during a near-total eclipse. With the moon blocking much of the Sun's visible radiation, Hess still measured rising radiation at rising altitudes. He concluded "The results of my observation are best explained by the assumption that a radiation of very great penetrating power enters our atmosphere from above." In 1913–1914, Werner Kolhörster confirmed Victor Hess' earlier results by measuring the increased ionization rate at an altitude of 9 km. Hess received the Nobel Prize in Physics in 1936 for his discovery. Homi J. Bhabha derived an expression for the probability of scattering positrons by electrons, a process now

known as Bhabha scattering. His classic paper, jointly with Walter Heitler, published in 1937 described how primary cosmic rays from space interact with the upper atmosphere to produce particles observed at the ground level. Bhabha and Heitler explained the cosmic ray shower formation by the cascade production of gamma rays and positive and negative electron pairs. Soviet physicist Sergey Vernov was the first to use radiosondes to perform cosmic ray readings with an instrument carried to high altitude by a balloon. On 1 April 1935, he took measurements at heights up to 13.6 kilometers using a pair of Geiger counters in an anti-coincidence circuit to avoid counting secondary ray showers.]

See https://en.wikipedia.org/wiki/Cosmic_ray

http://web.mit.edu/8.13/www/JLExperiments/JLExp14.pdf

http://web.mit.edu/lululiu/Public/pixx/not-pixx/muons.pdf

https://en.wikipedia.org/wiki/Time_dilation

http://www.physics.rutgers.edu/ugrad/389/muon/muonphysics.pdf

http://www2.fisica.unlp.edu.ar/~veiga/experiments.html

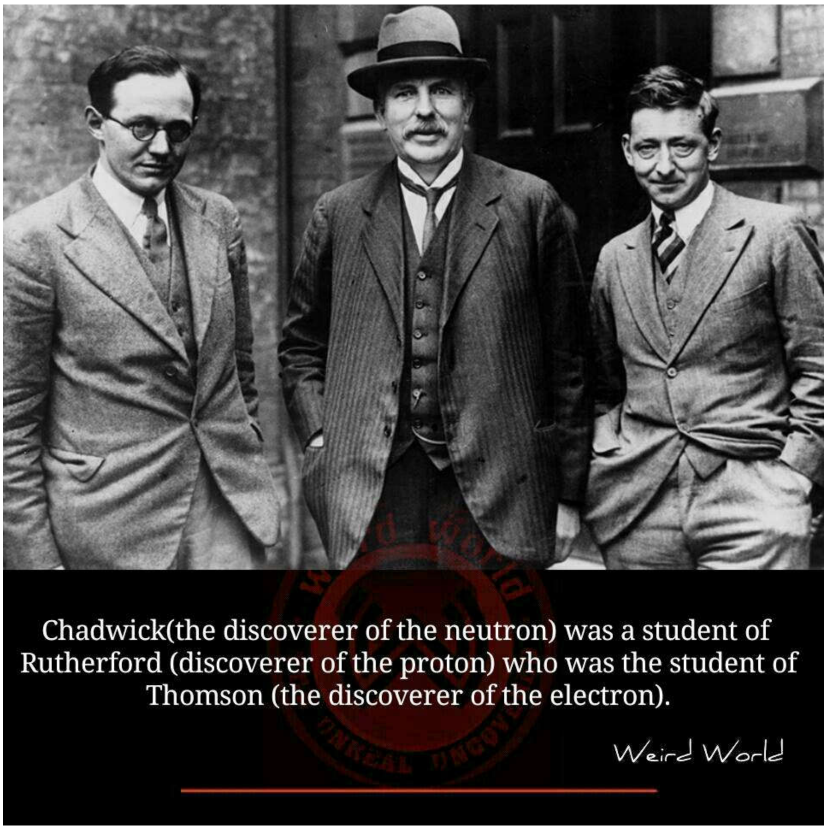
Detecting Neutrons

Rutherford predicted the existence of the neutron in 1920. Twelve years later, his assistant James Chadwick found it. At Cambridge, Chadwick searched for the neutron. He tried in 1923, but did not find it. He tried again in 1928, with no success. In 1930, the German physicists Walther Bothe and Herbert Becker noticed something odd. When they shot alpha rays at beryllium (atomic number 4) the beryllium emitted a neutral radiation that could penetrate 200 millimeters of lead. In contrast, it takes less than one millimeter of lead to stop a proton. Bothe and Becker assumed the neutral radiation was high-energy gamma rays.

Marie Curie's daughter, Irene Joliot-Curie, and Irene's husband, Frederic, put a block of paraffin wax in front of the beryllium rays. They observed high-speed protons coming from the paraffin. They knew that gamma rays could eject electrons from metals. Theythought the same thing was happening to the protons in the paraffin. Chadwick said the radiation could not be gamma rays. To eject protons at such a high velocity, the rays must have an energy of 50 million electron volts. An electron volt is a tiny amount of energy, only enough to keep a 75-watt light bulb burning for a tenth of a trillionth of a second. The alpha particles colliding with beryllium nuclei could produce only 14 million electron volts.

The law of conservation of energy states that energy can neither be created nor destroyed. It certainly looked as if energy was being created along with the neutral radiation. Chadwick had another explanation for the beryllium rays. He thought they were neutrons. He set up an experiment to test his hypothesis.

Chadwick put a piece of beryllium in a vacuum chamber with some polonium. The polonium emitted alpha rays, which struck the beryllium. When struck, the beryllium emitted the mysterious neutral rays.



In the path of the rays, Chadwick put a target. When the rays hit the target, they knocked atoms out of it. The atoms, which became electrically charged in the collision, flew into a detector. Chadwick's detector was a chamber filled with gas. When a charged particle passed through the chamber, it ionized the gas molecules. The ions drifted toward an electrode. Chadwick measured the current flowing through the electrode. Knowing the current, he could count the atoms and estimate their speed. Chadwick used targets of different elements, measuring the energy needed to eject the atoms of each. Gamma rays could not explain the speed of the atoms. The only good explanation for his result was a neutral particle. To prove that the particle was indeed the neutron, Chadwick measured its mass. He could not weigh it directly. Instead he measured everything else in the collision and used that information to calculate the mass.

For his mass measurement, Chadwick bombarded boron with alpha particles. Like beryllium, boron emitted neutral rays. Chadwick placed a hydrogen target in the path of the rays. When the rays struck the target, protons flew out. Chadwick measured the velocity of the protons.

Using the laws of conservation of momentum and energy, Chadwick calculated the mass of the neutral particle. It was 1.0067 times the mass of the proton. The neutral radiation was indeed the long-sought neutron.

http://ansnuclearcafe.org/2011/10/19/pioneers102011/

100 Greatest Discoveries of Physics

https://www.youtube.com/watch?v=Bpid0LBTqWg

(**As I write these words { 2016 } GUT [General Unified Theory] is being modified to introduce a 5th fundamental force, because some heavy particles have been observed at CERN and various other experiments and Producing Gravitational waves at will, without mass,** Madala Bosons to explain Dark Matter)

Learn Science from https://www.youtube.com/user/cassiopeiaproject/videos

Some easy Physics (much easier than IIT-JEE)
https://www.youtube.com/channel/UCliSRiiRVQuDfgxI_QN_Fmw/videos

https://www.youtube.com/watch?v=VCVTK5yzo0g&list=PLB03A41EA88A8DE65

https://www.youtube.com/user/diggitydev/playlists

https://www.youtube.com/user/onlearningcurve/playlists

https://www.youtube.com/watch?v=qWu82nJS42I&list=PLF71B362214423F9D

https://www.youtube.com/user/FizziksGuy/playlists

https://www.youtube.com/watch?v=gIOTFjq76tM&list=PL3pIurvIhuSANBIZa3u0RP9GFQprlSN11

https://www.youtube.com/watch?v=y7fXEKCP2XU&list=PL3pIurvIhuSDjUvzNZwC1HBW9eY1qldno

https://www.youtube.com/channel/UCiEHVhv0SBMpP75JbzJShqw/playlists

(Pradeep Kshetrapal Sir's Videos are at -
https://www.youtube.com/user/PradeepKshetrapal/videos)

Lectures by Professor Robert Riggs

https://www.youtube.com/watch?v=RWqAjKFKH3o&list=PL01771E7CE99097F8

Lectures by Professor Jerzy Wrobel

https://www.youtube.com/watch?v=DFhdUQ9AZw4&list=PLEEB9EC9DD59D6D85

Lectures by Yuri-Kolomensky

https://www.youtube.com/watch?v=KEiYSQnMHHQ&list=PL-XXv-cvA_iAKxxGD1tlWLS0DcieGLHh0

Physics Videos from Berkeley

https://www.youtube.com/watch?v=a-0h-9KCGjo&list=PLr11xUV7FM0EDu3u28Zp3d4ffjpqROm5Y

Lectures by Professor Muller

https://www.youtube.com/watch?v=6ysbZ_j2xi0&list=PL09717125E8C05BFC

Lectures by Steven W. Stahler

https://www.youtube.com/watch?v=Uc9Q5hNpv4Q&list=PL-XXv-cvA_iB1lYkU1YcdLCranBB0woKX

Lectures by Michel van Biezen

https://www.youtube.com/watch?v=FkO6vyMqo8E&list=PLX2gX-ftPVXVCw9WxxEA4yD14k8yskTSj

Dr. Don Lincoln of Fermilab https://www.youtube.com/user/fermilab/videos

Advance Physics Lectures by Leonard Susskind

https://www.youtube.com/watch?v=pyX8kQ-JzHI&list=PLQrxduI9Pds1fm91Dmn8x1lo-O_kpZGk8

A kid who wants more fun

https://www.youtube.com/watch?v=p_o4aY7xkXg&list=PL908547EAA7E4AE74

https://www.youtube.com/watch?v=51GNAET2zFU&list=PLlVwaZQkS2rxqMXTH-cdE0LIX9Zi_oS1

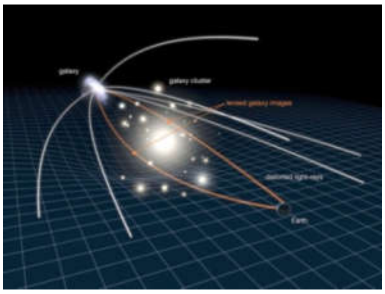
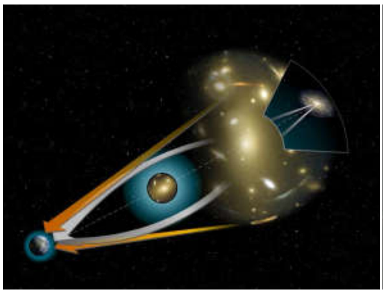
https://www.youtube.com/watch?v=h0hwuyOmd4k&list=PLSBNC6ROBP12PUanbUNaVLhNbJR6rgbmm

https://www.youtube.com/user/dramaticphysics/playlists

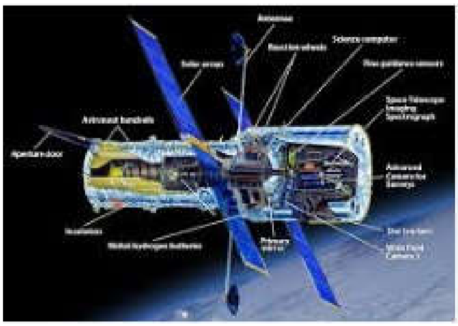
IIT-JEE is extremely tough for most humans. A productive PhD in Physics, or actually contributing to growth of the subject is much more tougher (than IIT JEE). { **I personally know quite a few IIT-JEE single or double digit rankers, joining for PhD and then dropped out due to performance** }. Most people have an illusion that they can argue with Scientists and imagine to ask some " smart " questions which the Scientists will not able to answer, so the argument is won, and existence of God is proved. As if Scientist are eagerly sitting or waiting to answer every crap asked. I can only say; that most scientists (since more than 100 years) have stopped wasting their time arguing or convincing fools. I am not a Scientist. Even being a simple teacher, I do not try to teach fools, or argue with anyone.

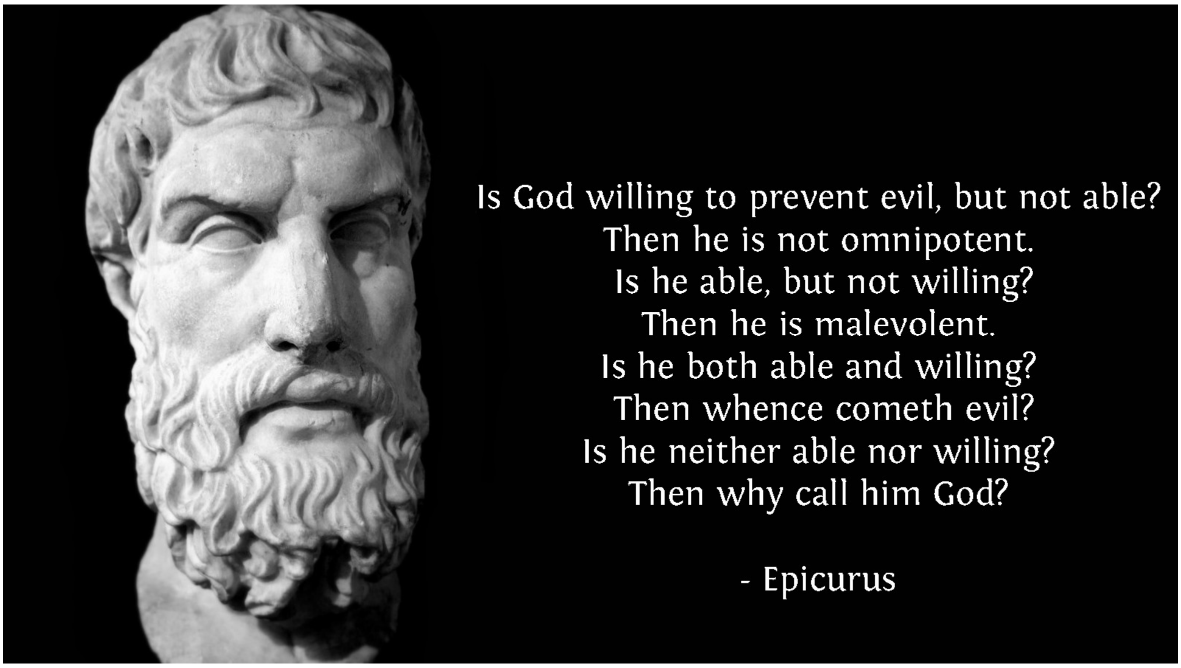
[**For History of Physics I recommend** http://www.historyworld.net/wrldhis/PlainTextHistories.asp?ParagraphID=kqq]

[Gravitational lens and Einstein ring due to bending of light by mass]

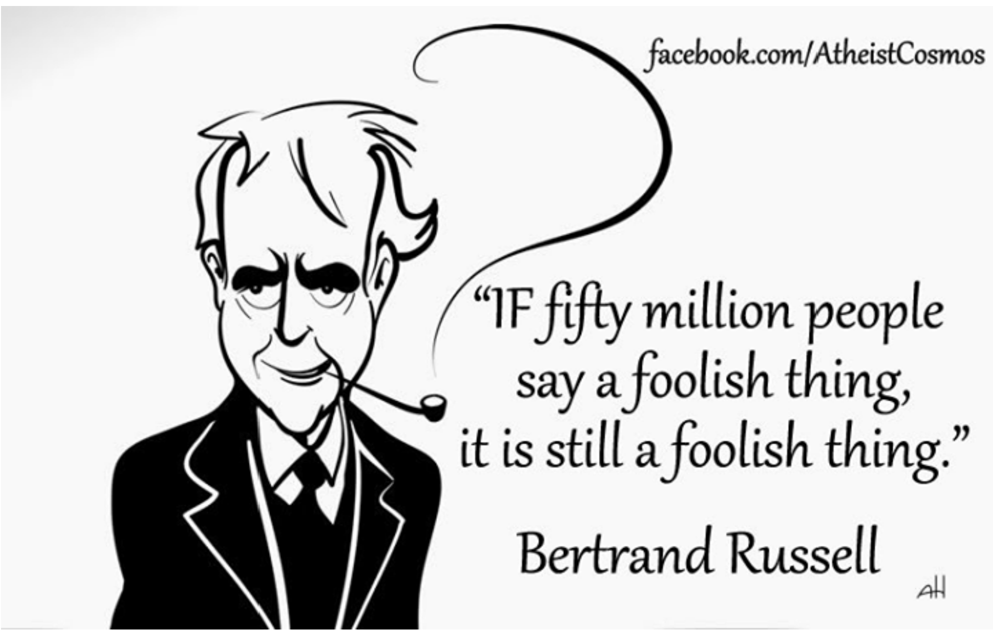

Recall what I said at the beginning of the book .... " **Someone will learn only by his hard work, his desire to learn.** " No arguments or no ‘time wasting’ with fools. There is too much of good material (data, books, videos etc) out and free in this world. If someone wants to learn, can learn; **instead of wasting time arguing**. Since centuries stupids and/or fools are being eliminated in various exams. Entrance exam, is a misnomer. These are elimination tests. The society has systems of Interviews, Peer reviews, appraisals, Thesis evaluation etc... to eliminate crap, foolish things, and nonsense.





Religion and/or " war between religions " mostly to decide whose God is better; have killed millions. Instead of fighting and killing; to decide which custom to follow; how to dress; what rituals to do on a daily basis; better to spend time experimenting and developing new things, new technologies, new ideas. Scientists (**the men**) are busy; and **always will be busy**! **Rather, in war**; **with new frontiers of knowledge**; not in arguments, verbal wars, or physical wars. **Atheism is the most peaceful Doctrine**.

"**Bertrand Arthur William Russell**" the famous Philosopher, Mathematician, Logician, received 1950 Nobel Prize for Literature.



So those who want to learn can continue learning ...

See https://www.youtube.com/results?search_query=History+of+science

See
https://www.youtube.com/results?search_query=history+of+science+the+complete+full+documentary+

I will choose only two extreme examples of what Human beings have " seen " by now ...

For far and big) Very powerful cameras ready with video recording facilities were scanning the sky. Coincidentally the "place or region " a camera was looking had an event (many million years back though) of a black hole devouring a star.

https://www.youtube.com/watch?v=O3Z5AS3TTS4

https://www.youtube.com/watch?v=x7ZX10UbMus

For small) Photographs of molecules and subsequently atoms

https://www.youtube.com/watch?v=yqLlgIaz1L0

https://www.youtube.com/watch?v=ofp-OHIq6Wo

https://www.youtube.com/watch?v=oSCX78-8-q0

https://www.youtube.com/watch?v=RTLeWIqynW4

https://www.youtube.com/watch?v=J3xLuZNKhlY

https://www.youtube.com/watch?v=SMgi2j9Ks9k

https://www.youtube.com/watch?v=V0KjXsGRvoA&list=PLC3E0tG-9im_kuMwYIM7-NZR62VyWZ6rl

-

Entertainment and relaxed mind is required. Students can improve Visual Presentation skills by watching "Two men and wardrobe" by Roman Polanski

https://www.youtube.com/watch?v=Cs2RZewMuAg

Imagine a world where Millions of People have "**better**" Visual story telling or Visual presentation skills than **Roman Polanski** or say **Jim Jarmusch** ...

https://www.youtube.com/watch?v=wJS2mC-7LSM

Enjoy

-

Spoon Feeding Series - Indefinite Integrals Survival Guide

Some concepts which are topic or chapter independent and must know for all.

1) **First Law of Statistics** – Larger is the dataset for analysis better is the result. Also the data must be widely varying from widely spread sources. Else biased or concentrated data will not give correct results / conclusions. There are elaborate mathematical rules to select the sample size, select distributions, sampling techniques, measure of Biases and / or confidence level of the conclusions. The best possible result can be obtained by measuring all / total population. Often this is just not possible. Opposite to this is Hasty Generalization.

2) Several things just can't be measured. Say for example if a shopkeeper wants to know why people are not coming to his shop; he only has to guess. May be people do not like the outside look of the shop, may be they don't like the lighting, may be they don't like his face or caste … We can only guess. A small shopkeeper can never go to all people in the town to ask why they are not coming to his shop.

3) Various kinds of analysis can be given / produced / projected in a biased way. Say for example in a Engineering class there are 50 Boys and 4 girls. 2 boys and 2 girls marry. Someone reports that 50% girls fall in love and have love marriages with batch-mates while only 4% Boys do so. Even if the data-size was 5000 Boys and 400 girls where 200 girls marry their batch-mates; the conclusions are not correct.

Regarding Probability

1) For most events (close to 100%) in this world the Probability just cannot be measured. We have no data regarding the probability. We will never have. There is an obscure theorem regarding probability… "if the data to calculate probability is not available then the probability should be considered as 0.5 meaning 50% i.e. either it can happen or cannot happen. "

Let me elaborate this with some examples. If you just now go out of your home to the street, what is the probability of seeing a Man with a Green T-Shirt ?

If a truck is carrying 23 sheep and a sheep jumps out of the truck when the truck is crossing near your home ?

I had seen a radioactivity problem asked in an exam… if the rate of decay now is 31 disintegrations per microsecond, then in next 1 second what is the probability of a particular atom getting disintegrated ? [Let as assume it is told that we have 8.7 moles of the radioactive material. Though it may be given or not, it is irrelevant]

The answer in this case is 50%. A particular atom may or may not decay.

It is 50% probability that a Man is wearing Blue T-Shirt or Black … We have no data whom we will meet, how often he wears T-Shirt, How many T-Shirt of what colour he has, or what is considered as a T-Shirt and what is not. No one ever will have any data of this kind.

2) In some rare cases even if we have data we cannot conclude anything on the probability. Say you want to hire a car to go to the airport. The car rental owner is a great friend of yours and shares all data. He says… on average the cars get a puncture every 345 hours (**as per data of several decades**). I have 7 cars which did not get any puncture even though these are running for 400, 500 etc hours. All more than 345 hours. And 2 more cars, one 200 hours since last puncture and another one 10 hours since last puncture.

Which car do you choose ?

First of all there will be no car rental guy who will tell you this kind of data. Also it makes no difference to the journey regarding the car you choose. Probability and data do not help on specific future events.

3) In some very small number of cases even if you have long term probability data, the Probability values mean nothing. Say you know that when you eat out on average for every 13 eating you get a stomach upset. Now your friends want to treat you tomorrow and all will eat out. Will you go ?

The probability questions that we see in standard 10 to 12 with dice, coins horses etc are limited to give you some concepts. Even the Bay's rule etc. The probability concepts are valid only for large number of events such as patient inflow in a very large hospital, Quality defects in millions of things being manufactured, or say in Quantum Mechanics Probability of events where 10 to the power 25 particles involved in every nano second.

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Regarding Holistic picture and Systems thinking

Human beings are in general prone to reductionism; assuming repeated specific / fixed outcome; assuming predictability etc. "Systems Thinking" is only few decades old idea. We often miss out the Holistic picture for boundaries of chaos, Strange attractors; effects of small perturbations etc.

See https://www.youtube.com/watch?v=lhbLNBqhQkc&list=PLhsIdCVDmWaOoNsTnVYzr-HuS-lR11Zei

See https://www.youtube.com/watch?v=WrdSkqRypsg

See https://www.youtube.com/watch?v=c0gDLEHbYCk

See https://www.youtube.com/watch?v=R6NnCOs20GQ&list=PL66DBF862753B9A75&index=7

See https://www.youtube.com/watch?v=aAJkLh76QnM

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God is not an intelligent Designer

Several parts of the bodies human and animals have imperfections.

See https://www.youtube.com/watch?v=cO1a1Ek-HD0

See https://www.youtube.com/watch?v=lIEoO5KdPvg

See https://www.youtube.com/watch?v=-OCMx2VuP1U

See https://www.youtube.com/watch?v=dzYgScf47EI

See https://www.youtube.com/watch?v=ujYNSDYIZKw

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Spoon Feeding - Indefinite Integrals

Recall the various tricks, formulae, and rules of solving Indefinite Integrals

(i) $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

(ii) $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C$

(iii) $\int \frac{dx}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C = -\frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C$

(iv) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$

(v) $\int \frac{dx}{\sqrt{x^2-a^2}} = \log|x+\sqrt{x^2-a^2}| + C = \cosh^{-1}\left(\frac{x}{a}\right) + C$

(vi) $\int \frac{dx}{\sqrt{x^2+a^2}} = \log|x+\sqrt{x^2+a^2}| + C = \sinh^{-1}\left(\frac{x}{a}\right) + C$

(vii) $\int \sqrt{x^2+a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2+a^2} + a^2\log|x+\sqrt{x^2+a^2}|\right] + C$

(viii) $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\sin^{-1}\left(\frac{x}{a}\right)\right] + C$

(ix) $\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2-a^2} - a^2\log|x+\sqrt{x^2-a^2}|\right] + C$

(x) $\int (px+q)\sqrt{ax^2+bx+c}\,dx = \frac{p}{2a}\int (2ax+b)\sqrt{ax^2+bx+c}\,dx$

$$+\left(\frac{q-pb}{2a}\right)\int \sqrt{ax^2+bx+c}\,dx$$

- $\int e^x dx = e^x$
- $\int e^{ax} dx = \frac{1}{a} e^{ax}$
- $\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2+b^2} (a \cos bx + b \sin bx)$
- $\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2+b^2} (a \sin bx - b \cos bx)$
- $\int a^x dx = \frac{a^x}{lna} + c$

- $\int \log x dx = x(\log x - 1) + c$
- $\int \frac{1}{x} dx = \log |x| + c$
- $\int a^x dx = a^x \log x + c$
- $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$
- $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} + c$

- $\int \csc x \cot x dx = -\csc x + c$
- $\int \csc^2 x dx = -\cot x + c$
- $\int \sec x \tan x dx = \sec x + c$
- $\int \sec^2 x dx = \tan x + c$
- $\int \sin x dx = -\cos x + c$
- $\int \cos x dx = \sin x + c$

- $\int (ax+b)^n = \frac{1}{a} \frac{(ax+b)^{n+1}}{n+1} + C$, $n \neq 1$
- $\int \frac{dx}{(ax+b)} = \frac{1}{a} \log |ax+b| + C$
- $\int e^{ax+b} = \frac{1}{a} e^{ax+b} + C$
- $\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + C$
- $\int \sec^2(ax+b) dx = \frac{1}{a} \tan(ax+b) + C$
- $\int \csc^2(ax+b) dx = \frac{-1}{a} \cot(ax+b) + C$
- $\int \csc(ax+b) \cot(ax+b) dx = \frac{-1}{a} \csc(ax+b) + C$

For **Integrals of the form**

(i) $\int \frac{dx}{a + b \sin x}$ (ii) $\int \frac{dx}{a + b \cos x}$ (iii) $\int \frac{dx}{a \sin x + b \cos x + c}$

Put $\cos x = \frac{1 - \tan^2 x/2}{1 + \tan^2 x/2}$, $\sin x = \frac{2 \tan x/2}{1 + \tan^2 x/2}$

Some advanced procedures….

$\int \frac{x^m}{(a+bx)^p}\,dx$ — Put $a + bx = z$

m is a + ve integer

$\int \frac{dx}{x^m (a+bx)^p}$, — Put $a + bx = zx$

where either (m and p positive integers) or (m and p are fractions, but $m + p$ = integers > 1)

$\int x^m \left(a + bx^n\right)^p\, dx$,

where m, n, p are rationals.

(i) p is a + ve integer — Apply Binomial theorem to $(a + bx^n)^p$

(ii) p is a − ve integer — Put $x = z^k$ where k = common denominator of m and n.

(iii) $\frac{m+1}{n}$ is an integer — Put $(a + bx^n) = z^k$ where k = denominator of p.

(iv) $\frac{m+1}{n} + p$ is an integer — Put $a + bx^n = x^n z^k$ where k = denominator of fraction p.

$$\int \frac{x^2\,dx}{x^4 + k x^2 + a^4} = \frac{1}{2}\int \frac{(x^2 + a^2)\,dx}{(x^4 + k x^2 + x^4)} + \frac{1}{2}\int \frac{(x^2 - a^2)\,dx}{(x^4 + k x^2 + a^4)}$$

$$\int \frac{dx}{(x^4 + k x^2 + a^4)} = \frac{1}{2a^2}\int \frac{(x^2 + a^2)\,dx}{(x^4 + k x^2 + a^4)} - \frac{1}{2a^2}\int \frac{(x^2 - a^2)\,dx}{(x^4 + k x^2 + a^4)}$$

$$\int \frac{dx}{(x^2 + k)^n} = \frac{x}{k(2n-2)(x^2+k)^{n-1}} + \frac{(2n-3)}{k(2n-2)}\int \frac{dx}{(x^2+k)^{n-1}}$$

For $\int \frac{dx}{(Ax^2 + Bx + C)\sqrt{(ax^2 + bx + c)}}$ we need to substitute $\frac{ax^2 + bx + c}{Ax^2 + Bx + C} = t^2$

We have hundreds of books for Calculus or Integration (both Indefinite or Definite). Then why am I writing another book ?

In general one of the limitations of books is to miss the discussions. While teaching in a class, a teacher faces many kinds of questions, many kinds of discussions happen. All of that cannot be given in a book. Even if someone gives all the discussions possible, the book will become very fat. The students will not read such a thick book. A video with explanations is one way better than books as we see the intermediate steps being written. There are several examples where in a book the final step is written, while the parts how it was developed is not shown or cannot be shown.

This book is backed-up with hundreds of videos. That makes the book uniquely different from previous or other books.

In the Industry the Indefinite Integrals that we get are generally very complicated. All of those are solved after expanding. We will cover Tayler series expansion, McLaurin's expansion (or Maclaurin series) etc later. Since 1970s, i.e. beginning of modern computers; we have Mathematical packages which can give you the series expansion of any function in milliseconds.

The following problems have to be solved by series expansions only

$\int \frac{dx}{\sqrt{x^3 + a}}$ or say $\int \sqrt{\text{Sec } x}\, dx$

Though definite integral of 0 to π/2 of root sec x can be analytically done.

The Questions students get in books, or exams, or class discussions are easy, cooked ones. These are known to be solved, rather easily. The Teacher will not able to solve any random integral.

There are around 30 or say max 40 " Patterns " which we know how to solve. **Indefinite Integration is a " Pattern Matching " approach. A given problem is modified towards a known pattern, and then finally we say the solution is** …..

If an Indefinite Integral can be solved in x then it also can be solved for x being replaced by x + k or ax+b etc.

So if we know how to solve $\int \sqrt{x^2 + a^2}\, dx$ then we can also solve $\int \sqrt{(x + k)^2 + a^2}\, dx$ as well

There have been several integration problems which depend on the trick of x being interchanged with ax+b Integration of Cos (ax + b) / Cos x is easy. Expand the Numerator and then simplify to integrate. But I have met lot of senior people in my life who just could not integrate cos x by cos (ax + b). Even after several hours of trying, or open book attempts. ☺

$$\int \frac{1}{\sqrt{x^2+2x+2}}dx = \int \frac{1}{\sqrt{(x+1)^2+(1)^2}}dx$$

Let $x+1=t$

$\therefore\ dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{x^2+2x+2}}dx = \int \frac{1}{\sqrt{t^2+1}}dt$$

$$= \log\left|t+\sqrt{t^2+1}\right|+C$$

$$= \log\left|(x+1)+\sqrt{(x+1)^2+1}\right|+C$$

$$= \log\left|(x+1)+\sqrt{x^2+2x+2}\right|+C$$

Some examples are highlighted in the video given below

https://archive.org/details/4Integrations6CosSquareIITJEEMath

https://archive.org/details/AIEEEIntegralCalculusSinXBySinXPiBy4FlipByLinearSubstitution2008

IIT JEE 1979 Integration x square by a plus bx whole square

https://archive.org/details/IITJEE1979IntegrationXSquareByAPlusBxSquare

See the following Videos to learn the basic steps.

I taught several batches in my life. I know how repetitive and boring it is. These are easy concepts and we gain nothing by typing the whole thing once again. I am keeping the study videos, which were recorded while I was doing classroom teaching, at archive.org

Concept of unknown constant. In Indefinite Integral there is an unknown constant which we always write as + c

Integration of Sin6x dx can have infinite possible values

https://archive.org/details/1IntegrationOfSin6xDxCanHaveInfinitePossibleValues

https://archive.org/details/2IntegrationOfSin6xDxCanHaveInfinitePossibleValues

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How do you integrate Sin 2x ?

We know that

$$\frac{d}{dx}(\cos 2x) = -2\sin 2x$$
$$\Rightarrow \sin 2x = -\frac{1}{2}\frac{d}{dx}(\cos 2x)$$
$$\therefore \sin 2x = \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$$

So ∫Sin 2x dx = (-1/2) Cos 2x + c

-

How do you integrate Cos 3x ?

We know that

$$\frac{d}{dx}(\sin 3x) = 3\cos 3x$$
$$\Rightarrow \cos 3x = \frac{1}{3}\frac{d}{dx}(\sin 3x)$$
$$\therefore \cos 3x = \frac{d}{dx}\left(\frac{1}{3}\sin 3x\right)$$

So ∫Cos 3x dx = (1/3) Sin 3x + c

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Find $\int e^{2x}$ dx ?

We know that

$$\frac{d}{dx}\left(e^{2x}\right) = 2e^{2x}$$
$$\Rightarrow e^{2x} = \frac{1}{2}\frac{d}{dx}\left(e^{2x}\right)$$
$$\therefore e^{2x} = \frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$$

So $\int e^{2x}$ dx = (½) e^{2x} + c

Find $\int (ax + b)^2\, dx$

The anti derivative of $(ax+b)^2$ is the function of x whose derivative is $(ax+b)^2$.

It is known that,

$$\frac{d}{dx}(ax+b)^3 = 3a(ax+b)^2$$
$$\Rightarrow (ax+b)^2 = \frac{1}{3a}\frac{d}{dx}(ax+b)^3$$
$$\therefore (ax+b)^2 = \frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right)$$

Therefore, the anti derivative of $(ax+b)^2$ is $\frac{1}{3a}(ax+b)^3$.

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Find $\int (\sin 2x - 4e^{3x})\,dx$

The anti derivative of $(\sin 2x - 4e^{3x})$ is the function of x whose derivative is $(\sin 2x - 4e^{3x})$

It is known that,

$$\frac{d}{dx}\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right) = \sin 2x - 4e^{3x}$$

Therefore, the anti derivative of $(\sin 2x - 4e^{3x})$ is $\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right)$.

-

$$\int (4e^{3x} + 1)dx$$
$$= 4\int e^{3x}dx + \int 1 dx$$
$$= 4\left(\frac{e^{3x}}{3}\right) + x + C$$
$$= \frac{4}{3}e^{3x} + x + C$$

Find $\int x^2\left(1-\frac{1}{x^2}\right)dx$

Answer :

$$\int x^2\left(1-\frac{1}{x^2}\right)dx$$
$$=\int\left(x^2-1\right)dx$$
$$=\int x^2dx-\int 1dx$$
$$=\frac{x^3}{3}-x+C$$

-

$$\int\left(ax^2+bx+c\right)dx$$
$$=a\int x^2dx+b\int xdx+c\int 1.dx$$
$$=a\left(\frac{x^3}{3}\right)+b\left(\frac{x^2}{2}\right)+cx+C$$
$$=\frac{ax^3}{3}+\frac{bx^2}{2}+cx+C$$

-

$$\int\left(2x^2+e^x\right)dx$$
$$=2\int x^2dx+\int e^xdx$$
$$=2\left(\frac{x^3}{3}\right)+e^x+C$$
$$=\frac{2}{3}x^3+e^x+C$$

-

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$$

$$= \int\left(x+\frac{1}{x}-2\right)dx$$

$$= \int x dx + \int\frac{1}{x}dx - 2\int 1.dx$$

$$= \frac{x^2}{2} + \log|x| - 2x + C$$

-

$$\int\frac{x^3+5x^2-4}{x^2}dx$$

$$= \int\left(x+5-4x^{-2}\right)dx$$

$$= \int x dx + 5\int 1.dx - 4\int x^{-2}dx$$

$$= \frac{x^2}{2} + 5x - 4\left(\frac{x^{-1}}{-1}\right) + C$$

$$= \frac{x^2}{2} + 5x + \frac{4}{x} + C$$

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$$\int \frac{x^3+3x+4}{\sqrt{x}}dx$$

$$= \int\left(x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}\right)dx$$

$$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{3\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}} + \frac{4\left(x^{\frac{1}{2}}\right)}{\frac{1}{2}} + C$$

$$= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + C$$

$$= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8\sqrt{x} + C$$

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$$\int \frac{x^3-x^2+x-1}{x-1}dx$$

On dividing, we obtain

$$= \int\left(x^2+1\right)dx$$

$$= \int x^2 dx + \int 1 dx$$

$$= \frac{x^3}{3} + x + C$$

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$$\int(1-x)\sqrt{x}dx$$

$$= \int\left(\sqrt{x}-x^{\frac{3}{2}}\right)dx$$

$$= \int x^{\frac{1}{2}}dx - \int x^{\frac{3}{2}}dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$= \frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + C$$

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$$\int\sqrt{x}\left(3x^2+2x+3\right)dx$$

$$= \int\left(3x^{\frac{5}{2}}+2x^{\frac{3}{2}}+3x^{\frac{1}{2}}\right)dx$$

$$= 3\int x^{\frac{5}{2}}dx + 2\int x^{\frac{3}{2}}dx + 3\int x^{\frac{1}{2}}dx$$

$$= 3\left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right) + 2\left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right) + 3\frac{\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}} + C$$

$$= \frac{6}{7}x^{\frac{7}{2}} + \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C$$

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$$\int\left(2x-3\cos x+e^x\right)dx$$

$$= 2\int xdx - 3\int\cos xdx + \int e^x dx$$

$$= \frac{2x^2}{2} - 3(\sin x) + e^x + C$$

$$= x^2 - 3\sin x + e^x + C$$

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$$\int\left(2x^2 - 3\sin x + 5\sqrt{x}\right)dx$$

$$= 2\int x^2 dx - 3\int \sin x dx + 5\int x^{\frac{1}{2}} dx$$

$$= \frac{2x^3}{3} - 3(-\cos x) + 5\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{\frac{3}{2}} + C$$

-

$$\int \sec x(\sec x + \tan x)dx$$

$$= \int\left(\sec^2 x + \sec x \tan x\right)dx$$

$$= \int \sec^2 x dx + \int \sec x \tan x dx$$

$$= \tan x + \sec x + C$$

-

$$\int \frac{\sec^2 x}{\cos ec^2 x}dx$$

$$= \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}}dx$$

$$= \int \frac{\sin^2 x}{\cos^2 x}dx$$

$$= \int \tan^2 x dx$$

$$= \int\left(\sec^2 x - 1\right)dx$$

$$= \int \sec^2 x dx - \int 1 dx$$

$$= \tan x - x + C$$

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$$\int \frac{2-3\sin x}{\cos^2 x}dx$$
$$= \int\left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x}\right)dx$$
$$= \int 2\sec^2 x dx - 3\int \tan x \sec x dx$$
$$= 2\tan x - 3\sec x + C$$

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Find $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx$

$$= \int x^{\frac{1}{2}}dx + \int x^{-\frac{1}{2}}dx$$
$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$$
$$= \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$$

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If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$, then $f(x)$ is

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (B) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ (D) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

It is given that

$$\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$$

$\therefore$ Anti derivative of $4x^3 - \frac{3}{x^4} = f(x)$

$$\therefore f(x) = \int 4x^3 - \frac{3}{x^4}\, dx$$

$$f(x) = 4\int x^3 dx - 3\int (x^{-4})dx$$

$$\therefore f(x) = 4\left(\frac{x^4}{4}\right) - 3\left(\frac{x^{-3}}{-3}\right) + C$$

$$f(x) = x^4 + \frac{1}{x^3} + C$$

Also

$$f(2) = 0$$

$$\therefore f(2) = (2)^4 + \frac{1}{(2)^3} + C = 0$$

$$\Rightarrow 16 + \frac{1}{8} + C = 0$$

$$\Rightarrow C = -\left(16 + \frac{1}{8}\right)$$

$$\Rightarrow C = \frac{-129}{8}$$

$$\therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$$

Hence, the correct answer is A.

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Find $\int \frac{2x}{1+x^2}$ dx

Let $1+x^2 = t$

$\therefore 2x\, dx = dt$

$$\Rightarrow \int \frac{2x}{1+x^2}dx = \int \frac{1}{t}dt$$

$$= \log|t| + C$$

$$= \log|1+x^2| + C$$

$$= \log(1+x^2) + C$$

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Find $\int \frac{(\log x)^2}{x}$ dx

Let $\log|x| = t$

$\therefore \frac{1}{x}dx = dt$

$$\Rightarrow \int \frac{(\log|x|)^2}{x}dx = \int t^2 dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{(\log|x|)^3}{3} + C$$

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Find $\int \frac{1}{x+x\log x}$ dx

$$\frac{1}{x+x\log x}=\frac{1}{x(1+\log x)}$$

Let $1 + \log x = t$

$$\therefore \frac{1}{x}dx=dt$$

$$\Rightarrow \int\frac{1}{x(1+\log x)}dx=\int\frac{1}{t}dt$$

$$=\log|t|+C$$
$$=\log|1+\log x|+C$$

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Find $\int$ Sin x Sin (Cos x) dx

Let $\cos x = t$

$\therefore -\sin x\, dx = dt$

$$\Rightarrow \int\sin x\cdot\sin(\cos x)dx=-\int\sin t\,dt$$
$$=-[-\cos t]+C$$
$$=\cos t+C$$
$$=\cos(\cos x)+C$$

-

Find $\int$ Sin (ax + b) Cos (ax + b) dx

$$\sin(ax+b)\cos(ax+b) = \frac{2\sin(ax+b)\cos(ax+b)}{2} = \frac{\sin 2(ax+b)}{2}$$

Let $2(ax+b) = t$

$\therefore 2adx = dt$

$$\Rightarrow \int \frac{\sin 2(ax+b)}{2} dx = \frac{1}{2}\int \frac{\sin t \; dt}{2a}$$
$$= \frac{1}{4a}\left[-\cos t\right] + C$$
$$= \frac{-1}{4a}\cos 2(ax+b) + C$$

-

Find $\int \sqrt{ax+b}$ dx

Let $ax + b = t$

$\Rightarrow adx = dt$

$$\therefore dx = \frac{1}{a} dt$$

$$\Rightarrow \int (ax+b)^{\frac{1}{2}} dx = \frac{1}{a}\int t^{\frac{1}{2}} dt$$

$$= \frac{1}{a}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{2}{3a}(ax+b)^{\frac{3}{2}} + C$$

-

Linear under root not necessarily need to be substituted as t^2 In the problem below substituting t can exchange the complexity, and simplify the problem

Let $(x+2)=t$

$\therefore dx = dt$

$$\Rightarrow \int x\sqrt{x+2}\,dx = \int (t-2)\sqrt{t}\,dt$$
$$= \int \left(t^{\frac{3}{2}} - 2t^{\frac{1}{2}}\right) dt$$
$$= \int t^{\frac{3}{2}} dt - 2\int t^{\frac{1}{2}} dt$$
$$= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$
$$= \frac{2}{5}t^{\frac{5}{2}} - \frac{4}{3}t^{\frac{3}{2}} + C$$
$$= \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$$

-

Find $\int x\sqrt{1+2x^2}\,dx$

Let $1 + 2x^2 = t$

$\therefore 4x\,dx = dt$

$$\Rightarrow \int x\sqrt{1+2x^2}\,dx = \int \frac{\sqrt{t}\,dt}{4}$$
$$= \frac{1}{4}\int t^{\frac{1}{2}} dt$$
$$= \frac{1}{4}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$
$$= \frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}} + C$$

Find $\int(4x+2)\sqrt{x^2+x+1}\,dx$

Let $x^2+x+1=t$

$\therefore (2x+1)dx = dt$

$$\int(4x+2)\sqrt{x^2+x+1}\,dx$$

$$= \int 2\sqrt{t}\,dt$$

$$= 2\int\sqrt{t}\,dt$$

$$= 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$

$$= \frac{4}{3}\left(x^2+x+1\right)^{\frac{3}{2}}+C$$

-

Find $\int \frac{dx}{x-\sqrt{x}}$

$$\frac{1}{x-\sqrt{x}} = \frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}$$

Let $\left(\sqrt{x}-1\right)=t$

$$\therefore \frac{1}{2\sqrt{x}}dx = dt$$

$$\Rightarrow \int\frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}dx = \int\frac{2}{t}\,dt$$

$$= 2\log|t|+C$$

$$= 2\log\left|\sqrt{x}-1\right|+C$$

-

Find $\int \frac{x}{\sqrt{x+4}}$ dx

Put x + 4 = t^2 => dx = 2t dt and x = t^2 - 4

So $\int$ (t^2 - 4)/t 2tdt = 2 $\int$ (t^2 - 4) dt = (1/3) t^3 -4t + c put t = root (x + 4)

-

Find $\int (x^3-1)^{\frac{1}{3}} x^5 dx$

Let $x^3 - 1 = t$

$\therefore 3x^2 dx = dt$

$$\Rightarrow \int (x^3-1)^{\frac{1}{3}} x^5 dx = \int (x^3-1)^{\frac{1}{3}} x^3 \cdot x^2 dx$$
$$= \int t^{\frac{1}{3}} (t+1) \frac{dt}{3}$$
$$= \frac{1}{3} \int \left(t^{\frac{4}{3}} + t^{\frac{1}{3}} \right) dt$$
$$= \frac{1}{3} \left[\frac{t^{\frac{7}{3}}}{\frac{7}{3}} + \frac{t^{\frac{4}{3}}}{\frac{4}{3}} \right] + C$$
$$= \frac{1}{3} \left[\frac{3}{7} t^{\frac{7}{3}} + \frac{3}{4} t^{\frac{4}{3}} \right] + C$$
$$= \frac{1}{7} (x^3-1)^{\frac{7}{3}} + \frac{1}{4} (x^3-1)^{\frac{4}{3}} + C$$

-

Find $\int \frac{x^2}{\left(2+3x^3\right)^3} dx$

Let $2+3x^3 = t$

$\therefore 9x^2\, dx = dt$

$$\Rightarrow \int \frac{x^2}{\left(2+3x^3\right)^3} dx = \frac{1}{9}\int \frac{dt}{(t)^3}$$

$$= \frac{1}{9}\left[\frac{t^{-2}}{-2}\right] + C$$

$$= \frac{-1}{18}\left(\frac{1}{t^2}\right) + C$$

$$= \frac{-1}{18\left(2+3x^3\right)^2} + C$$

-

Find $\int \frac{1}{x(\log x)^m} dx$

Let $\log x = t$

$\therefore \frac{1}{x} dx = dt$

$$\Rightarrow \int \frac{1}{x(\log x)^m} dx = \int \frac{dt}{(t)^m}$$

$$= \left(\frac{t^{-m+1}}{1-m}\right) + C$$

$$= \frac{(\log x)^{1-m}}{(1-m)} + C$$

-

Find $\int \frac{x}{9-4x^2}\,dx$

Let $9-4x^2 = t$

$\therefore -8x\,dx = dt$

$$\Rightarrow \int \frac{x}{9-4x^2}\,dx = \frac{-1}{8}\int \frac{1}{t}\,dt$$
$$= \frac{-1}{8}\log|t| + C$$
$$= \frac{-1}{8}\log\left|9-4x^2\right| + C$$

-

Find $\int e^{2x+3}\,dx$

Let $2x+3 = t$

$\therefore 2dx = dt$

$$\Rightarrow \int e^{2x+3}\,dx = \frac{1}{2}\int e^t\,dt$$
$$= \frac{1}{2}\left(e^t\right) + C$$
$$= \frac{1}{2}e^{(2x+3)} + C$$

-

Find $\int \frac{x}{e^{x^2}} dx$

Let $x^2 = t$

$\therefore 2xdx = dt$

$\Rightarrow \int \frac{x}{e^{x^2}} dx = \frac{1}{2} \int \frac{1}{e^t} dt$

$= \frac{1}{2} \int e^{-t} dt$

$= \frac{1}{2} \left(\frac{e^{-t}}{-1} \right) + C$

$= -\frac{1}{2} e^{-x^2} + C$

$= \frac{-1}{2e^{x^2}} + C$

-

Find $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx$

Let $\tan^{-1} x = t$

$\therefore \frac{1}{1+x^2} dx = dt$

$\Rightarrow \int \frac{e^{\tan^{-1} x}}{1+x^2} dx = \int e^t dt$

$= e^t + C$

$= e^{\tan^{-1} x} + C$

-

Find $\int \frac{e^{2x}-1}{e^{2x}+1}dx$

Dividing numerator and denominator by e^x, we obtain

$$\frac{\frac{(e^{2x}-1)}{e^x}}{\frac{(e^{2x}+1)}{e^x}} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Let $e^x + e^{-x} = t$

$$\therefore (e^x - e^{-x})dx = dt$$

$$\Rightarrow \int \frac{e^{2x}-1}{e^{2x}+1}dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$$

$$= \int \frac{dt}{t}$$
$$= \log|t| + C$$
$$= \log\left|e^x + e^{-x}\right| + C$$

-

Find $\int\left(\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\right)dx$

Let $e^{2x}+e^{-2x}=t$

$\therefore\ \left(2e^{2x}-2e^{-2x}\right)dx=dt$

$\Rightarrow 2\left(e^{2x}-e^{-2x}\right)dx=dt$

$$\Rightarrow \int\left(\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\right)dx=\int\frac{dt}{2t}$$
$$=\frac{1}{2}\int\frac{1}{t}dt$$
$$=\frac{1}{2}\log|t|+C$$
$$=\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+C$$

-

Find $\int\tan^2(2x-3)\,dx$

$\tan^2(2x-3)=\sec^2(2x-3)-1$

Let $2x-3=t$

$\therefore\ 2dx=dt$

$$\Rightarrow \int\tan^2(2x-3)\,dx=\int\left[\left(\sec^2(2x-3)\right)-1\right]dx$$
$$=\frac{1}{2}\int\left(\sec^2 t\right)dt-\int 1dx$$
$$=\frac{1}{2}\int\sec^2 t\,dt-\int 1dx$$
$$=\frac{1}{2}\tan t-x+C$$
$$=\frac{1}{2}\tan(2x-3)-x+C$$

Find $\int \sec^2(7-4x)\,dx$

Let $7 - 4x = t$

$\therefore -4dx = dt$

$$\therefore \int \sec^2(7-4x)\,dx = \frac{-1}{4}\int \sec^2 t\,dt$$
$$= \frac{-1}{4}(\tan t) + C$$
$$= \frac{-1}{4}\tan(7-4x) + C$$

-

Find $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx$

Let $\sin^{-1} x = t$

$$\therefore \frac{1}{\sqrt{1-x^2}}\,dx = dt$$

$$\Rightarrow \int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx = \int t\,dt$$

$$= \frac{t^2}{2} + C$$
$$= \frac{(\sin^{-1} x)^2}{2} + C$$

-

Find $\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}\, dx$

$$\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} = \frac{2\cos x - 3\sin x}{2(3\cos x + 2\sin x)}$$

Let $3\cos x + 2\sin x = t$

$\therefore\ (-3\sin x + 2\cos x)dx = dt$

$$\begin{aligned}\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}\, dx &= \int \frac{dt}{2t} \\ &= \frac{1}{2}\int \frac{1}{t}\, dt \\ &= \frac{1}{2}\log|t| + C \\ &= \frac{1}{2}\log|2\sin x + 3\cos x| + C\end{aligned}$$

-

Find $\int \frac{\sec^2 x}{(1-\tan x)^2}\, dx$

$$\frac{1}{\cos^2 x(1-\tan x)^2} = \frac{\sec^2 x}{(1-\tan x)^2}$$

Let $(1-\tan x) = t$

$\therefore\ -\sec^2 x dx = dt$

$$\begin{aligned}\Rightarrow \int \frac{\sec^2 x}{(1-\tan x)^2}\, dx &= \int \frac{-dt}{t^2} \\ &= -\int t^{-2} dt \\ &= +\frac{1}{t} + C \\ &= \frac{1}{(1-\tan x)} + C\end{aligned}$$

Find $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

Let $\sqrt{x} = t$

$\therefore\ \frac{1}{2\sqrt{x}}\,dx = dt$

$$\Rightarrow \int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = 2\int \cos t\,dt$$
$$= 2\sin t + C$$
$$= 2\sin\sqrt{x} + C$$

-

Find $\int \sqrt{\sin 2x}\ \cos 2x\,dx$

Let $\sin 2x = t$

$\therefore\ 2\cos 2x\,dx = dt$

$$\Rightarrow \int \sqrt{\sin 2x}\ \cos 2x\,dx = \frac{1}{2}\int \sqrt{t}\,dt$$
$$= \frac{1}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$
$$= \frac{1}{3}t^{\frac{3}{2}} + C$$
$$= \frac{1}{3}(\sin 2x)^{\frac{3}{2}} + C$$

-

Find $\int \frac{\cos x}{\sqrt{1+\sin x}} dx$

Let $1+\sin x = t$

$\therefore \cos x \, dx = dt$

$$\Rightarrow \int \frac{\cos x}{\sqrt{1+\sin x}} dx = \int \frac{dt}{\sqrt{t}}$$
$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{1+\sin x} + C$$

-

Find $\int \cot x \, \log \sin x \, dx$

Let $\log \sin x = t$

$$\Rightarrow \frac{1}{\sin x} \cdot \cos x \, dx = dt$$
$$\therefore \cot x \, dx = dt$$

$$\Rightarrow \int \cot x \, \log \sin x \, dx = \int t \, dt$$
$$= \frac{t^2}{2} + C$$
$$= \frac{1}{2}(\log \sin x)^2 + C$$

-

Find $\int \frac{\sin x}{1+\cos x}\,dx$

Let $1 + \cos x = t$

$\therefore -\sin x\,dx = dt$

$$\Rightarrow \int \frac{\sin x}{1+\cos x}\,dx = \int -\frac{dt}{t}$$
$$= -\log|t| + C$$
$$= -\log|1+\cos x| + C$$

-

Find $\int \frac{\sin x}{(1+\cos x)^2}\,dx$

Let $1 + \cos x = t$

$\therefore -\sin x\,dx = dt$

$$\Rightarrow \int \frac{\sin x}{(1+\cos x)^2}\,dx = \int -\frac{dt}{t^2}$$
$$= -\int t^{-2}dt$$
$$= \frac{1}{t} + C$$
$$= \frac{1}{1+\cos x} + C$$

-

$$\text{Let } I = \int \frac{1}{1+\cot x} dx$$
$$= \int \frac{1}{1+\frac{\cos x}{\sin x}} dx$$
$$= \int \frac{\sin x}{\sin x + \cos x} dx$$
$$= \frac{1}{2} \int \frac{2\sin x}{\sin x + \cos x} dx$$
$$= \frac{1}{2} \int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x + \cos x)} dx$$
$$= \frac{1}{2} \int 1\, dx + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$
$$= \frac{1}{2}(x) + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

Let $\sin x + \cos x = t \Rightarrow (\cos x - \sin x)\, dx = dt$

$$\therefore I = \frac{x}{2} + \frac{1}{2} \int \frac{-(dt)}{t}$$
$$= \frac{x}{2} - \frac{1}{2} \log|t| + C$$
$$= \frac{x}{2} - \frac{1}{2} \log|\sin x + \cos x| + C$$

-

$$\text{Let } I = \int \frac{1}{1-\tan x}dx$$
$$= \int \frac{1}{1-\frac{\sin x}{\cos x}}dx$$
$$= \int \frac{\cos x}{\cos x - \sin x}dx$$
$$= \frac{1}{2}\int \frac{2\cos x}{\cos x - \sin x}dx$$
$$= \frac{1}{2}\int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{(\cos x - \sin x)}dx$$
$$= \frac{1}{2}\int 1\,dx + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x}dx$$
$$= \frac{x}{2} + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x}dx$$

Put $\cos x - \sin x = t \Rightarrow (-\sin x - \cos x)\,dx = dt$

$$\therefore I = \frac{x}{2} + \frac{1}{2}\int \frac{-(dt)}{t}$$
$$= \frac{x}{2} - \frac{1}{2}\log|t| + C$$
$$= \frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + C$$

-

$$\text{Let } I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$$
$$= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx$$
$$= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx$$
$$= \int \frac{\sec^2 x \, dx}{\sqrt{\tan x}}$$

Let $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$$\therefore I = \int \frac{dt}{\sqrt{t}}$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{\tan x} + C$$

-

Find $\int \frac{(1+\log x)^2}{x} dx$

Let $1 + \log x = t$

$$\therefore \frac{1}{x} dx = dt$$

$$\Rightarrow \int \frac{(1+\log x)^2}{x} dx = \int t^2 dt$$
$$= \frac{t^3}{3} + C$$
$$= \frac{(1+\log x)^3}{3} + C$$

-

Find $\int\left(1+\frac{1}{x}\right)(x+\log x)^2\,dx$

$$\frac{(x+1)(x+\log x)^2}{x}=\left(\frac{x+1}{x}\right)(x+\log x)^2=\left(1+\frac{1}{x}\right)(x+\log x)^2$$

Let $(x+\log x)=t$

$$\therefore\ \left(1+\frac{1}{x}\right)dx=dt$$

$$\begin{aligned}\Rightarrow \int\left(1+\frac{1}{x}\right)(x+\log x)^2\,dx &= \int t^2\,dt\\ &=\frac{t^3}{3}+C\\ &=\frac{1}{3}(x+\log x)^3+C\end{aligned}$$

-

Find $\int \frac{x^3 \sin\left(\tan^{-1} x^4\right)}{1+x^8} dx$

Let $x^4 = t$

$\therefore 4x^3 dx = dt$

$\Rightarrow \int \frac{x^3 \sin\left(\tan^{-1} x^4\right)}{1+x^8} dx = \frac{1}{4}\int \frac{\sin\left(\tan^{-1} t\right)}{1+t^2} dt$ (1)

Let $\tan^{-1} t = u$

$\therefore \frac{1}{1+t^2} dt = du$

From (1), we obtain

$$\int \frac{x^3 \sin\left(\tan^{-1} x^4\right) dx}{1+x^8} = \frac{1}{4}\int \sin u \, du$$
$$= \frac{1}{4}(-\cos u) + C$$

$$= \frac{-1}{4}\cos\left(\tan^{-1} t\right) + C$$
$$= \frac{-1}{4}\cos\left(\tan^{-1} x^4\right) + C$$

-

Find $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$ equals

(A) $10^x - x^{10} + C$ (B) $10^x + x^{10} + C$

(C) $(10^x - x^{10})^{-1} + C$ (D) $\log(10^x + x^{10}) + C$

Answer :

Let $x^{10} + 10^x = t$

$\therefore (10x^9 + 10^x \log_e 10) dx = dt$

$\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$

$= \log t + C$

$= \log(10^x + x^{10}) + C$

Hence, the correct answer is D.

-

Find $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$ equals

(A) $10^x - x^{10} + C$ (B) $10^x + x^{10} + C$

(C) $\left(10^x - x^{10}\right)^{-1} + C$ (D) $\log\left(10^x + x^{10}\right) + C$

Answer :

Let $x^{10} + 10^x = t$

$\therefore \left(10x^9 + 10^x \log_e 10\right) dx = dt$

$$\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$$

$$= \log t + C$$

$$= \log\left(10^x + x^{10}\right) + C$$

Hence, the correct answer is D.

-

$$\sin^2(2x+5) = \frac{1-\cos 2(2x+5)}{2} = \frac{1-\cos(4x+10)}{2}$$

$$\Rightarrow \int \sin^2(2x+5)\,dx = \int \frac{1-\cos(4x+10)}{2} dx$$

$$= \frac{1}{2}\int 1\,dx - \frac{1}{2}\int \cos(4x+10)\,dx$$

$$= \frac{1}{2}x - \frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right) + C$$

$$= \frac{1}{2}x - \frac{1}{8}\sin(4x+10) + C$$

-

It is known that, $\sin A\cos B = \frac{1}{2}\left\{\sin(A+B)+\sin(A-B)\right\}$

$$\begin{aligned}
\therefore \int \sin 3x\cos 4x\,dx &= \frac{1}{2}\int\left\{\sin(3x+4x)+\sin(3x-4x)\right\}dx\\
&= \frac{1}{2}\int\left\{\sin 7x+\sin(-x)\right\}dx\\
&= \frac{1}{2}\int\left\{\sin 7x-\sin x\right\}dx\\
&= \frac{1}{2}\int\sin 7x\,dx-\frac{1}{2}\int\sin x\,dx\\
&= \frac{1}{2}\left(\frac{-\cos 7x}{7}\right)-\frac{1}{2}(-\cos x)+C\\
&= \frac{-\cos 7x}{14}+\frac{\cos x}{2}+C
\end{aligned}$$

-

It is known that, $\cos A\cos B = \frac{1}{2}\left\{\cos(A+B)+\cos(A-B)\right\}$

$$\begin{aligned}
\therefore \int\cos 2x(\cos 4x\cos 6x)\,dx &= \int\cos 2x\left[\frac{1}{2}\left\{\cos(4x+6x)+\cos(4x-6x)\right\}\right]dx\\
&= \frac{1}{2}\int\left\{\cos 2x\cos 10x+\cos 2x\cos(-2x)\right\}dx\\
&= \frac{1}{2}\int\left\{\cos 2x\cos 10x+\cos^2 2x\right\}dx\\
&= \frac{1}{2}\int\left[\left\{\frac{1}{2}\cos(2x+10x)+\cos(2x-10x)\right\}+\left(\frac{1+\cos 4x}{2}\right)\right]dx\\
&= \frac{1}{4}\int(\cos 12x+\cos 8x+1+\cos 4x)\,dx\\
&= \frac{1}{4}\left[\frac{\sin 12x}{12}+\frac{\sin 8x}{8}+x+\frac{\sin 4x}{4}\right]+C
\end{aligned}$$

-

Let $I = \int \sin^3(2x+1)$

$$\Rightarrow \int \sin^3(2x+1)\,dx = \int \sin^2(2x+1)\cdot\sin(2x+1)\,dx$$
$$= \int \left(1-\cos^2(2x+1)\right)\sin(2x+1)\,dx$$

Let $\cos(2x+1) = t$

$$\Rightarrow -2\sin(2x+1)\,dx = dt$$

$$\Rightarrow \sin(2x+1)\,dx = \frac{-dt}{2}$$

$$\Rightarrow I = \frac{-1}{2}\int\left(1-t^2\right)dt$$
$$= \frac{-1}{2}\left\{t-\frac{t^3}{3}\right\}$$
$$= \frac{-1}{2}\left\{\cos(2x+1)-\frac{\cos^3(2x+1)}{3}\right\}$$
$$= \frac{-\cos(2x+1)}{2}+\frac{\cos^3(2x+1)}{6}+C$$

-

$$\text{Let } I = \int \sin^3 x \cos^3 x \cdot dx$$
$$= \int \cos^3 x \cdot \sin^2 x \cdot \sin x \cdot dx$$
$$= \int \cos^3 x \left(1-\cos^2 x\right) \sin x \cdot dx$$

$$\text{Let } \cos x = t$$
$$\Rightarrow -\sin x \cdot dx = dt$$
$$\Rightarrow I = -\int t^3 \left(1-t^2\right) dt$$
$$= -\int \left(t^3 - t^5\right) dt$$
$$= -\left\{\frac{t^4}{4} - \frac{t^6}{6}\right\} + C$$
$$= -\left\{\frac{\cos^4 x}{4} - \frac{\cos^6 x}{6}\right\} + C$$
$$= \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + C$$

-

Find $\int \frac{1-\cos x}{1+\cos x}dx$

$$\left[2\sin^2\frac{x}{2} = 1-\cos x \text{ and } 2\cos^2\frac{x}{2} = 1+\cos x\right]$$

$$\frac{1-\cos x}{1+\cos x} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$= \tan^2\frac{x}{2}$$

$$= \left(\sec^2\frac{x}{2} - 1\right)$$

$$\therefore \int \frac{1-\cos x}{1+\cos x}dx = \int\left(\sec^2\frac{x}{2} - 1\right)dx$$

$$= \left[\frac{\tan\frac{x}{2}}{\frac{1}{2}} - x\right] + C$$

$$= 2\tan\frac{x}{2} - x + C$$

-

$$\left[\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} \text{ and } \cos x = 2\cos^2\frac{x}{2} - 1\right]$$

$$\frac{\cos x}{1+\cos x} = \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$= \frac{1}{2}\left[1 - \tan^2\frac{x}{2}\right]$$

$$\therefore \int \frac{\cos x}{1+\cos x}dx = \frac{1}{2}\int\left(1 - \tan^2\frac{x}{2}\right)dx$$

$$= \frac{1}{2}\int\left(1 - \sec^2\frac{x}{2} + 1\right)dx$$

$$= \frac{1}{2}\int\left(2 - \sec^2\frac{x}{2}\right)dx$$

$$= \frac{1}{2}\left[2x - \frac{\tan\frac{x}{2}}{\frac{1}{2}}\right] + C$$

$$= x - \tan\frac{x}{2} + C$$

-

$$\begin{aligned}
\sin^4 x &= \sin^2 x \sin^2 x \\
&= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right) \\
&= \frac{1}{4}(1-\cos 2x)^2 \\
&= \frac{1}{4}\left[1+\cos^2 2x - 2\cos 2x\right] \\
&= \frac{1}{4}\left[1+\left(\frac{1+\cos 4x}{2}\right)-2\cos 2x\right] \\
&= \frac{1}{4}\left[1+\frac{1}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right] \\
&= \frac{1}{4}\left[\frac{3}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right]
\end{aligned}$$

$$\begin{aligned}
\therefore \int \sin^4 x \, dx &= \frac{1}{4}\int\left[\frac{3}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right] dx \\
&= \frac{1}{4}\left[\frac{3}{2}x+\frac{1}{2}\left(\frac{\sin 4x}{4}\right)-\frac{2\sin 2x}{2}\right]+C \\
&= \frac{1}{8}\left[3x+\frac{\sin 4x}{4}-2\sin 2x\right]+C
\end{aligned}$$

-

$$\begin{aligned}
\cos^4 2x &= \left(\cos^2 2x\right)^2 \\
&= \left(\frac{1+\cos 4x}{2}\right)^2 \\
&= \frac{1}{4}\left[1+\cos^2 4x + 2\cos 4x\right] \\
&= \frac{1}{4}\left[1+\left(\frac{1+\cos 8x}{2}\right)+2\cos 4x\right] \\
&= \frac{1}{4}\left[1+\frac{1}{2}+\frac{\cos 8x}{2}+2\cos 4x\right] \\
&= \frac{1}{4}\left[\frac{3}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]
\end{aligned}$$

$$\begin{aligned}
\therefore \int \cos^4 2x \, dx &= \int\left(\frac{3}{8}+\frac{\cos 8x}{8}+\frac{\cos 4x}{2}\right)dx \\
&= \frac{3}{8}x+\frac{\sin 8x}{64}+\frac{\sin 4x}{8}+C
\end{aligned}$$

-

$$\left[\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2};\ \cos x = 2\cos^2\frac{x}{2}-1\right]$$

$$\begin{aligned}
\frac{\sin^2 x}{1+\cos x} &= \frac{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}} \\
&= \frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \\
&= 2\sin^2\frac{x}{2} \\
&= 1-\cos x
\end{aligned}$$

$$\begin{aligned}
\therefore \int \frac{\sin^2 x}{1+\cos x}dx &= \int(1-\cos x)dx \\
&= x-\sin x + C
\end{aligned}$$

$$\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}=\frac{-2\sin\frac{2x+2\alpha}{2}\sin\frac{2x-2\alpha}{2}}{-2\sin\frac{x+\alpha}{2}\sin\frac{x-\alpha}{2}}\quad\left[\cos C-\cos D=-2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$=\frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$=\frac{\left[2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\right]\left[2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)\right]}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$=4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$=2\left[\cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right)+\cos\frac{x+\alpha}{2}-\frac{x-\alpha}{2}\right]$$

$$=2\left[\cos(x)+\cos\alpha\right]$$

$$=2\cos x+2\cos\alpha$$

$$\therefore\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx=\int 2\cos x+2\cos\alpha$$

$$=2\left[\sin x+x\cos\alpha\right]+\mathrm{C}$$

-

$$\frac{\cos x-\sin x}{1+\sin 2x}=\frac{\cos x-\sin x}{\left(\sin^2 x+\cos^2 x\right)+2\sin x\cos x}$$

$$\left[\sin^2 x+\cos^2 x=1;\ \ \sin 2x=2\sin x\cos x\right]$$

$$=\frac{\cos x-\sin x}{\left(\sin x+\cos x\right)^2}$$

Let $\sin x+\cos x=t$

$\therefore\left(\cos x-\sin x\right)dx=dt$

$$\Rightarrow\int\frac{\cos x-\sin x}{1+\sin 2x}dx=\int\frac{\cos x-\sin x}{\left(\sin x+\cos x\right)^2}dx$$

$$=\int\frac{dt}{t^2}$$

$$=\int t^{-2}dt$$

$$=-t^{-1}+C$$

$$=-\frac{1}{t}+C$$

$$=\frac{-1}{\sin x+\cos x}+C$$

-

$$\tan^3 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$$
$$= \left(\sec^2 2x - 1\right) \tan 2x \sec 2x$$
$$= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x$$
$$\therefore \int \tan^3 2x \sec 2x \, dx = \int \sec^2 2x \tan 2x \sec 2x \, dx - \int \tan 2x \sec 2x \, dx$$
$$= \int \sec^2 2x \tan 2x \sec 2x \, dx - \frac{\sec 2x}{2} + C$$

Let $\sec 2x = t$

$\therefore 2 \sec 2x \tan 2x \, dx = dt$

$$\therefore \int \tan^3 2x \sec 2x \, dx = \frac{1}{2} \int t^2 dt - \frac{\sec 2x}{2} + C$$
$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$$
$$= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C$$

-

$$\begin{aligned}&\tan^4 x\\&=\tan^2 x\cdot\tan^2 x\\&=\left(\sec^2 x-1\right)\tan^2 x\\&=\sec^2 x\tan^2 x-\tan^2 x\\&=\sec^2 x\tan^2 x-\left(\sec^2 x-1\right)\\&=\sec^2 x\tan^2 x-\sec^2 x+1\end{aligned}$$

$$\begin{aligned}\therefore\ \int\tan^4 x\,dx&=\int\sec^2 x\tan^2 x\,dx-\int\sec^2 x\,dx+\int 1\cdot dx\\&=\int\sec^2 x\tan^2 x\,dx-\tan x+x+C\qquad\ldots(1)\end{aligned}$$

Consider $\int\sec^2 x\tan^2 x\,dx$

Let $\tan x=t\Rightarrow\sec^2 x\,dx=dt$

$$\Rightarrow\int\sec^2 x\tan^2 x dx=\int t^2 dt=\frac{t^3}{3}=\frac{\tan^3 x}{3}$$

From equation (1), we obtain

$$\int\tan^4 x\,dx=\frac{1}{3}\tan^3 x-\tan x+x+C$$

-

$$\begin{aligned}\frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}&=\frac{\sin^3 x}{\sin^2 x\cos^2 x}+\frac{\cos^3 x}{\sin^2 x\cos^2 x}\\&=\frac{\sin x}{\cos^2 x}+\frac{\cos x}{\sin^2 x}\\&=\tan x\sec x+\cot x\operatorname{cosec} x\end{aligned}$$

$$\begin{aligned}\therefore\ \int\frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}\,dx&=\int\left(\tan x\sec x+\cot x\operatorname{cosec} x\right)dx\\&=\sec x-\operatorname{cosec} x+C\end{aligned}$$

-

$$\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$$

$$= \frac{\cos 2x + (1 - \cos 2x)}{\cos^2 x} \qquad \left[\cos 2x = 1 - 2\sin^2 x\right]$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$\therefore \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\, dx = \int \sec^2 x\, dx = \tan x + C$$

-

$$\frac{\cos 2x}{(\cos x + \sin x)^2} = \frac{\cos 2x}{\cos^2 x + \sin^2 x + 2\sin x \cos x} = \frac{\cos 2x}{1 + \sin 2x}$$

$$\therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2}\, dx = \int \frac{\cos 2x}{(1 + \sin 2x)}\, dx$$

Let $1 + \sin 2x = t$

$\Rightarrow 2\cos 2x\, dx = dt$

$$\therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2}\, dx = \frac{1}{2}\int \frac{1}{t} dt$$

$$= \frac{1}{2}\log|t| + C$$

$$= \frac{1}{2}\log|1 + \sin 2x| + C$$

$$= \frac{1}{2}\log\left|(\sin x + \cos x)^2\right| + C$$

$$= \log|\sin x + \cos x| + C$$

-

$\sin^{-1}(\cos x)$

Let $\cos x = t$

Then, $\sin x = \sqrt{1-t^2}$

$$\Rightarrow (-\sin x)dx = dt$$

$$dx = \frac{-dt}{\sin x}$$

$$dx = \frac{-dt}{\sqrt{1-t^2}}$$

$$\therefore \int \sin^{-1}(\cos x)dx = \int \sin^{-1} t\left(\frac{-dt}{\sqrt{1-t^2}}\right)$$

$$= -\int \frac{\sin^{-1} t}{\sqrt{1-t^2}}dt$$

$$\left[\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

Substituting in (1) we get

$$\int \sin^{-1}(\cos x)\, dx = \frac{-\left[\frac{\pi}{2}-x\right]^2}{2} + C$$

$$= -\frac{1}{2}\left(\frac{\pi^2}{2} + x^2 - \pi x\right) + C$$

$$= -\frac{\pi^2}{8} - \frac{x^2}{2} + \frac{1}{2}\pi x + C$$

$$= \frac{\pi x}{2} - \frac{x^2}{2} + \left(C - \frac{\pi^2}{8}\right)$$

$$= \frac{\pi x}{2} - \frac{x^2}{2} + C_1$$

-

$$\frac{1}{\cos(x-a)\cos(x-b)}=\frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\frac{\sin\left[(x-b)-(x-a)\right]}{\cos(x-a)\cos(x-b)}\right]$$

$$=\frac{1}{\sin(a-b)}\frac{\left[\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)\right]}{\cos(x-a)\cos(x-b)}$$

$$=\frac{1}{\sin(a-b)}\left[\tan(x-b)-\tan(x-a)\right]$$

$$\Rightarrow\int\frac{1}{\cos(x-a)\cos(x-b)}dx=\frac{1}{\sin(a-b)}\int\left[\tan(x-b)-\tan(x-a)\right]dx$$

$$=\frac{1}{\sin(a-b)}\left[-\log\left|\cos(x-b)\right|+\log\left|\cos(x-a)\right|\right]$$

$$=\frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+C$$

-

$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$ is equal to

A. tan x + cot x + C

B. tan x + cosec x + C

C. - tan x + cot x + C

D. tan x + sec x + C

Answer :

$$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$$
$$= \int \left(\sec^2 x - \operatorname{cosec}^2 x \right) dx$$
$$= \tan x + \cot x + C$$

Hence, the correct answer is A.

-

$$\int \frac{e^x (1+x)}{\cos^2 (e^x x)} dx$$

Let $e^x x = t$

$$\Rightarrow \left(e^x \cdot x + e^x \cdot 1 \right) dx = dt$$
$$e^x (x+1) dx = dt$$

$$\therefore \int \frac{e^x (1+x)}{\cos^2 (e^x x)} dx = \int \frac{dt}{\cos^2 t}$$
$$= \int \sec^2 t \, dt$$
$$= \tan t + C$$
$$= \tan \left(e^x \cdot x \right) + C$$

-

$$\sin^2(2x+5) = \frac{1-\cos 2(2x+5)}{2} = \frac{1-\cos(4x+10)}{2}$$

$$\Rightarrow \int \sin^2(2x+5)\,dx = \int \frac{1-\cos(4x+10)}{2}dx$$

$$= \frac{1}{2}\int 1\,dx - \frac{1}{2}\int \cos(4x+10)\,dx$$

$$= \frac{1}{2}x - \frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right) + C$$

$$= \frac{1}{2}x - \frac{1}{8}\sin(4x+10) + C$$

-

It is known that, $\sin A \cos B = \frac{1}{2}\{\sin(A+B) + \sin(A-B)\}$

$$\therefore \int \sin 3x \cos 4x\,dx = \frac{1}{2}\int\{\sin(3x+4x) + \sin(3x-4x)\}\,dx$$

$$= \frac{1}{2}\int\{\sin 7x + \sin(-x)\}\,dx$$

$$= \frac{1}{2}\int\{\sin 7x - \sin x\}\,dx$$

$$= \frac{1}{2}\int \sin 7x\,dx - \frac{1}{2}\int \sin x\,dx$$

$$= \frac{1}{2}\left(\frac{-\cos 7x}{7}\right) - \frac{1}{2}(-\cos x) + C$$

$$= \frac{-\cos 7x}{14} + \frac{\cos x}{2} + C$$

-

AIEEE 2007 - Integration of a Sin x + b Cos x form in denominator easy

$$\int \frac{dx}{a \operatorname{Sin} x + b \operatorname{Cos} x}$$

https://archive.org/details/AIEEEIntegrationOfASinXBCosXFormInDenominatorEasy2007

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AIEEE 2008 - Nature of cubic curve, x^3 - p x - q where is maxima, where is minima

https://archive.org/details/AIEEENatureOfCubicCurveX3PXQWhereMaximaWhereMinima2008

AIEEE 2008 - Tricks with monotonously increasing curves, How many real roots ?

https://archive.org/details/AIEEETricksWithMonotonouslyIncreasingCurvesHowManyRealRoots2008

AIEEE 2009 - Tricks with monotonously increasing curves, How many times crosses x-axis

https://archive.org/details/AIEEETricksWithMonotonouslyIncreasingCurvesHowManyTimesCrossesXAxis2009

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It is known that, $\cos A \cos B = \frac{1}{2}\{\cos(A+B)+\cos(A-B)\}$

$$\therefore \int \cos 2x(\cos 4x \cos 6x)\,dx = \int \cos 2x\left[\frac{1}{2}\{\cos(4x+6x)+\cos(4x-6x)\}\right]dx$$

$$= \frac{1}{2}\int\{\cos 2x \cos 10x + \cos 2x \cos(-2x)\}\,dx$$

$$= \frac{1}{2}\int\{\cos 2x \cos 10x + \cos^2 2x\}\,dx$$

$$= \frac{1}{2}\int\left[\left\{\frac{1}{2}\cos(2x+10x)+\cos(2x-10x)\right\}+\left(\frac{1+\cos 4x}{2}\right)\right]dx$$

$$= \frac{1}{4}\int(\cos 12x + \cos 8x + 1 + \cos 4x)\,dx$$

$$= \frac{1}{4}\left[\frac{\sin 12x}{12}+\frac{\sin 8x}{8}+x+\frac{\sin 4x}{4}\right]+C$$

CBSE 2012 Integral Calculus Sin x Sin 2x Sin 3x dx Just convert Multiple angle and subtractions

$\int$ Sin x Sin 2x Sin 3x dx

It is known that, $\sin A \sin B = \frac{1}{2}\{\cos(A-B) - \cos(A+B)\}$

$$\therefore \int \sin x \sin 2x \sin 3x \, dx = \int \left[\sin x \cdot \frac{1}{2}\{\cos(2x-3x) - \cos(2x+3x)\} \right] dx$$

$$= \frac{1}{2}\int (\sin x \cos(-x) - \sin x \cos 5x) \, dx$$

$$= \frac{1}{2}\int (\sin x \cos x - \sin x \cos 5x) \, dx$$

$$= \frac{1}{2}\int \frac{\sin 2x}{2} \, dx - \frac{1}{2}\int \sin x \cos 5x \, dx$$

$$= \frac{1}{4}\left[\frac{-\cos 2x}{2}\right] - \frac{1}{2}\int \left\{\frac{1}{2}\sin(x+5x) + \sin(x-5x)\right\} dx$$

$$= \frac{-\cos 2x}{8} - \frac{1}{4}\int (\sin 6x + \sin(-4x)) \, dx$$

$$= \frac{-\cos 2x}{8} - \frac{1}{4}\left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{4}\right] + C$$

$$= \frac{-\cos 2x}{8} - \frac{1}{8}\left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{2}\right] + C$$

$$= \frac{1}{8}\left[\frac{\cos 6x}{3} - \frac{\cos 4x}{2} - \cos 2x\right] + C$$

https://archive.org/details/CBSE2012IntegralCalculusSinXSin2xSin3xDxJustConvertMultipleAngleSubtractions

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Differentiation of Sec inverse and then Integration. Integration as reverse of Differentiation.

https://archive.org/details/2DiffSecInverseVariousIntegrationsIITJEEMaths

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Integration of 1 by (Sin x + Cos x)

$$\int \frac{dx}{\text{Sin } x + \text{Cos } x}$$

https://archive.org/details/2Integration1BySinXCosXIITJEEMaths

-

Integration of 1 by (x^2 - a^2)

$$\int \frac{dx}{x^2 - a^2}$$

https://archive.org/details/2Integration2ByX2A2IITJEEMaths

https://archive.org/details/Integral1ByA2X2CanBeEasilySplitIntoPartialFractionsAndDone

Again Integration of 1 by root (x^2 - a^2)

Again $\displaystyle\int \frac{dx}{\sqrt{x^2 - a^2}}$

https://archive.org/details/4IntegrationRootXSquareASquareIITJEEMath

-

Integration of Ln x by x plus 1 dx with limits and without limits

$$\int \frac{\ln x \, dx}{x + 1}$$

https://archive.org/details/1IntegrationOfLnXByXPlus1DxWithLimitsAndWithoutLimits

https://archive.org/details/2IntegrationOfLnXByXPlus1DxWithLimitsAndWithoutLimits

-

Integrate sec x by (sec x + tan x)

$$\int \frac{\operatorname{Sec} x}{\operatorname{Sec} x + \tan x} dx$$

https://archive.org/details/IntegralStupidAndSmartWayOfDoingSecXBySecXTanX

https://archive.org/details/2Integration3BySecXTanXIITJEEMaths

-

Integration 1 by (Cos x + Cosec x)

$$\int \frac{dx}{\operatorname{Cos} x + \operatorname{Cosec} x}$$

https://archive.org/details/2Integration4ByCosXCosecXIITJEEMaths

-

Integration of 1 by (Cot x + Cosec x)

$$\int \frac{dx}{\operatorname{Cot} x + \operatorname{Cosec} x}$$

https://archive.org/details/2IntegrationByCotXCosecXIITJEEMaths

-

Integration 1 by (Cot x + Sec x)

$$\int \frac{dx}{\operatorname{Cot} x + \sec x}$$

https://archive.org/details/2IntegrationByCotXSecXIITJEEMaths

-

Integration of 1 by (Tan x + Cosec x)

$$\int \frac{dx}{\tan x + \text{Cosec } x}$$

https://archive.org/details/2IntegrationByTanXCosecXIITJEEMath1

https://archive.org/details/2VariousIntegrationsP5ByTanXCosecXIITJEEMaths

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CBSE 2012 Integrate ∫ Sin x Sin 2x Sin 3x dx

https://archive.org/details/CBSE2012IntegralSinXSin2xSin3xSplitIntoTerms

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Many examples of Integrals discussed

https://archive.org/details/2VariousIntegrationsP4IITJEEMaths

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A student learns only if he practices a lot. No one else, say a teacher, can practice in his behalf. Only by repeated practice can the student remember.

$\int \frac{1}{x^4 - 1} dx$ can be solved by partial fraction or by dividing both Numerator and Denominator by x^2

Solution by partial fraction

$$\frac{1}{(x^4-1)} = \frac{1}{(x^2-1)(x^2+1)} = \frac{1}{(x+1)(x-1)(1+x^2)}$$

$$\text{Let } \frac{1}{(x+1)(x-1)(1+x^2)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{Cx+D}{(x^2+1)}$$

$$1 = A(x-1)(x^2+1) + B(x+1)(x^2+1) + (Cx+D)(x^2-1)$$

$$1 = A(x^3+x-x^2-1) + B(x^3+x+x^2+1) + Cx^3 + Dx^2 - Cx - D$$

$$1 = (A+B+C)x^3 + (-A+B+D)x^2 + (A+B-C)x + (-A+B-D)$$

Equating the coefficient of x^3, x^2, x, and constant term, we obtain

$$A+B+C=0$$
$$-A+B+D=0$$
$$A+B-C=0$$
$$-A+B-D=1$$

On solving these equations, we obtain

$$A = -\frac{1}{4}, B = \frac{1}{4}, C = 0, \text{ and } D = -\frac{1}{2}$$

$$\therefore \frac{1}{x^4-1} = \frac{-1}{4(x+1)} + \frac{1}{4(x-1)} - \frac{1}{2(x^2+1)}$$

$$\Rightarrow \int \frac{1}{x^4-1}\,dx = -\frac{1}{4}\log|x-1| + \frac{1}{4}\log|x-1| - \frac{1}{2}\tan^{-1} x + C$$

$$= \frac{1}{4}\log\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\tan^{-1} x + C$$

Important Integration of x^2 by (x^4 + 1)

$$\int \frac{x^2}{x^4+1}\,dx$$

https://archive.org/details/2VariousIntegrationsP6X2ByX41IITJEEMaths

https://archive.org/details/4IntegrationModificationOfDenominatorIITJEEMath

Find $\int \frac{1}{x\left(x^n+1\right)} dx$

Multiplying numerator and denominator by x^{n-1}, we obtain

$$\frac{1}{x\left(x^n+1\right)} = \frac{x^{n-1}}{x^{n-1}x\left(x^n+1\right)} = \frac{x^{n-1}}{x^n\left(x^n+1\right)}$$

Let $x^n = t \Rightarrow x^{n-1}dx = dt$

$$\therefore \int \frac{1}{x\left(x^n+1\right)} dx = \int \frac{x^{n-1}}{x^n\left(x^n+1\right)} dx = \frac{1}{n}\int \frac{1}{t(t+1)} dt$$

Let $\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{(t+1)}$

$$1 = A(1+t) + Bt \qquad \ldots(1)$$

Substituting $t = 0, -1$ in equation (1), we obtain

$A = 1$ and $B = -1$

$$\therefore \frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{(1+t)}$$

$$\Rightarrow \int \frac{1}{x\left(x^n+1\right)} dx = \frac{1}{n}\int \left\{\frac{1}{t} - \frac{1}{(t+1)}\right\} dx$$

$$= \frac{1}{n}\left[\log|t| - \log|t+1|\right] + C$$

$$= -\frac{1}{n}\left[\log\left|x^n\right| - \log\left|x^n+1\right|\right] + C$$

$$= \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$$

-

Rationalize and Simplify the Integral then split and solve individually

https://archive.org/details/RationalizeAndSimplifyTheIntegralThenSplitAndSolveIndividuallyPart1

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Slightly advanced …. Integration 1 by (a Cos x + b)

$$\int \frac{dx}{a \cos x + b}$$

https://archive.org/details/4Integrations7ByACosXBIITJEEMath

-

Slightly advanced …. Integration 1 by (Sec x + Cosec x)

$$\int \frac{dx}{\sec x + \operatorname{cosec} x}$$

https://archive.org/details/4Integrations8VariousStd1112IITJEEMath

-

$$\text{Let } I = \int \sin^3(2x+1)$$

$$\Rightarrow \int \sin^3(2x+1)\,dx = \int \sin^2(2x+1)\cdot\sin(2x+1)\,dx$$

$$= \int \left(1-\cos^2(2x+1)\right)\sin(2x+1)\,dx$$

$$\text{Let} \cos(2x+1) = t$$

$$\Rightarrow -2\sin(2x+1)\,dx = dt$$

$$\Rightarrow \sin(2x+1)\,dx = \frac{-dt}{2}$$

$$\Rightarrow I = \frac{-1}{2}\int \left(1-t^2\right)dt$$

$$= \frac{-1}{2}\left\{t - \frac{t^3}{3}\right\}$$

$$= \frac{-1}{2}\left\{\cos(2x+1) - \frac{\cos^3(2x+1)}{3}\right\}$$

$$= \frac{-\cos(2x+1)}{2} + \frac{\cos^3(2x+1)}{6} + C$$

-

$$\begin{aligned}\text{Let } I &= \int \sin^3 x \cos^3 x \cdot dx \\ &= \int \cos^3 x \cdot \sin^2 x \cdot \sin x \cdot dx \\ &= \int \cos^3 x\left(1-\cos^2 x\right)\sin x \cdot dx\end{aligned}$$

$$\begin{aligned}&\text{Let } \cos x = t \\ &\Rightarrow -\sin x \cdot dx = dt \\ &\Rightarrow I = -\int t^3\left(1-t^2\right)dt \\ &\quad = -\int\left(t^3-t^5\right)dt \\ &\quad = -\left\{\frac{t^4}{4}-\frac{t^6}{6}\right\}+\mathrm{C} \\ &\quad = -\left\{\frac{\cos^4 x}{4}-\frac{\cos^6 x}{6}\right\}+\mathrm{C} \\ &\quad = \frac{\cos^6 x}{6}-\frac{\cos^4 x}{4}+\mathrm{C}\end{aligned}$$

-

$$\frac{\cos x}{1+\cos x} = \frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \qquad \left[\cos x = \cos^2\frac{x}{2}-\sin^2\frac{x}{2} \text{ and } \cos x = 2\cos^2\frac{x}{2}-1\right]$$

$$= \frac{1}{2}\left[1-\tan^2\frac{x}{2}\right]$$

$$\begin{aligned}\therefore \int\frac{\cos x}{1+\cos x}dx &= \frac{1}{2}\int\left(1-\tan^2\frac{x}{2}\right)dx \\ &= \frac{1}{2}\int\left(1-\sec^2\frac{x}{2}+1\right)dx \\ &= \frac{1}{2}\int\left(2-\sec^2\frac{x}{2}\right)dx \\ &= \frac{1}{2}\left[2x-\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]+\mathrm{C}\end{aligned}$$

$$= x - \tan\frac{x}{2} + C$$

-

$$\begin{aligned}
\sin^4 x &= \sin^2 x \sin^2 x \\
&= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right) \\
&= \frac{1}{4}(1-\cos 2x)^2 \\
&= \frac{1}{4}\left[1+\cos^2 2x - 2\cos 2x\right] \\
&= \frac{1}{4}\left[1+\left(\frac{1+\cos 4x}{2}\right) - 2\cos 2x\right] \\
&= \frac{1}{4}\left[1+\frac{1}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right] \\
&= \frac{1}{4}\left[\frac{3}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right]
\end{aligned}$$

$$\begin{aligned}
\therefore \int \sin^4 x\, dx &= \frac{1}{4}\int\left[\frac{3}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right] dx \\
&= \frac{1}{4}\left[\frac{3}{2}x+\frac{1}{2}\left(\frac{\sin 4x}{4}\right) - \frac{2\sin 2x}{2}\right] + C \\
&= \frac{1}{8}\left[3x+\frac{\sin 4x}{4} - 2\sin 2x\right] + C
\end{aligned}$$

-

$$\begin{aligned}
\cos^4 2x &= \left(\cos^2 2x\right)^2 \\
&= \left(\frac{1+\cos 4x}{2}\right)^2 \\
&= \frac{1}{4}\left[1+\cos^2 4x + 2\cos 4x\right] \\
&= \frac{1}{4}\left[1+\left(\frac{1+\cos 8x}{2}\right)+2\cos 4x\right] \\
&= \frac{1}{4}\left[1+\frac{1}{2}+\frac{\cos 8x}{2}+2\cos 4x\right] \\
&= \frac{1}{4}\left[\frac{3}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]
\end{aligned}$$

$$\begin{aligned}
\therefore \int \cos^4 2x\, dx &= \int\left(\frac{3}{8}+\frac{\cos 8x}{8}+\frac{\cos 4x}{2}\right)dx \\
&= \frac{3}{8}x+\frac{\sin 8x}{64}+\frac{\sin 4x}{8}+C
\end{aligned}$$

-

$$\frac{\sin^2 x}{1+\cos x} = \frac{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}} \quad \left[\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2};\ \cos x = 2\cos^2\frac{x}{2}-1\right]$$

$$\begin{aligned}
&= \frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \\
&= 2\sin^2\frac{x}{2} \\
&= 1-\cos x
\end{aligned}$$

$$\begin{aligned}
\therefore \int \frac{\sin^2 x}{1+\cos x}dx &= \int(1-\cos x)dx \\
&= x-\sin x+C
\end{aligned}$$

-

$$\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}=\frac{-2\sin\frac{2x+2\alpha}{2}\sin\frac{2x-2\alpha}{2}}{-2\sin\frac{x+\alpha}{2}\sin\frac{x-\alpha}{2}} \qquad \left[\cos C-\cos D=-2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$=\frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$=\frac{\left[2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\right]\left[2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)\right]}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$=4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$=2\left[\cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right)+\cos\frac{x+\alpha}{2}-\frac{x-\alpha}{2}\right]$$

$$=2\left[\cos(x)+\cos\alpha\right]$$

$$=2\cos x+2\cos\alpha$$

$$\therefore\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx=\int 2\cos x+2\cos\alpha$$

$$=2\left[\sin x+x\cos\alpha\right]+C$$

-

$$\frac{\cos x-\sin x}{1+\sin 2x}=\frac{\cos x-\sin x}{\left(\sin^2 x+\cos^2 x\right)+2\sin x\cos x}$$

$$\left[\sin^2 x+\cos^2 x=1;\ \sin 2x=2\sin x\cos x\right]$$

$$=\frac{\cos x-\sin x}{\left(\sin x+\cos x\right)^2}$$

Let $\sin x+\cos x=t$

$\therefore \left(\cos x-\sin x\right)dx=dt$

$$\Rightarrow \int\frac{\cos x-\sin x}{1+\sin 2x}dx=\int\frac{\cos x-\sin x}{\left(\sin x+\cos x\right)^2}dx$$

$$=\int\frac{dt}{t^2}$$

$$=\int t^{-2}dt$$

$$=-t^{-1}+C$$

$$=-\frac{1}{t}+C$$

$$=\frac{-1}{\sin x+\cos x}+C$$

-

$$\tan^3 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$$
$$= \left(\sec^2 2x - 1\right) \tan 2x \sec 2x$$
$$= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x$$
$$\therefore \int \tan^3 2x \sec 2x \, dx = \int \sec^2 2x \tan 2x \sec 2x \, dx - \int \tan 2x \sec 2x \, dx$$
$$= \int \sec^2 2x \tan 2x \sec 2x \, dx - \frac{\sec 2x}{2} + C$$

Let $\sec 2x = t$

$\therefore 2 \sec 2x \tan 2x \, dx = dt$

$$\therefore \int \tan^3 2x \sec 2x \, dx = \frac{1}{2} \int t^2 dt - \frac{\sec 2x}{2} + C$$
$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$$
$$= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C$$

-

$\tan^4 x$

$= \tan^2 x \cdot \tan^2 x$

$= \left(\sec^2 x - 1\right) \tan^2 x$

$= \sec^2 x \tan^2 x - \tan^2 x$

$= \sec^2 x \tan^2 x - \left(\sec^2 x - 1\right)$

$= \sec^2 x \tan^2 x - \sec^2 x + 1$

$$\therefore \int \tan^4 x \, dx = \int \sec^2 x \tan^2 x \, dx - \int \sec^2 x \, dx + \int 1 \cdot dx$$
$$= \int \sec^2 x \tan^2 x \, dx - \tan x + x + C \qquad \ldots(1)$$

Consider $\int \sec^2 x \tan^2 x \, dx$

Let $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$$\Rightarrow \int \sec^2 x \tan^2 x dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$$

From equation (1), we obtain

$$\int \tan^4 x \, dx = \frac{1}{3}\tan^3 x - \tan x + x + C$$

-

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x}$$
$$= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}$$
$$= \tan x \sec x + \cot x \operatorname{cosec} x$$
$$\therefore \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int (\tan x \sec x + \cot x \operatorname{cosec} x) dx$$
$$= \sec x - \operatorname{cosec} x + C$$

-

$$\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$$
$$= \frac{\cos 2x + (1 - \cos 2x)}{\cos^2 x} \qquad [\cos 2x = 1 - 2\sin^2 x]$$
$$= \frac{1}{\cos^2 x}$$
$$= \sec^2 x$$
$$\therefore \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx = \int \sec^2 x \, dx = \tan x + C$$

-

$$\frac{\cos 2x}{(\cos x+\sin x)^2}=\frac{\cos 2x}{\cos^2 x+\sin^2 x+2\sin x\cos x}=\frac{\cos 2x}{1+\sin 2x}$$

$$\therefore \int\frac{\cos 2x}{(\cos x+\sin x)^2}\,dx=\int\frac{\cos 2x}{(1+\sin 2x)}\,dx$$

Let $1+\sin 2x=t$

$\Rightarrow 2\cos 2x\,dx=dt$

$$\therefore \int\frac{\cos 2x}{(\cos x+\sin x)^2}\,dx=\frac{1}{2}\int\frac{1}{t}dt$$

$$=\frac{1}{2}\log|t|+C$$

$$=\frac{1}{2}\log|1+\sin 2x|+C$$

$$=\frac{1}{2}\log\left|(\sin x+\cos x)^2\right|+C$$

$$=\log|\sin x+\cos x|+C$$

-

$$\frac{1}{\cos(x-a)\cos(x-b)}=\frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)}\right]$$

$$=\frac{1}{\sin(a-b)}\frac{[\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)]}{\cos(x-a)\cos(x-b)}$$

$$=\frac{1}{\sin(a-b)}[\tan(x-b)-\tan(x-a)]$$

$$\Rightarrow \int\frac{1}{\cos(x-a)\cos(x-b)}\,dx=\frac{1}{\sin(a-b)}\int[\tan(x-b)-\tan(x-a)]\,dx$$

$$=\frac{1}{\sin(a-b)}\left[-\log|\cos(x-b)|+\log|\cos(x-a)|\right]$$

$$=\frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+C$$

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$$

Let $e^x x = t$

$$\Rightarrow (e^x \cdot x + e^x \cdot 1)dx = dt$$

$$e^x(x+1)dx = dt$$

$$\begin{aligned} \therefore \int \frac{e^x(1+x)}{\cos^2(e^x x)}\, dx &= \int \frac{dt}{\cos^2 t} \\ &= \int \sec^2 t\ dt \\ &= \tan t + C \\ &= \tan(e^x \cdot x) + C \end{aligned}$$

-

Let $x^3 = t$

$\therefore 3x^2\ dx = dt$

$$\begin{aligned} \Rightarrow \int \frac{3x^2}{x^6+1}dx &= \int \frac{dt}{t^2+1} \\ &= \tan^{1} t + C \\ &= \tan^{-1}(x^3) + C \end{aligned}$$

-

Let $2x = t$

$\therefore 2dx = dt$

$$\begin{aligned} \Rightarrow \int \frac{1}{\sqrt{1+4x^2}}dx &= \frac{1}{2}\int \frac{dt}{\sqrt{1+t^2}} \\ &= \frac{1}{2}\left[\log\left|t+\sqrt{t^2+1}\right|\right]+C \qquad \left[\int \frac{1}{\sqrt{x^2+a^2}}dt = \log\left|x+\sqrt{x^2+a^2}\right|\right] \\ &= \frac{1}{2}\log\left|2x+\sqrt{4x^2+1}\right|+C \end{aligned}$$

Let $2 - x = t$

$\Rightarrow -dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{(2-x)^2+1}}dx = -\int \frac{1}{\sqrt{t^2+1}}dt$$

$$= -\log\left|t+\sqrt{t^2+1}\right| + C \qquad \left[\int \frac{1}{\sqrt{x^2+a^2}}dt = \log\left|x+\sqrt{x^2+a^2}\right|\right]$$

$$= -\log\left|2-x+\sqrt{(2-x)^2+1}\right| + C$$

$$= \log\left|\frac{1}{(2-x)+\sqrt{x^2-4x+5}}\right| + C$$

-

Let $5x = t$

$\therefore 5dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{9-25x^2}}\,dx = \frac{1}{5}\int \frac{1}{9-t^2}\,dt$$

$$= \frac{1}{5}\int \frac{1}{\sqrt{3^2-t^2}}\,dt$$

$$= \frac{1}{5}\sin^{-1}\left(\frac{t}{3}\right) + C$$

$$= \frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + C$$

-

Let $\sqrt{2}x^2 = t$

$\therefore\ 2\sqrt{2}x\ dx = dt$

$$\Rightarrow \int \frac{3x}{1+2x^4}dx = \frac{3}{2\sqrt{2}}\int \frac{dt}{1+t^2}$$
$$= \frac{3}{2\sqrt{2}}\left[\tan^{-1} t\right] + C$$
$$= \frac{3}{2\sqrt{2}}\tan^{-1}\left(\sqrt{2}x^2\right) + C$$

-

Let $x^3 = t$

$\therefore 3x^2\ dx = dt$

$$\Rightarrow \int \frac{x^2}{1-x^6}dx = \frac{1}{3}\int \frac{dt}{1-t^2}$$
$$= \frac{1}{3}\left[\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|\right] + C$$
$$= \frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + C$$

-

$$\int \frac{x-1}{\sqrt{x^2-1}}\,dx = \int \frac{x}{\sqrt{x^2-1}}\,dx - \int \frac{1}{\sqrt{x^2-1}}\,dx \qquad \ldots(1)$$

For $\int \frac{x}{\sqrt{x^2-1}}dx$, let $x^2-1=t \Rightarrow 2x\,dx = dt$

$$\therefore \int \frac{x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$
$$= \frac{1}{2}\int t^{-\frac{1}{2}}dt$$
$$= \frac{1}{2}\left[2t^{\frac{1}{2}}\right]$$
$$= \sqrt{t}$$
$$= \sqrt{x^2-1}$$

From (1), we obtain

$$\int \frac{x-1}{\sqrt{x^2-1}}dx = \int \frac{x}{\sqrt{x^2-1}}dx - \int \frac{1}{\sqrt{x^2-1}}dx \qquad \left[\int \frac{1}{\sqrt{x^2-a^2}}dt = \log\left|x+\sqrt{x^2-a^2}\right|\right]$$
$$= \sqrt{x^2-1} - \log\left|x+\sqrt{x^2-1}\right| + C$$

-

Let $x^3 = t$

$\Rightarrow 3x^2\,dx = dt$

$$\therefore \int \frac{x^2}{\sqrt{x^6+a^6}}dx = \frac{1}{3}\int \frac{dt}{\sqrt{t^2+\left(a^3\right)^2}}$$
$$= \frac{1}{3}\log\left|t+\sqrt{t^2+a^6}\right| + C$$
$$= \frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right| + C$$

-

Let $\tan x = t$

$\therefore \sec^2 x\, dx = dt$

$$\Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx = \int \frac{dt}{\sqrt{t^2 + 2^2}}$$

$$= \log\left|t + \sqrt{t^2 + 4}\right| + C$$

$$= \log\left|\tan x + \sqrt{\tan^2 x + 4}\right| + C$$

-

$7 - 6x - x^2$ can be written as $7 - \left(x^2 + 6x + 9 - 9\right)$.

Therefore,

$$7 - \left(x^2 + 6x + 9 - 9\right)$$

$$= 16 - \left(x^2 + 6x + 9\right)$$

$$= 16 - (x+3)^2$$

$$= (4)^2 - (x+3)^2$$

$$\therefore \int \frac{1}{\sqrt{7 - 6x - x^2}} dx = \int \frac{1}{\sqrt{(4)^2 - (x+3)^2}} dx$$

Let $x + 3 = t$

$\Rightarrow dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{(4)^2 - (x+3)^2}} dx = \int \frac{1}{\sqrt{(4)^2 - (t)^2}} dt$$

$$= \sin^{-1}\left(\frac{t}{4}\right) + C$$

$$= \sin^{-1}\left(\frac{x+3}{4}\right) + C$$

-

$(x-1)(x-2)$ can be written as x^2-3x+2.

Therefore,

$$x^2-3x+2$$

$$=x^2-3x+\frac{9}{4}-\frac{9}{4}+2$$

$$=\left(x-\frac{3}{2}\right)^2-\frac{1}{4}$$

$$=\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2$$

$$\therefore \int \frac{1}{\sqrt{(x-1)(x-2)}}\,dx=\int \frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}\,dx$$

-

Let $x-\frac{3}{2}=t$

$\therefore\ dx=dt$

$$\Rightarrow \int \frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}\,dx=\int \frac{1}{\sqrt{t^2-\left(\frac{1}{2}\right)^2}}\,dt$$

$$=\log\left|t+\sqrt{t^2-\left(\frac{1}{2}\right)^2}\right|+C$$

$$=\log\left|\left(x-\frac{3}{2}\right)+\sqrt{x^2-3x+2}\right|+C$$

-

$8+3x-x^2$ can be written as $8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$.

Therefore,

$$8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$$

$$=\frac{41}{4}-\left(x-\frac{3}{2}\right)^2$$

$$\Rightarrow \int \frac{1}{\sqrt{8+3x-x^2}}dx = \int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx$$

Let $x-\frac{3}{2}=t$

$\therefore\ dx=dt$

$$\Rightarrow \int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx = \int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2-t^2}}dt$$

$$=\sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{2}}\right)+C$$

$$=\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}}\right)+C$$

$$=\sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right)+C$$

-

$(x-a)(x-b)$ can be written as $x^2-(a+b)x+ab$.

Therefore,

$$x^2-(a+b)x+ab$$

$$=x^2-(a+b)x+\frac{(a+b)^2}{4}-\frac{(a+b)^2}{4}+ab$$

$$=\left[x-\left(\frac{a+b}{2}\right)\right]^2-\frac{(a-b)^2}{4}$$

$$\Rightarrow\int\frac{1}{\sqrt{(x-a)(x-b)}}dx=\int\frac{1}{\sqrt{\left\{x-\left(\frac{a+b}{2}\right)\right\}^2-\left(\frac{a-b}{2}\right)^2}}dx$$

Let $x-\left(\frac{a+b}{2}\right)=t$

$\therefore\ dx=dt$

$$\Rightarrow\int\frac{1}{\sqrt{\left\{x-\left(\frac{a+b}{2}\right)\right\}^2-\left(\frac{a-b}{2}\right)^2}}dx=\int\frac{1}{\sqrt{t^2-\left(\frac{a-b}{2}\right)^2}}dt$$

$$=\log\left|t+\sqrt{t^2-\left(\frac{a-b}{2}\right)^2}\right|+C$$

$$=\log\left|\left\{x-\left(\frac{a+b}{2}\right)\right\}+\sqrt{(x-a)(x-b)}\right|+C$$

-

$$\text{Let } 4x+1 = A\frac{d}{dx}\left(2x^2+x-3\right)+B$$
$$\Rightarrow 4x+1 = A\left(4x+1\right)+B$$
$$\Rightarrow 4x+1 = 4Ax+A+B$$

Equating the coefficients of x and constant term on both sides, we obtain

$4A = 4 \Rightarrow A = 1$

$A + B = 1 \Rightarrow B = 0$

Let $2x^2 + x - 3 = t$

$\therefore (4x + 1)\ dx = dt$

$$\Rightarrow \int\frac{4x+1}{\sqrt{2x^2+x-3}}dx = \int\frac{1}{\sqrt{t}}dt$$
$$= 2\sqrt{t}+C$$
$$= 2\sqrt{2x^2+x-3}+C$$

-

$$\text{Let } x+2 = A\frac{d}{dx}\left(x^2-1\right)+B \qquad ...(1)$$
$$\Rightarrow x+2 = A(2x)+B$$

Equating the coefficients of x and constant term on both sides, we obtain

$$2A = 1 \Rightarrow A = \frac{1}{2}$$
$$B = 2$$

From (1), we obtain

$$(x+2) = \frac{1}{2}(2x)+2$$

$$\text{Then, } \int \frac{x+2}{\sqrt{x^2-1}}dx = \int \frac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}}dx$$
$$= \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx + \int \frac{2}{\sqrt{x^2-1}}dx \qquad ...(2)$$

$$\text{In } \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx, \text{ let } x^2-1 = t \Rightarrow 2x\,dx = dt$$
$$\frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$= \frac{1}{2}\left[2\sqrt{t}\right]$$
$$= \sqrt{t}$$
$$= \sqrt{x^2-1}$$

$$\text{Then, } \int \frac{2}{\sqrt{x^2-1}}dx = 2\int \frac{1}{\sqrt{x^2-1}}dx = 2\log\left|x+\sqrt{x^2-1}\right|$$

From equation (2), we obtain

$$\int \frac{x+2}{\sqrt{x^2-1}}dx = \sqrt{x^2-1} + 2\log\left|x+\sqrt{x^2-1}\right| + C$$

-

$$\frac{6x+7}{\sqrt{(x-5)(x-4)}}=\frac{6x+7}{\sqrt{x^2-9x+20}}$$

$$\text{Let } 6x+7=A\frac{d}{dx}\left(x^2-9x+20\right)+B$$
$$\Rightarrow 6x+7=A(2x-9)+B$$

Equating the coefficients of x and constant term, we obtain

$2A = 6 \Rightarrow A = 3$

$-9A + B = 7 \Rightarrow B = 34$

$\therefore 6x + 7 = 3(2x - 9) + 34$

$$\int\frac{6x+7}{\sqrt{x^2-9x+20}}=\int\frac{3(2x-9)+34}{\sqrt{x^2-9x+20}}dx$$
$$=3\int\frac{2x-9}{\sqrt{x^2-9x+20}}dx+34\int\frac{1}{\sqrt{x^2-9x+20}}dx$$

$$\text{Let } I_1=\int\frac{2x-9}{\sqrt{x^2-9x+20}}dx \text{ and } I_2=\int\frac{1}{\sqrt{x^2-9x+20}}dx$$

$$\therefore \int\frac{6x+7}{\sqrt{x^2-9x+20}}=3I_1+34I_2 \qquad \ldots(1)$$

Then,

$$I_1 = \int \frac{2x-9}{\sqrt{x^2-9x+20}}dx$$

Let $x^2 - 9x + 20 = t$

$$\Rightarrow (2x-9)dx = dt$$

$$\Rightarrow I_1 = \frac{dt}{\sqrt{t}}$$

$$I_1 = 2\sqrt{t}$$

$$I_1 = 2\sqrt{x^2-9x+20} \qquad (2)$$

and $I_2 = \int \frac{1}{\sqrt{x^2-9x+20}}dx$

$x^2 - 9x + 20$ can be written as $x^2 - 9x + 20 + \frac{81}{4} - \frac{81}{4}$.

Therefore,

$$x^2 - 9x + 20 + \frac{81}{4} - \frac{81}{4}$$

$$= \left(x - \frac{9}{2}\right)^2 - \frac{1}{4}$$

$$= \left(x - \frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$\Rightarrow I_2 = \int \frac{1}{\sqrt{\left(x-\frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}dx$$

$$I_2 = \log\left|\left(x - \frac{9}{2}\right) + \sqrt{x^2-9x+20}\right| \qquad ...(3)$$

Substituting equations (2) and (3) in (1), we obtain

$$\int \frac{6x+7}{\sqrt{x^2-9x+20}}dx = 3\left[2\sqrt{x^2-9x+20}\right] + 34\log\left[\left(x-\frac{9}{2}\right) + \sqrt{x^2-9x+20}\right] + C$$

$$= 6\sqrt{x^2-9x+20} + 34\log\left[\left(x-\frac{9}{2}\right) + \sqrt{x^2-9x+20}\right] + C$$

Find $\int \frac{x+2}{\sqrt{4x-x^2}}dx$

Let $x+2 = A\frac{d}{dx}\left(4x-x^2\right)+B$

$\Rightarrow x+2 = A(4-2x)+B$

Equating the coefficients of x and constant term on both sides, we obtain

$-2A = 1 \Rightarrow A = -\frac{1}{2}$

$4A+B = 2 \Rightarrow B = 4$

$\Rightarrow (x+2) = -\frac{1}{2}(4-2x)+4$

$$\therefore \int \frac{x+2}{\sqrt{4x-x^2}}dx = \int \frac{-\frac{1}{2}(4-2x)+4}{\sqrt{4x-x^2}}dx$$

$$= -\frac{1}{2}\int \frac{4-2x}{\sqrt{4x-x^2}}dx + 4\int \frac{1}{\sqrt{4x-x^2}}dx$$

Let $I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}}dx$ and $I_2 \int \frac{1}{\sqrt{4x-x^2}}dx$

$$\therefore \int \frac{x+2}{\sqrt{4x-x^2}}dx = -\frac{1}{2}I_1 + 4I_2 \qquad ...(1)$$

Then, $I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}}dx$

Let $4x - x^2 = t$

$\Rightarrow (4-2x)dx = dt$

$$\Rightarrow I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{4x-x^2} \qquad ...(2)$$

$$I_2 = \int \frac{1}{\sqrt{4x-x^2}}dx$$

$$\begin{aligned}\Rightarrow 4x - x^2 &= -\left(-4x + x^2\right) \\ &= \left(-4x + x^2 + 4 - 4\right) \\ &= 4 - (x-2)^2 \\ &= (2)^2 - (x-2)^2\end{aligned}$$

$$\therefore I_2 = \int \frac{1}{\sqrt{(2)^2-(x-2)^2}}dx = \sin^{-1}\left(\frac{x-2}{2}\right) \qquad ...(3)$$

Using equations (2) and (3) in (1), we obtain

$$\begin{aligned}\int \frac{x+2}{\sqrt{4x-x^2}}dx &= -\frac{1}{2}\left(2\sqrt{4x-x^2}\right) + 4\sin^{-1}\left(\frac{x-2}{2}\right) + C \\ &= -\sqrt{4x-x^2} + 4\sin^{-1}\left(\frac{x-2}{2}\right) + C\end{aligned}$$

-

$$\int \frac{(x+2)}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}\int \frac{2(x+2)}{\sqrt{x^2+2x+3}}dx$$
$$= \frac{1}{2}\int \frac{2x+4}{\sqrt{x^2+2x+3}}dx$$
$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx + \frac{1}{2}\int \frac{2}{\sqrt{x^2+2x+3}}dx$$
$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx + \int \frac{1}{\sqrt{x^2+2x+3}}dx$$

Let $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$ and $I_2 = \int \frac{1}{\sqrt{x^2+2x+3}}dx$

$$\therefore \int \frac{x+2}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}I_1 + I_2 \qquad ...(1)$$

Then, $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$

Let $x^2 + 2x + 3 = t$

$\Rightarrow (2x+2)\, dx = dt$

$$I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{x^2+2x+3} \qquad ...(2)$$

$$I_2 = \int \frac{1}{\sqrt{x^2+2x+3}}dx$$

$$\Rightarrow x^2+2x+3 = x^2+2x+1+2 = (x+1)^2 + \left(\sqrt{2}\right)^2$$

$$\therefore I_2 = \int \frac{1}{\sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}}dx = \log\left|(x+1)+\sqrt{x^2+2x+3}\right| \qquad ...(3)$$

Using equations (2) and (3) in (1), we obtain

$$\int \frac{x+2}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}\left[2\sqrt{x^2+2x+3}\right] + \log\left|(x+1)+\sqrt{x^2+2x+3}\right| + C$$
$$= \sqrt{x^2+2x+3} + \log\left|(x+1)+\sqrt{x^2+2x+3}\right| + C$$

-

Find $\int \frac{x+3}{x^2-2x-5}dx$

$$\text{Let } (x+3) = A\frac{d}{dx}\left(x^2-2x-5\right)+B$$
$$(x+3) = A(2x-2)+B$$

Equating the coefficients of x and constant term on both sides, we obtain

$$2A = 1 \Rightarrow A = \frac{1}{2}$$
$$-2A+B = 3 \Rightarrow B = 4$$
$$\therefore (x+3) = \frac{1}{2}(2x-2)+4$$
$$\Rightarrow \int \frac{x+3}{x^2-2x-5}dx = \int \frac{\frac{1}{2}(2x-2)+4}{x^2-2x-5}dx$$
$$= \frac{1}{2}\int \frac{2x-2}{x^2-2x-5}dx + 4\int \frac{1}{x^2-2x-5}dx$$

$$\text{Let } I_1 = \int \frac{2x-2}{x^2-2x-5}dx \text{ and } I_2 = \int \frac{1}{x^2-2x-5}dx$$
$$\therefore \int \frac{x+3}{\left(x^2-2x-5\right)}dx = \frac{1}{2}I_1 + 4I_2 \qquad ...(1)$$
$$\text{Then, } I_1 = \int \frac{2x-2}{x^2-2x-5}dx$$

$$\text{Let } x^2 - 2x - 5 = t$$
$$\Rightarrow (2x-2)\,dx = dt$$
$$\Rightarrow I_1 = \int \frac{dt}{t} = \log|t| = \log\left|x^2 - 2x - 5\right| \qquad ...(2)$$

$$I_2 = \int \frac{1}{x^2 - 2x - 5}dx$$
$$= \int \frac{1}{(x^2 - 2x + 1) - 6}dx$$
$$= \int \frac{1}{(x-1)^2 + \left(\sqrt{6}\right)^2}dx$$
$$= \frac{1}{2\sqrt{6}}\log\left(\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right) \qquad ...(3)$$

Substituting (2) and (3) in (1), we obtain

$$\int \frac{x+3}{x^2-2x-5}dx = \frac{1}{2}\log\left|x^2 - 2x - 5\right| + \frac{4}{2\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$
$$= \frac{1}{2}\log\left|x^2 - 2x - 5\right| + \frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$

-

Find $\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$

$$\text{Let } 5x+3 = A\frac{d}{dx}\left(x^2+4x+10\right)+B$$
$$\Rightarrow 5x+3 = A\left(2x+4\right)+B$$

Equating the coefficients of x and constant term, we obtain

$$2A = 5 \Rightarrow A = \frac{5}{2}$$
$$4A + B = 3 \Rightarrow B = -7$$
$$\therefore 5x+3 = \frac{5}{2}(2x+4)-7$$
$$\Rightarrow \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = \int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}dx$$
$$= \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}}dx - 7\int \frac{1}{\sqrt{x^2+4x+10}}dx$$
$$\text{Let } I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}}dx \text{ and } I_2 = \int \frac{1}{\sqrt{x^2+4x+10}}dx$$
$$\therefore \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = \frac{5}{2}I_1 - 7I_2 \qquad ...(1)$$
$$\text{Then, } I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}}dx$$

Let $x^2+4x+10=t$

$\therefore (2x+4)dx=dt$

$$\Rightarrow I_1=\int\frac{dt}{t}=2\sqrt{t}=2\sqrt{x^2+4x+10} \qquad ...(2)$$

$$I_2=\int\frac{1}{\sqrt{x^2+4x+10}}dx$$

$$=\int\frac{1}{\sqrt{(x^2+4x+4)+6}}dx$$

$$=\int\frac{1}{(x+2)^2+\left(\sqrt{6}\right)^2}dx$$

$$=\log\left|(x+2)\sqrt{x^2+4x+10}\right| \qquad ...(3)$$

Using equations (2) and (3) in (1), we obtain

$$\int\frac{5x+3}{\sqrt{x^2+4x+10}}dx=\frac{5}{2}\left[2\sqrt{x^2+4x+10}\right]-7\log\left|(x+2)+\sqrt{x^2+4x+10}\right|+C$$

$$=5\sqrt{x^2+4x+10}-7\log\left|(x+2)+\sqrt{x^2+4x+10}\right|+C$$

-

$$\int\frac{dx}{x^2+2x+2}=\int\frac{dx}{(x^2+2x+1)+1}$$

$$=\int\frac{1}{(x+1)^2+(1)^2}dx$$

$$=\left[\tan^{-1}(x+1)\right]+C$$

Hence, the correct answer is B.

-

$$\int \frac{dx}{\sqrt{9x-4x^2}}$$

$$= \int \frac{1}{\sqrt{-4\left(x^2-\frac{9}{4}x\right)}}dx$$

$$= \int \frac{1}{-4\left(x^2-\frac{9}{4}x+\frac{81}{64}-\frac{81}{64}\right)}dx$$

$$= \int \frac{1}{\sqrt{-4\left[\left(x-\frac{9}{8}\right)^2-\left(\frac{9}{8}\right)^2\right]}}dx$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{\left(\frac{9}{8}\right)^2-\left(x-\frac{9}{8}\right)^2}}dx$$

$$= \frac{1}{2}\left[\sin^{-1}\left(\frac{x-\frac{9}{8}}{\frac{9}{8}}\right)\right]+C \qquad \left(\int \frac{dy}{\sqrt{a^2-y^2}} = \sin^{-1}\frac{y}{a}+C\right)$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$$

-

Find $\int \frac{x}{(x+1)(x+2)} dx$

Let $\frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$

$\Rightarrow x = A(x+2) + B(x+1)$

Equating the coefficients of x and constant term, we obtain

$A + B = 1$

$2A + B = 0$

On solving, we obtain

$A = -1$ and $B = 2$

$$\therefore \frac{x}{(x+1)(x+2)} = \frac{-1}{(x+1)} + \frac{2}{(x+2)}$$

$$\begin{aligned}
\Rightarrow \int \frac{x}{(x+1)(x+2)} dx &= \int \frac{-1}{(x+1)} + \frac{2}{(x+2)} dx \\
&= -\log|x+1| + 2\log|x+2| + C \\
&= \log(x+2)^2 - \log|x+1| + C \\
&= \log \frac{(x+2)^2}{(x+1)} + C
\end{aligned}$$

-

Find $\int \frac{1}{(x^2-9)} dx$

Let $\frac{1}{(x+3)(x-3)} = \frac{A}{(x+3)} + \frac{B}{(x-3)}$

$1 = A(x-3) + B(x+3)$

Equating the coefficients of x and constant term, we obtain

$A + B = 0$

$-3A + 3B = 1$

On solving, we obtain

$A = -\frac{1}{6}$ and $B = \frac{1}{6}$

$$\therefore \frac{1}{(x+3)(x-3)} = \frac{-1}{6(x+3)} + \frac{1}{6(x-3)}$$

$$\Rightarrow \int \frac{1}{(x^2-9)} dx = \int \left(\frac{-1}{6(x+3)} + \frac{1}{6(x-3)} \right) dx$$

$$= -\frac{1}{6} \log|x+3| + \frac{1}{6} \log|x-3| + C$$

$$= \frac{1}{6} \log\left| \frac{(x-3)}{(x+3)} \right| + C$$

-

Find $\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx$

Let $\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) \qquad \ldots(1)$$

Substituting x = 1, 2, and 3 respectively in equation (1), we obtain

A = 1, B = - 5, and C = 4

$$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}$$

$$\Rightarrow \int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \left\{ \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)} \right\} dx$$

$$= \log|x-1| - 5\log|x-2| + 4\log|x-3| + C$$

-

Find $\int \frac{x}{(x-1)(x-2)(x-3)} dx$

Let $\frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$$x = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) \qquad \ldots(1)$$

Substituting x = 1, 2, and 3 respectively in equation (1), we obtain $A = \frac{1}{2}$, $B = -2$, and $C = \frac{3}{2}$

$$\therefore \frac{x}{(x-1)(x-2)(x-3)} = \frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)}$$

$$\Rightarrow \int \frac{x}{(x-1)(x-2)(x-3)} dx = \int \left\{ \frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)} \right\} dx$$

$$= \frac{1}{2}\log|x-1| - 2\log|x-2| + \frac{3}{2}\log|x-3| + C$$

-

Find $\int \frac{2x}{(x+1)(x+2)} dx$

Let $\frac{2x}{x^2+3x+2} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$

$$2x = A(x+2) + B(x+1) \qquad ...(1)$$

Substituting x = - 1 and - 2 in equation (1), we obtain

A = - 2 and B = 4

$$\therefore \frac{2x}{(x+1)(x+2)} = \frac{-2}{(x+1)} + \frac{4}{(x+2)}$$

$$\Rightarrow \int \frac{2x}{(x+1)(x+2)} dx = \int \left\{ \frac{4}{(x+2)} - \frac{2}{(x+1)} \right\} dx$$

$$= 4\log|x+2| - 2\log|x+1| + C$$

-

Find $\int \frac{1-x^2}{x(1-2x)} dx$

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $(1 - x^2)$ by $x(1 - 2x)$, we obtain

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2}\left(\frac{2-x}{x(1-2x)}\right)$$

Let $\frac{2-x}{x(1-2x)} = \frac{A}{x} + \frac{B}{(1-2x)}$

$$\Rightarrow (2-x) = A(1-2x) + Bx \qquad ...(1)$$

Substituting $x = 0$ and $\frac{1}{2}$ in equation (1), we obtain

$A = 2$ and $B = 3$

$$\therefore \frac{2-x}{x(1-2x)} = \frac{2}{x} + \frac{3}{1-2x}$$

Substituting in equation (1), we obtain

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2}\left\{\frac{2}{x} + \frac{3}{(1-2x)}\right\}$$

$$\Rightarrow \int \frac{1-x^2}{x(1-2x)} dx = \int \left\{\frac{1}{2} + \frac{1}{2}\left(\frac{2}{x} + \frac{3}{1-2x}\right)\right\} dx$$

$$= \frac{x}{2} + \log|x| + \frac{3}{2(-2)} \log|1-2x| + C$$

$$= \frac{x}{2} + \log|x| - \frac{3}{4} \log|1-2x| + C$$

-

Find $\int \frac{x \, dx}{(x^2+1)(x-1)}$

Let $\frac{x}{(x^2+1)(x-1)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{(x-1)}$

$x = (Ax+B)(x-1) + C(x^2+1)$

$x = Ax^2 - Ax + Bx - B + Cx^2 + C$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + C = 0$

$-A + B = 1$

$-B + C = 0$

On solving these equations, we obtain

$A = -\frac{1}{2}, B = \frac{1}{2}, \text{ and } C = \frac{1}{2}$

From equation (1), we obtain

$$\therefore \frac{x}{(x^2+1)(x-1)} = \frac{\left(-\frac{1}{2}x+\frac{1}{2}\right)}{x^2+1} + \frac{\frac{1}{2}}{(x-1)}$$

$$\Rightarrow \int \frac{x}{(x^2+1)(x-1)} = -\frac{1}{2}\int \frac{x}{x^2+1}dx + \frac{1}{2}\int \frac{1}{x^2+1}dx + \frac{1}{2}\int \frac{1}{x-1}dx$$

$$= -\frac{1}{4}\int \frac{2x}{x^2+1}dx + \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log|x-1| + C$$

Consider $\int \frac{2x}{x^2+1}dx$, let $(x^2+1) = t \Rightarrow 2x\,dx = dt$

$$\Rightarrow \int \frac{2x}{x^2+1}dx = \int \frac{dt}{t} = \log|t| = \log|x^2+1|$$

$$\therefore \int \frac{x\ dx}{(x^2+1)(x-1)} = -\frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log|x-1| + C$$

$$= \frac{1}{2}\log|x-1| - \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + C$$

-

Find $\int \frac{x}{(x-1)^2(x+2)} dx$

Let $\frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$

$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$

Substituting $x = 1$, we obtain

$B = \frac{1}{3}$

Equating the coefficients of x^2 and constant term, we obtain

$A + C = 0$

$-2A + 2B + C = 0$

On solving, we obtain

$A = \frac{2}{9}$ and $C = \frac{-2}{9}$

$$\therefore \frac{x}{(x-1)^2(x+2)} = \frac{2}{9(x-1)} + \frac{1}{3(x-1)^2} - \frac{2}{9(x+2)}$$

$$\Rightarrow \int \frac{x}{(x-1)^2(x+2)} dx = \frac{2}{9}\int \frac{1}{(x-1)} dx + \frac{1}{3}\int \frac{1}{(x-1)^2} dx - \frac{2}{9}\int \frac{1}{(x+2)} dx$$

$$= \frac{2}{9}\log|x-1| + \frac{1}{3}\left(\frac{-1}{x-1}\right) - \frac{2}{9}\log|x+2| + C$$

$$= \frac{2}{9}\log\left|\frac{x-1}{x+2}\right| - \frac{1}{3(x-1)} + C$$

-

Find $\int \frac{3x+5}{(x-1)^2(x+1)}dx$

$$\frac{3x+5}{x^3-x^2-x+1}=\frac{3x+5}{(x-1)^2(x+1)}$$

Let $\frac{3x+5}{(x-1)^2(x+1)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x+1)}$

$$3x+5=A(x-1)(x+1)+B(x+1)+C(x-1)^2$$
$$3x+5=A(x^2-1)+B(x+1)+C(x^2+1-2x) \qquad ...(1)$$

Substituting $x = 1$ in equation (1), we obtain

$B = 4$

Equating the coefficients of x^2 and x, we obtain

$A + C = 0$

$B - 2C = 3$

On solving, we obtain

$A=-\frac{1}{2}$ and $C=\frac{1}{2}$

$$\therefore \frac{3x+5}{(x-1)^2(x+1)}=\frac{-1}{2(x-1)}+\frac{4}{(x-1)^2}+\frac{1}{2(x+1)}$$

$$\Rightarrow \int\frac{3x+5}{(x-1)^2(x+1)}dx=-\frac{1}{2}\int\frac{1}{x-1}dx+4\int\frac{1}{(x-1)^2}dx+\frac{1}{2}\int\frac{1}{(x+1)}dx$$
$$=-\frac{1}{2}\log|x-1|+4\left(\frac{-1}{x-1}\right)+\frac{1}{2}\log|x+1|+C$$
$$=\frac{1}{2}\log\left|\frac{x+1}{x-1}\right|-\frac{4}{(x-1)}+C$$

-

Find $\int \frac{2x-3}{(x^2-1)(2x+3)}dx$

$$\frac{2x-3}{(x^2-1)(2x+3)} = \frac{2x-3}{(x+1)(x-1)(2x+3)}$$

Let $\frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(2x+3)}$

$$\Rightarrow (2x-3) = A(x-1)(2x+3) + B(x+1)(2x+3) + C(x+1)(x-1)$$
$$\Rightarrow (2x-3) = A(2x^2+x-3) + B(2x^2+5x+3) + C(x^2-1)$$
$$\Rightarrow (2x-3) = (2A+2B+C)x^2 + (A+5B)x + (-3A+3B-C)$$

Equating the coefficients of x^2 and x, we obtain

$B = -\frac{1}{10}$, $A = \frac{5}{2}$, and $C = -\frac{24}{5}$

$$\therefore \frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{5}{2(x+1)} - \frac{1}{10(x-1)} - \frac{24}{5(2x+3)}$$
$$\Rightarrow \int \frac{2x-3}{(x^2-1)(2x+3)}dx = \frac{5}{2}\int \frac{1}{(x+1)}dx - \frac{1}{10}\int \frac{1}{x-1}dx - \frac{24}{5}\int \frac{1}{(2x+3)}dx$$
$$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{24}{5\times 2}\log|2x+3|$$
$$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{12}{5}\log|2x+3| + C$$

-

Find $\int \frac{5x}{(x+1)(x^2-4)} dx$

$$\frac{5x}{(x+1)(x^2-4)} = \frac{5x}{(x+1)(x+2)(x-2)}$$

Let $\frac{5x}{(x+1)(x+2)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{C}{(x-2)}$

$$5x = A(x+2)(x-2) + B(x+1)(x-2) + C(x+1)(x+2) \qquad ...(1)$$

Substituting $x = -1, -2$, and 2 respectively in equation (1), we obtain

$$A = \frac{5}{3},\ B = -\frac{5}{2}, \text{ and } C = \frac{5}{6}$$

$$\therefore \frac{5x}{(x+1)(x+2)(x-2)} = \frac{5}{3(x+1)} - \frac{5}{2(x+2)} + \frac{5}{6(x-2)}$$

$$\Rightarrow \int \frac{5x}{(x+1)(x^2-4)} dx = \frac{5}{3}\int \frac{1}{(x+1)} dx - \frac{5}{2}\int \frac{1}{(x+2)} dx + \frac{5}{6}\int \frac{1}{(x-2)} dx$$

$$= \frac{5}{3}\log|x+1| - \frac{5}{2}\log|x+2| + \frac{5}{6}\log|x-2| + C$$

-

Find $\int \frac{x^3+x+1}{x^2-1}dx$

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $(x^3 + x + 1)$ by $x^2 - 1$, we obtain

$$\frac{x^3+x+1}{x^2-1} = x + \frac{2x+1}{x^2-1}$$

Let $\frac{2x+1}{x^2-1} = \frac{A}{(x+1)} + \frac{B}{(x-1)}$

$$2x+1 = A(x-1)+B(x+1) \qquad ...(1)$$

Substituting $x = 1$ and -1 in equation (1), we obtain

$$A = \frac{1}{2} \text{ and } B = \frac{3}{2}$$

$$\therefore \frac{x^3+x+1}{x^2-1} = x + \frac{1}{2(x+1)} + \frac{3}{2(x-1)}$$

$$\Rightarrow \int \frac{x^3+x+1}{x^2-1}dx = \int x\,dx + \frac{1}{2}\int\frac{1}{(x+1)}dx + \frac{3}{2}\int\frac{1}{(x-1)}dx$$

$$= \frac{x^2}{2} + \frac{1}{2}\log|x+1| + \frac{3}{2}\log|x-1| + C$$

-

Find $\int \frac{2}{(1-x)(1+x^2)} dx$

$$\text{Let } \frac{2}{(1-x)(1+x^2)} = \frac{A}{(1-x)} + \frac{Bx+C}{(1+x^2)}$$

$$2 = A(1+x^2) + (Bx+C)(1-x)$$

$$2 = A + Ax^2 + Bx - Bx^2 + C - Cx$$

Equating the coefficient of x^2, x, and constant term, we obtain

$A - B = 0$

$B - C = 0$

$A + C = 2$

On solving these equations, we obtain

$A = 1$, $B = 1$, and $C = 1$

$$\therefore \frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x+1}{1+x^2}$$

$$\Rightarrow \int \frac{2}{(1-x)(1+x^2)} dx = \int \frac{1}{1-x} dx + \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

$$= -\int \frac{1}{x-1} dx + \frac{1}{2} \int \frac{2x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

$$= -\log|x-1| + \frac{1}{2} \log|1+x^2| + \tan^{-1} x + C$$

-

Find $\int \frac{3x-1}{(x+2)^2}\,dx$

$$\text{Let } \frac{3x-1}{(x+2)^2} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$$

$$\Rightarrow 3x-1 = A(x+2)+B$$

Equating the coefficient of x and constant term, we obtain

$A = 3$

$2A + B = -1 \Rightarrow B = -7$

$$\therefore \frac{3x-1}{(x+2)^2} = \frac{3}{(x+2)} - \frac{7}{(x+2)^2}$$

$$\Rightarrow \int \frac{3x-1}{(x+2)^2}\,dx = 3\int \frac{1}{(x+2)}dx - 7\int \frac{x}{(x+2)^2}\,dx$$

$$= 3\log|x+2| - 7\left(\frac{-1}{(x+2)}\right) + C$$

$$= 3\log|x+2| + \frac{7}{(x+2)} + C$$

-

Find $\int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx$

$$\frac{\cos x}{(1-\sin x)(2-\sin x)}$$

Let $\sin x = t \Rightarrow \cos x\, dx = dt$

$$\therefore \int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx = \int \frac{dt}{(1-t)(2-t)}$$

Let $\frac{1}{(1-t)(2-t)} = \frac{A}{(1-t)} + \frac{B}{(2-t)}$

$$1 = A(2-t) + B(1-t) \qquad ...(1)$$

Substituting $t = 2$ and then $t = 1$ in equation (1), we obtain

$A = 1$ and $B = -1$

$$\therefore \frac{1}{(1-t)(2-t)} = \frac{1}{(1-t)} - \frac{1}{(2-t)}$$

$$\begin{aligned}\Rightarrow \int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx &= \int \left\{ \frac{1}{1-t} - \frac{1}{(2-t)} \right\} dt \\ &= -\log|1-t| + \log|2-t| + C \\ &= \log\left|\frac{2-t}{1-t}\right| + C \\ &= \log\left|\frac{2-\sin x}{1-\sin x}\right| + C\end{aligned}$$

-

Find $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx$

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \frac{(4x^2+10)}{(x^2+3)(x^2+4)}$$

$$\text{Let } \frac{4x^2+10}{(x^2+3)(x^2+4)} = \frac{Ax+B}{(x^2+3)} + \frac{Cx+D}{(x^2+4)}$$

$$4x^2+10 = (Ax+B)(x^2+4)+(Cx+D)(x^2+3)$$

$$4x^2+10 = Ax^3+4Ax+Bx^2+4B+Cx^3+3Cx+Dx^2+3D$$

$$4x^2+10 = (A+C)x^3+(B+D)x^2+(4A+3C)x+(4B+3D)$$

Equating the coefficients of x^3, x^2, x, and constant term, we obtain

$A + C = 0$

$B + D = 4$

$4A + 3C = 0$

$4B + 3D = 10$

On solving these equations, we obtain

$A = 0$, $B = -2$, $C = 0$, and $D = 6$

$$\therefore \frac{4x^2+10}{(x^2+3)(x^2+4)} = \frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}$$

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \left(\frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}\right)$$

$$\Rightarrow \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx = \int \left\{1 + \frac{2}{(x^2+3)} - \frac{6}{(x^2+4)}\right\}dx$$

$$= \int \left\{1 + \frac{2}{x^2+\left(\sqrt{3}\right)^2} - \frac{6}{x^2+2^2}\right\}$$

$$= x + 2\left(\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right) - 6\left(\frac{1}{2}\tan^{-1}\frac{x}{2}\right) + C$$
$$= x + \frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - 3\tan^{-1}\frac{x}{2} + C$$

-

Find $\int \frac{2x}{(x^2+1)(x^2+3)}dx$

$$\frac{2x}{(x^2+1)(x^2+3)}$$

Let $x^2 = t \Rightarrow 2x\,dx = dt$

$$\therefore \int \frac{2x}{(x^2+1)(x^2+3)}dx = \int \frac{dt}{(t+1)(t+3)} \qquad ...(1)$$

$$\text{Let } \frac{1}{(t+1)(t+3)} = \frac{A}{(t+1)} + \frac{B}{(t+3)}$$
$$1 = A(t+3) + B(t+1) \qquad ...(1)$$

Substituting $t = -3$ and $t = -1$ in equation (1), we obtain

$$A = \frac{1}{2} \text{ and } B = -\frac{1}{2}$$

$$\therefore \frac{1}{(t+1)(t+3)} = \frac{1}{2(t+1)} - \frac{1}{2(t+3)}$$

$$\Rightarrow \int \frac{2x}{(x^2+1)(x^2+3)}dx = \int\left\{\frac{1}{2(t+1)} - \frac{1}{2(t+3)}\right\}dt$$
$$= \frac{1}{2}\log|(t+1)| - \frac{1}{2}\log|t+3| + C$$
$$= \frac{1}{2}\log\left|\frac{t+1}{t+3}\right| + C$$
$$= \frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + C$$

Find $\int \frac{1}{x(x^4-1)}dx$

Multiplying numerator and denominator by x^3, we obtain

$$\frac{1}{x(x^4-1)} = \frac{x^3}{x^4(x^4-1)}$$

$$\therefore \int \frac{1}{x(x^4-1)}dx = \int \frac{x^3}{x^4(x^4-1)}dx$$

Let $x^4 = t \Rightarrow 4x^3 dx = dt$

$$\therefore \int \frac{1}{x(x^4-1)}dx = \frac{1}{4}\int \frac{dt}{t(t-1)}$$

$$\text{Let } \frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{(t-1)}$$

$$1 = A(t-1) + Bt \qquad \ldots(1)$$

Substituting $t = 0$ and 1 in (1), we obtain

$A = -1$ and $B = 1$

$$\Rightarrow \frac{1}{t(t+1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\Rightarrow \int \frac{1}{x(x^4-1)}dx = \frac{1}{4}\int\left\{\frac{-1}{t} + \frac{1}{t-1}\right\}dt$$

$$= \frac{1}{4}\left[-\log|t| + \log|t-1|\right] + C$$

$$= \frac{1}{4}\log\left|\frac{t-1}{t}\right| + C$$

$$= \frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right| + C$$

-

Find ∫ dx / (e^x - 1)

Let $e^x = t \Rightarrow e^x\,dx = dt$

$$\Rightarrow \int \frac{1}{e^x-1}dx = \int \frac{1}{t-1} \times \frac{dt}{t} = \int \frac{1}{t(t-1)}dt$$

$$\text{Let } \frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$$

$$1 = A(t-1) + Bt \qquad \ldots(1)$$

Substituting $t = 1$ and $t = 0$ in equation (1), we obtain

$A = -1$ and $B = 1$

$$\therefore \frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\Rightarrow \int \frac{1}{t(t-1)}dt = \log\left|\frac{t-1}{t}\right| + C$$

$$= \log\left|\frac{e^x-1}{e^x}\right| + C$$

-

$$\int \frac{x\,dx}{(x-1)(x-2)} \text{ equals}$$

A. $\log\left|\frac{(x-1)^2}{x-2}\right| + C$

B. $\log\left|\frac{(x-2)^2}{x-1}\right| + C$

C. $\log\left|\left(\frac{x-1}{x-2}\right)^2\right| + C$

D. $\log\left|(x-1)(x-2)\right| + C$

Let $\frac{x}{(x-1)(x-2)}=\frac{A}{(x-1)}+\frac{B}{(x-2)}$

$x=A(x-2)+B(x-1)$ ...(1)

Substituting $x = 1$ and 2 in (1), we obtain

$A = -1$ and $B = 2$

$$\therefore \frac{x}{(x-1)(x-2)}=-\frac{1}{(x-1)}+\frac{2}{(x-2)}$$

$$\Rightarrow \int\frac{x}{(x-1)(x-2)}dx=\int\left\{\frac{-1}{(x-1)}+\frac{2}{(x-2)}\right\}dx$$

$$=-\log|x-1|+2\log|x-2|+C$$

$$=\log\left|\frac{(x-2)^2}{x-1}\right|+C$$

Hence, the correct answer is B.

-

- $\int\frac{dx}{x(x^2+1)}$ equals

A. $\log|x|-\frac{1}{2}\log(x^2+1)+C$

B. $\log|x|+\frac{1}{2}\log(x^2+1)+C$

C. $-\log|x|+\frac{1}{2}\log(x^2+1)+C$

D. $\frac{1}{2}\log|x|+\log(x^2+1)+C$

Let $\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$1 = A(x^2+1) + (Bx+C)x$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + B = 0$

$C = 0$

$A = 1$

On solving these equations, we obtain

$A = 1$, $B = -1$, and $C = 0$

$$\therefore \frac{1}{x(x^2+1)} = \frac{1}{x} + \frac{-x}{x^2+1}$$

$$\Rightarrow \int \frac{1}{x(x^2+1)} dx = \int \left\{ \frac{1}{x} - \frac{x}{x^2+1} \right\} dx$$

$$= \log|x| - \frac{1}{2}\log|x^2+1| + C$$

-

Let $I = \int x \sin x \, dx$

Taking x as first function and $\sin x$ as second function and integrating by parts, we obtain

$$I = x\int \sin x \, dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sin x \, dx \right\} dx$$

$$= x(-\cos x) - \int 1 \cdot (-\cos x) dx$$

$$= -x\cos x + \sin x + C$$

-

Let $I = \int x \sin 3x \, dx$

Taking x as first function and sin $3x$ as second function and integrating by parts, we obtain

$$I = x\int \sin 3x \, dx - \int\left\{\left(\frac{d}{dx}x\right)\int \sin 3x \, dx\right\}$$

$$= x\left(\frac{-\cos 3x}{3}\right) - \int 1\cdot\left(\frac{-\cos 3x}{3}\right)dx$$

$$= \frac{-x\cos 3x}{3} + \frac{1}{3}\int \cos 3x \, dx$$

$$= \frac{-x\cos 3x}{3} + \frac{1}{9}\sin 3x + C$$

-

Let $I = \int x^2 e^x dx$

Taking x^2 as first function and e^x as second function and integrating by parts, we obtain

$$I = x^2\int e^x \, dx - \int\left\{\left(\frac{d}{dx}x^2\right)\int e^x dx\right\}dx$$

$$= x^2e^x - \int 2x\cdot e^x dx$$

$$= x^2e^x - 2\int x\cdot e^x dx$$

Again integrating by parts, we obtain

$$= x^2e^x - 2\left[x\cdot\int e^x dx - \int\left\{\left(\frac{d}{dx}x\right)\cdot\int e^x dx\right\}dx\right]$$

$$= x^2e^x - 2\left[xe^x - \int e^x dx\right]$$

$$= x^2e^x - 2\left[xe^x - e^x\right]$$

$$= x^2e^x - 2xe^x + 2e^x + C$$

$$= e^x\left(x^2 - 2x + 2\right) + C$$

-

Let $I = \int x \log x dx$

Taking $\log x$ as first function and x as second function and integrating by parts, we obtain

$$I = \log x \int x\, dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x\, dx \right\} dx$$

$$= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \log x}{2} - \int \frac{x}{2} dx$$

$$= \frac{x^2 \log x}{2} - \frac{x^2}{4} + C$$

-

Let $I = \int x \log 2x dx$

Taking $\log 2x$ as first function and x as second function and integrating by parts, we obtain

$$I = \log 2x \int x\, dx - \int \left\{ \left(\frac{d}{dx} 2 \log x \right) \int x\, dx \right\} dx$$

$$= \log 2x \cdot \frac{x^2}{2} - \int \frac{2}{2x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \log 2x}{2} - \int \frac{x}{2} dx$$

$$= \frac{x^2 \log 2x}{2} - \frac{x^2}{4} + C$$

-

Let $I = \int x^2 \log x\, dx$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$I = \log x \int x^2\, dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$

$$= \log x \left(\frac{x^3}{3} \right) - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \frac{x^3 \log x}{3} - \int \frac{x^2}{3} dx$$

$$= \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$$

Let $I = \int x \sin^{-1} x \, dx$

Taking $\sin^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \sin^{-1} x \int x \, dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1} x \right) \int x \, dx \right\} dx$$

$$= \sin^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} \, dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} \, dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \int \sqrt{1-x^2} \, dx - \int \frac{1}{\sqrt{1-x^2}} \, dx \right\}$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x - \sin^{-1} x \right\} + C$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4} \sqrt{1-x^2} + \frac{1}{4} \sin^{-1} x - \frac{1}{2} \sin^{-1} x + C$$

$$= \frac{1}{4} \left(2x^2 - 1 \right) \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} + C$$

-

Let $I = \int x \tan^{-1} x \, dx$

Taking $\tan^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \tan^{-1} x \int x\,dx - \int\left\{\left(\frac{d}{dx}\tan^{-1} x\right)\int x\,dx\right\}dx$$

$$= \tan^{-1} x\left(\frac{x^2}{2}\right) - \int \frac{1}{1+x^2}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\int \frac{x^2}{1+x^2}dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\int\left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\left(x - \tan^{-1} x\right) + C$$

$$= \frac{x^2}{2}\tan^{-1} x - \frac{x}{2} + \frac{1}{2}\tan^{-1} x + C$$

-

Let $I = \int x \cos^{-1} x dx$

Taking $\cos^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \cos^{-1} x \int x\,dx - \int\left\{\left(\frac{d}{dx}\cos^{-1} x\right)\int x\,dx\right\}dx$$

$$= \cos^{-1} x\frac{x^2}{2} - \int \frac{-1}{\sqrt{1-x^2}}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2}\int \frac{1-x^2-1}{\sqrt{1-x^2}}dx$$

$$= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2}\int\left\{\sqrt{1-x^2} + \left(\frac{-1}{\sqrt{1-x^2}}\right)\right\}dx$$

$$= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2}\int\sqrt{1-x^2}\,dx - \frac{1}{2}\int\left(\frac{-1}{\sqrt{1-x^2}}\right)dx$$

$$= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2}I_1 - \frac{1}{2}\cos^{-1} x \qquad \ldots(1)$$

where, $I_1 = \int \sqrt{1-x^2}\,dx$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int \frac{d}{dx}\sqrt{1-x^2}\int x\,dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int \frac{-2x}{2\sqrt{1-x^2}}.x\,dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int \frac{-x^2}{\sqrt{1-x^2}}dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int \frac{1-x^2-1}{\sqrt{1-x^2}}dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \left\{\int \sqrt{1-x^2}\,dx + \int \frac{-dx}{\sqrt{1-x^2}}\right\}$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \left\{I_1 + \cos^{-1} x\right\}$$

$$\Rightarrow 2I_1 = x\sqrt{1-x^2} - \cos^{-1} x$$

$$\therefore I_1 = \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\cos^{-1} x$$

Substituting in (1), we obtain

$$I = \frac{x\cos^{-1} x}{2} - \frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\cos^{-1} x\right) - \frac{1}{2}\cos^{-1} x$$

$$= \frac{\left(2x^2-1\right)}{4}\cos^{-1} x - \frac{x}{4}\sqrt{1-x^2} + C$$

Let $I = \int (\sin^{-1} x)^2 \cdot 1\, dx$

Taking $(\sin^{-1} x)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$I = (\sin^{-1} x) \int 1 dx - \int \left\{ \frac{d}{dx} (\sin^{-1} x)^2 \cdot \int 1 \cdot dx \right\} dx$$

$$= (\sin^{-1} x)^2 \cdot x - \int \frac{2 \sin^{-1} x}{\sqrt{1-x^2}} \cdot x\, dx$$

$$= x (\sin^{-1} x)^2 + \int \sin^{-1} x \cdot \left(\frac{-2x}{\sqrt{1-x^2}} \right) dx$$

$$= x (\sin^{-1} x)^2 + \left[\sin^{-1} x \int \frac{-2x}{\sqrt{1-x^2}} dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1} x \right) \int \frac{-2x}{\sqrt{1-x^2}} dx \right\} dx \right]$$

$$= x (\sin^{-1} x)^2 + \left[\sin^{-1} x \cdot 2\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2}\, dx \right]$$

$$= x (\sin^{-1} x)^2 + 2\sqrt{1-x^2} \sin^{-1} x - \int 2\, dx$$

$$= x (\sin^{-1} x)^2 + 2\sqrt{1-x^2} \sin^{-1} x - 2x + C$$

-

Let $I = \int \frac{x \cos^{-1} x}{\sqrt{1-x^2}} dx$

$$I = \frac{-1}{2} \int \frac{-2x}{\sqrt{1-x^2}} \cdot \cos^{-1} x dx$$

Taking $\cos^{-1} x$ as first function and $\left(\frac{-2x}{\sqrt{1-x^2}} \right)$ as second function and integrating by parts, we obtain

$$I = \frac{-1}{2} \left[\cos^{-1} x \int \frac{-2x}{\sqrt{1-x^2}} dx - \int \left\{ \left(\frac{d}{dx} \cos^{-1} x \right) \int \frac{-2x}{\sqrt{1-x^2}} dx \right\} dx \right]$$

$$= \frac{-1}{2} \left[\cos^{-1} x \cdot 2\sqrt{1-x^2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2}\, dx \right]$$

$$= \frac{-1}{2} \left[2\sqrt{1-x^2} \cos^{-1} x + \int 2\, dx \right]$$

$$= \frac{-1}{2} \left[2\sqrt{1-x^2} \cos^{-1} x + 2x \right] + C$$

$$= -\left[\sqrt{1-x^2} \cos^{-1} x + x \right] + C$$

-

Let $I = \int x \sec^2 x dx$

Taking x as first function and $\sec^2 x$ as second function and integrating by parts, we obtain

$$I = x \int \sec^2 x \, dx - \int \left\{ \left\{ \frac{d}{dx} x \right\} \int \sec^2 x \, dx \right\} dx$$
$$= x \tan x - \int 1 \cdot \tan x dx$$
$$= x \tan x + \log |\cos x| + C$$

-

Let $I = \int 1 \cdot \tan^{-1} x dx$

Taking $\tan^{-1} x$ as first function and 1 as second function and integrating by parts, we obtain

$$I = \tan^{-1} x \int 1 dx - \int \left\{ \left(\frac{d}{dx} \tan^{-1} x \right) \int 1 \cdot dx \right\} dx$$
$$= \tan^{-1} x \cdot x - \int \frac{1}{1+x^2} \cdot x \, dx$$
$$= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx$$
$$= x \tan^{-1} x - \frac{1}{2} \log |1+x^2| + C$$
$$= x \tan^{-1} x - \frac{1}{2} \log (1+x^2) + C$$

-

Let $I = \int (x^2+1)\log x\,dx = \int x^2 \log x\,dx + \int \log x\,dx$

Let $I = I_1 + I_2$ ... (1)

Where, $I_1 = \int x^2 \log x\,dx$ and $I_2 = \int \log x\,dx$

$I_1 = \int x^2 \log x dx$

Taking log x as first function and x^2 as second function and integrating by parts, we obtain

$$\begin{aligned}
I_1 &= \log x - \int x^2 dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx \\
&= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx \\
&= \frac{x^3}{3} \log x - \frac{1}{3} \left(\int x^2 dx \right) \\
&= \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 \qquad \text{... (2)}
\end{aligned}$$

$I_2 = \int \log x\,dx$

Taking log x as first function and 1 as second function and integrating by parts, we obtain

$$\begin{aligned}
I_2 &= \log x \int 1 \cdot dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int 1 \cdot dx \right\} \\
&= \log x \cdot x - \int \frac{1}{x} \cdot x dx \\
&= x \log x - \int 1 dx \\
&= x \log x - x + C_2 \qquad \text{... (3)}
\end{aligned}$$

Using equations (2) and (3) in (1), we obtain

$$\begin{aligned}
I &= \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 + x \log x - x + C_2 \\
&= \frac{x^3}{3} \log x - \frac{x^3}{9} + x \log x - x + (C_1 + C_2) \\
&= \left(\frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C
\end{aligned}$$

-

Let $I = \int e^x (\sin x + \cos x) dx$

Let $f(x) = \sin x$

$\Rightarrow f'(x) = \cos x$

$\therefore I = \int e^x \{f(x) + f'(x)\} dx$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$\therefore I = e^x \sin x + C$

-

Let $I = \int \frac{xe^x}{(1+x)^2} dx = \int e^x \left\{\frac{x}{(1+x)^2}\right\} dx$

$= \int e^x \left\{\frac{1+x-1}{(1+x)^2}\right\} dx$

$= \int e^x \left\{\frac{1}{1+x} - \frac{1}{(1+x)^2}\right\} dx$

Let $f(x) = \frac{1}{1+x} \Rightarrow f'(x) = \frac{-1}{(1+x)^2}$

$\Rightarrow \int \frac{xe^x}{(1+x)^2} dx = \int e^x \{f(x) + f'(x)\} dx$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$\therefore \int \frac{xe^x}{(1+x)^2} dx = \frac{e^x}{1+x} + C$

-

Find $\int \frac{e^x(1+\sin x)}{(1+\cos x)} dx$

$$e^x\left(\frac{1+\sin x}{1+\cos x}\right)$$

$$= e^x\left(\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$$

$$= \frac{e^x\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$$

$$= \frac{1}{2}e^x\cdot\left(\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}}\right)^2$$

$$= \frac{1}{2}e^x\left[\tan\frac{x}{2}+1\right]^2$$

$$= \frac{1}{2}e^2\left(1+\tan\frac{x}{2}\right)^2$$

$$=\frac{1}{2}e^x\left[1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$

$$=\frac{1}{2}e^x\left[\sec^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$

$$\frac{e^x(1+\sin x)dx}{(1+\cos x)}=e^x\left[\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right] \quad \ldots(1)$$

Let $\tan\frac{x}{2}=f(x) \Rightarrow f'(x)=\frac{1}{2}\sec^2\frac{x}{2}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+C$

From equation (1), we obtain

$$\int\frac{e^x(1+\sin x)}{(1+\cos x)}dx=e^x\tan\frac{x}{2}+C$$

-

Let $I=\int e^x\left[\frac{1}{x}-\frac{1}{x^2}\right]dx$

Also, let $\frac{1}{x}=f(x) \Rightarrow f'(x)=\frac{-1}{x^2}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+C$

$$\therefore I=\frac{e^x}{x}+C$$

-

$$\int e^x\left\{\frac{x-3}{(x-1)^3}\right\}dx=\int e^x\left\{\frac{x-1-2}{(x-1)^3}\right\}dx$$
$$=\int e^x\left\{\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right\}dx$$

Let $f(x)=\frac{1}{(x-1)^2}$ $\Rightarrow$ $f'(x)=\frac{-2}{(x-1)^3}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+C$

$$\therefore \int e^x\left\{\frac{(x-3)}{(x-1)^2}\right\}dx=\frac{e^x}{(x-1)^2}+C$$

-

Let $I=\int e^{2x}\sin x\,dx$ ...(1)

Integrating by parts, we obtain

$$I=\sin x\int e^{2x}dx-\int\left\{\left(\frac{d}{dx}\sin x\right)\int e^{2x}dx\right\}dx$$
$$\Rightarrow I=\sin x\cdot\frac{e^{2x}}{2}-\int\cos x\cdot\frac{e^{2x}}{2}dx$$
$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\int e^{2x}\cos x\,dx$$

Again integrating by parts, we obtain

$$I=\frac{e^{2x}\cdot\sin x}{2}-\frac{1}{2}\left[\cos x\int e^{2x}dx-\int\left\{\left(\frac{d}{dx}\cos x\right)\int e^{2x}dx\right\}dx\right]$$
$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\left[\cos x\cdot\frac{e^{2x}}{2}-\int(-\sin x)\frac{e^{2x}}{2}dx\right]$$
$$\Rightarrow I=\frac{e^{2x}\cdot\sin x}{2}-\frac{1}{2}\left[\frac{e^{2x}\cos x}{2}+\frac{1}{2}\int e^{2x}\sin x dx\right]$$
$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}-\frac{1}{4}I \quad \text{[From (1)]}$$

$$\Rightarrow I+\frac{1}{4}I=\frac{e^{2x}\cdot\sin x}{2}-\frac{e^{2x}\cos x}{4}$$

$$\Rightarrow \frac{5}{4}I=\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}$$

$$\Rightarrow I=\frac{4}{5}\left[\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}\right]+C$$

$$\Rightarrow I=\frac{e^{2x}}{5}\left[2\sin x-\cos x\right]+C$$

-

Find $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

Let $x=\tan\theta \Rightarrow dx=\sec^2\theta\, d\theta$

$$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right)=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)=\sin^{-1}(\sin 2\theta) = 2\theta$$

$$\Rightarrow \int\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx=\int 2\theta\cdot\sec^2\theta\, d\theta=2\int\theta\cdot\sec^2\theta\, d\theta$$

Integrating by parts, we obtain

$$2\left[\theta\cdot\int\sec^2\theta d\theta-\int\left\{\left(\frac{d}{d\theta}\theta\right)\int\sec^2\theta d\theta\right\}d\theta\right]$$

$$=2\left[\theta\cdot\tan\theta-\int\tan\theta d\theta\right]$$

$$=2\left[\theta\tan\theta+\log\left|\cos\theta\right|\right]+C$$

$$=2\left[x\tan^{-1}x+\log\left|\frac{1}{\sqrt{1+x^2}}\right|\right]+C$$

$$=2x\tan^{-1}x+2\log\left(1+x^2\right)^{-\frac{1}{2}}+C$$

$$=2x\tan^{-1}x+2\left[-\frac{1}{2}\log\left(1+x^2\right)\right]+C$$

$$=2x\tan^{-1}x-\log\left(1+x^2\right)+C$$

$\int x^2 e^{x^3}\, dx$ equals

(A) $\frac{1}{3}e^{x^3} + C$ (B) $\frac{1}{3}e^{x^2} + C$

(C) $\frac{1}{2}e^{x^3} + C$ (D) $\frac{1}{3}e^{x^2} + C$

Answer :

Let $I = \int x^2 e^{x^3}\, dx$

Also, let $x^3 = t \Rightarrow 3x^2 dx = dt$

$$\Rightarrow I = \frac{1}{3}\int e^t\, dt$$
$$= \frac{1}{3}\left(e^t\right) + C$$
$$= \frac{1}{3}e^{x^3} + C$$

Hence, the correct answer is A.

-

$\int e^x \sec x(1+\tan x)dx$ equals

(A)	$e^x \cos x + C$	(B)	$e^x \sec x + C$
(C)	$e^x \sin x + C$	(D)	$e^x \tan x + C$

Answer :

$\int e^x \sec x(1+\tan x)dx$

Let $I = \int e^x \sec x(1+\tan x)dx = \int e^x(\sec x + \sec x \tan x)dx$

Also, let $\sec x = f(x) \Rightarrow \sec x \tan x = f'(x)$

It is known that, $\int e^x \{f(x)+f'(x)\}dx = e^x f(x) + C$

$\therefore I = e^x \sec x + C$

Hence, the correct answer is B.

-

Let $I = \int \sqrt{4-x^2}\,dx = \int \sqrt{(2)^2-(x)^2}\,dx$

It is known that, $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$$\begin{aligned}\therefore I &= \frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2} + C \\ &= \frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2} + C\end{aligned}$$

-

Let $I = \int\sqrt{1-4x^2}\,dx = \int\sqrt{(1)^2-(2x)^2}\,dx$

Let $2x = t \Rightarrow 2\,dx = dt$

$\therefore I = \frac{1}{2}\int\sqrt{(1)^2-(t)^2}\,dt$

It is known that, $\int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$$\Rightarrow I = \frac{1}{2}\left[\frac{t}{2}\sqrt{1-t^2}+\frac{1}{2}\sin^{-1}t\right]+C$$

$$= \frac{t}{4}\sqrt{1-t^2}+\frac{1}{4}\sin^{-1}t+C$$

$$= \frac{2x}{4}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+C$$

$$= \frac{x}{2}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+C$$

-

Let $I = \int\sqrt{x^2+4x+6}\;dx$

$$= \int\sqrt{x^2+4x+4+2}\;dx$$

$$= \int\sqrt{\left(x^2+4x+4\right)+2}\;dx$$

$$= \int\sqrt{(x+2)^2+\left(\sqrt{2}\right)^2}\;dx$$

It is known that, $\int\sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x+6}+\frac{2}{2}\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$$

$$= \frac{(x+2)}{2}\sqrt{x^2+4x+6}+\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$$

-

$$\text{Let } I = \int\sqrt{x^2+4x+1}\,dx$$
$$= \int\sqrt{\left(x^2+4x+4\right)-3}\,dx$$
$$= \int\sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}\,dx$$

It is known that, $\int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right| + C$$

-

$$\text{Let } I = \int\sqrt{1-4x-x^2}\,dx$$
$$= \int\sqrt{1-\left(x^2+4x+4-4\right)}\,dx$$
$$= \int\sqrt{1+4-(x+2)^2}\,dx$$
$$= \int\sqrt{\left(\sqrt{5}\right)^2-(x+2)^2}\,dx$$

It is known that, $\int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{1-4x-x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right) + C$$

-

$$\text{Let } I = \int \sqrt{x^2 + 4x - 5}\, dx$$
$$= \int \sqrt{(x^2 + 4x + 4) - 9}\, dx$$
$$= \int \sqrt{(x+2)^2 - (3)^2}\, dx$$

It is known that, $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{x^2 + 4x - 5} - \frac{9}{2}\log\left|(x+2) + \sqrt{x^2 + 4x - 5}\right| + C$$

-

$$\text{Let } I = \int \sqrt{1 + 3x - x^2}\, dx$$
$$= \int \sqrt{1 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)}\, dx$$
$$= \int \sqrt{\left(1 + \frac{9}{4}\right) - \left(x - \frac{3}{2}\right)^2}\, dx$$
$$= \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2}\, dx$$

It is known that, $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$$\therefore I = \frac{x - \frac{3}{2}}{2}\sqrt{1 + 3x - x^2} + \frac{13}{4 \times 2}\sin^{-1}\left(\frac{x - \frac{3}{2}}{\frac{\sqrt{13}}{2}}\right) + C$$
$$= \frac{2x - 3}{4}\sqrt{1 + 3x - x^2} + \frac{13}{8}\sin^{-1}\left(\frac{2x - 3}{\sqrt{13}}\right) + C$$

-

Let $I = \int \sqrt{x^2+3x}\, dx$

$$= \int \sqrt{x^2+3x+\frac{9}{4}-\frac{9}{4}}\, dx$$

$$= \int \sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}\, dx$$

It is known that, $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$\therefore I = \frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2+3x}-\frac{\frac{9}{4}}{2}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$

$$= \frac{(2x+3)}{4}\sqrt{x^2+3x}-\frac{9}{8}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$

-

Let $I = \int \sqrt{1+\frac{x^2}{9}}\, dx = \frac{1}{3}\int \sqrt{9+x^2}\, dx = \frac{1}{3}\int \sqrt{(3)^2+x^2}\, dx$

It is known that, $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$$\therefore I = \frac{1}{3}\left[\frac{x}{2}\sqrt{x^2+9}+\frac{9}{2}\log\left|x+\sqrt{x^2+9}\right|\right]+C$$

$$= \frac{x}{6}\sqrt{x^2+9}+\frac{3}{2}\log\left|x+\sqrt{x^2+9}\right|+C$$

-

$\int \sqrt{1+x^2}\, dx$ is equal to

A. $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$

B. $\frac{2}{3}\left(1+x^2\right)^{\frac{2}{3}}+C$

C. $\frac{2}{3}x\left(1+x^2\right)^{\frac{3}{2}}+C$

D. $\frac{x^2}{2}\sqrt{1+x^2}+\frac{1}{2}x^2\log\left|x+\sqrt{1+x^2}\right|+C$

Answer :

It is known that, $\int\sqrt{a^2+x^2}\,dx=\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$\therefore \int\sqrt{1+x^2}\,dx=\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$

Hence, the correct answer is A.

-

$\int\sqrt{x^2-8x+7}dx$ is equal to

A. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

B. $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log\left|x+4+\sqrt{x^2-8x+7}\right|+C$

C. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

D. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

Answer :

$$\text{Let } I=\int\sqrt{x^2-8x+7}\,dx$$
$$=\int\sqrt{\left(x^2-8x+16\right)-9}\,dx$$
$$=\int\sqrt{(x-4)^2-(3)^2}\,dx$$

It is known that, $\int\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$\therefore I=\frac{(x-4)}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|(x-4)+\sqrt{x^2-8x+7}\right|+C$$

Hence, the correct answer is D.

-

Solve a Problem

Evaluate $\int \cos 2x \log(1+\tan x)dx$.

Solution:

Integrating by parts taking cos 2x as the 2nd function, the given integral

$$= \{\log(1+\tan x)\}\frac{\sin 2x}{2} - \int \frac{\sec^2 x}{1+\tan x}\cdot\frac{\sin 2x}{2}dx$$

$$= \frac{1}{2}\sin 2x \log(1+\tan x) - \int \frac{\sin x}{\sin x + \cos x}dx.$$

Now $\int \frac{\sin x\, dx}{\sin x + \cos x}$

$$= \frac{1}{2}\int \frac{(\sin x + \cos x) - (\cos x - \sin x)}{\sin x + \cos x}dx,$$

$$= \frac{1}{2}\int\left[1 - \frac{\cos x - \sin x}{\sin x + \cos x}\right]dx = \frac{1}{2}[x - \log(\sin x + \cos x)].$$

Hence the given integral

$$= \frac{1}{2}\sin 2x \log(1+\tan x) - \frac{1}{2}[x - \log(\sin x + \cos x)].$$

Recall how to integrate Linear X root Quadratic in denominator

Find the value of the $\int \frac{dx}{(x+1)\sqrt{(1+2x-x^2)}}$

Putting $(x + 1) = \frac{1}{t}$, so that $dx = -\frac{1}{t^2}\, dt$, $x = \frac{1-t}{t}$ and

$$(1 + 2x - x^2) = 1 + 2\left(\frac{1-t}{t}\right) - \frac{(1-t)^2}{t^2} = \frac{2}{t^2}\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (t-1)^2\right],$$

we get the value of the given integral transformed as'

$$\int \frac{-\frac{1}{t^2}\,dt}{\frac{1}{t}\frac{2}{\sqrt{t}}\left[\left(\frac{1}{\sqrt{2}}\right)^2-(t-1)^2\right]} = -\frac{1}{\sqrt{2}}\sin^{-1}\frac{t-1}{\left(\frac{1}{\sqrt{2}}\right)} + C$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}\frac{\sqrt{2}\,x}{(x+1)} + C$$

Remember -

For the form $\int \frac{dx}{(Ax+B)^r\sqrt{(ax^2+bx+c)}}$ where r is a positive integer

we can substitute $Ax+B = \frac{1}{t}$

But for $\int \frac{dx}{(Ax+B)\sqrt{(ax+b)}}$ $\int \frac{dx}{(Ax^2+Bx+C)\sqrt{(ax+b)}}$ we have to substitute ax + b = t^2

So the Linear expression that in inside the root will be substituted

See https://archive.org/details/4Integrations91ByUIsAPowerfulSubstitutionIITJEEMath

Another advanced example

Example Evaluate $\int \frac{dx}{x\sqrt{(1+x^n)}}$

Make the substitution $(1 + x^n) = t^2$ or $x^n = (t^2 - 1)$, so that $n\,x^{n-1}\,dx = 2t\,dt$, we get

$$\int \frac{2t\,dt}{n x^n t} = \frac{2}{n}\int \frac{dt}{(t^2-1)} = \frac{1}{n}\ln\left|\frac{t-1}{t+1}\right|$$

$$= \frac{1}{n}\ln\left|\frac{\sqrt{(1+x^n)}-1}{\sqrt{(1+x^n)}+1}\right| + C$$

Similarly

The value of integral $\int \frac{dx}{x\sqrt{1-x^3}}$ is given by

(a) $\frac{1}{3}\log\left|\frac{\sqrt{1-x^3}+1}{\sqrt{1-x^3}-1}\right|+C$ (b) $\frac{1}{3}\log\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^2}+1}\right|+C$

(c) $\frac{2}{3}\log\left|\frac{1}{\sqrt{1-x^3}}\right|+C$ (d) $\frac{1}{3}\log|1-x^3|+C$

Ans. (b)

Solution Put $1-x^3=t^2$. Then $-3x^2dx=2t\,dt$ and the integral becomes

$$-\frac{1}{3}\int\frac{-3x^2dx}{x^3\sqrt{1-x^3}}=-\frac{1}{3}\int\frac{2t\,dt}{(1-t^2)t}=\frac{2}{3}\int\frac{dt}{t^2-1}$$

$$=\frac{2}{3}\left(\frac{1}{2}\log\left|\frac{t-1}{t+1}\right|\right)+C=\frac{1}{3}\log\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right|+C$$

-

Integration of 1 by (Cos x + Cot x)

$$\int\frac{dx}{\text{Cot } x+\text{Cos } x}$$

https://archive.org/details/4Integrations10ByCosXCotXIITJEEMath

-

Solve a Problem

$\int \sqrt{\sec x - 1}\ dx$ is equal to

(a) $2 \log\left(\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C$

(b) $\log\left(\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C$

(c) $-2 \log\left(\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C$

(d) none of these

(c). $\int \sqrt{\sec x - 1}\ dx = \int \sqrt{\frac{1-\cos x}{\cos x}}\ dx$

$$= \sqrt{2} \int \frac{\sin\frac{x}{2}}{\sqrt{2\cos^2\frac{x}{2} - 1}}\ dx = -2\sqrt{2} \int \frac{dz}{\sqrt{2z^2 - 1}}$$

$$\left(\text{Putting } \cos\frac{x}{2} = z \Rightarrow \sin\frac{x}{2}\ dx = -2dz\right)$$

$$= -2 \int \frac{dz}{\sqrt{z^2 - \left(\frac{1}{\sqrt{2}}\right)^2}}$$

$$= -2 \log\left[z + \sqrt{z^2 - \left(\frac{1}{\sqrt{2}}\right)^2}\right] + C$$

$$= -2 \log\left(\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C$$

Solve a tricky problem

Solve $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$

Solution: $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$

$$= \int \sqrt{\frac{\sin x}{\cos x \sin^2 x \cos^2 x} dx}$$

$$\int \frac{1}{\sqrt{\sin x \cos^3 x}} dx$$

$$\int \frac{1}{\sqrt{\sin^4 x \cot^3 x}} dx$$

$$-\int -cosec^2 x \; cot^{-3/2} x \, dx$$

$$= \frac{2}{\sqrt{\cot x}} + C$$

-

IIT JEE 2014 VIT Indefinite Integral e to the power x + 1 by x a tricky problem

$$\int \left(1 + x - \frac{1}{x}\right) e^{x + \frac{1}{x}} \, dx$$

https://archive.org/details/IITJEE2014VITIndefiniteIntegralEToThePowerX1ByXDezrinaBangaloreTricky

IIT-JEE-Integral with d (x - floor (x)) strange interpretation

$$\int d(x - [x])$$

https://archive.org/details/IITJEEIntegralWithDXFloorXStrangeInterpretation

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IIT-JEE-Integral x^2(0 to root 2) split into appropriate floors

$$\int_0^{\sqrt{2}} [\,x^2\,]\,dx$$

https://archive.org/details/IITJEEIntegralX20ToRoot2SplitIntoAppropriateFloors

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Integration Tricks (63 examples)

$$\int \frac{1+x^4}{(1-x^4)^{3/2}}\,dx$$

https://archive.org/details/IndefiniteIntegralISEET2013QuestionLeakPowersOfXFactorisationSinLogXByParts

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Integrate 1 by (x plus root x square plus 1) rationalize and proceed

$\int 1/(x + \sqrt{(x^2 + 1)})\ dx$

https://archive.org/details/Integrate1ByXPlusRootXSquarePlus1RationalizeAndProceed

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Integrate 1 by x square into Cos 1 by x dx

$\int (1/x^2) \cos (1 / x)\ dx$

https://archive.org/details/Integrate1ByXSquareIntoCos1ByXDx

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Beautiful Integrals explained step by step 1

$\int \frac{dx}{x^3 \sqrt[3]{x^3+5}}$ is easy but what about $\int \frac{dx}{x^3 \sqrt{x^2-1}}$

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard1

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Beautiful Integrals explained step by step 2

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard2

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Beautiful Integrals explained step by step 3

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard3

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Beautiful Integrals explained step by step 4

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard4

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Beautiful Integrals explained step by step 5

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard5

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Beautiful Integrals explained step by step 6

$$\int \frac{x^2 \quad dx}{\sqrt{x^2 - 1}}$$

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard6

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Beautiful Integrals explained step by step 7

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard7

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Beautiful Integrals explained step by step 8

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard8

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Beautiful Integrals explained step by step 9

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard9

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Beautiful Integrals explained step by step 10

$$\int \frac{2x \quad dx}{(1 - x^2)\sqrt{x^4 - 1}}$$

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard10

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Beautiful Integrals explained step by step 11

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard11

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Beautiful Integrals explained step by step 12

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Beautiful Integrals explained step by step 13

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Beautiful Integrals explained step by step 16

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Beautiful Integrals explained step by step 17

$$\int \frac{dx}{(x^2 - 1)\sqrt{x^2 + 1}}$$

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Beautiful Integrals explained step by step 18

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Beautiful Integrals explained step by step 19

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Beautiful Integrals explained step by step 20

$$\int \frac{(x + 2)\ dx}{(x^2 + 3x + 3)\sqrt{x + 1}}$$

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Beautiful Integrals explained step by step 29

e to the power Matrix …. A rare problem

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Beautiful Integrals explained step by step 30

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Beautiful Integrals explained step by step 31

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Beautiful Integrals explained step by step 32

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Beautiful Integrals explained step by step 33

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Beautiful Integrals explained step by step 34

Try solving $\int \sqrt{1 + \text{Cosec } x}\, dx$

$$I = \int \sqrt{1 + \operatorname{cosec} x}\ .\ dx$$

$$= \int \sqrt{1 + \frac{1}{\sin x}}\ .\ dx = \int \sqrt{\frac{\sin x + 1}{\sin x}}\ .\ dx$$

$$= \int \sqrt{\frac{(1 + \sin x)(1 - \sin x)}{\sin x\,(1 - \sin x)}}\ .\ dx \qquad \text{[On rationalization]}$$

$$= \int \sqrt{\frac{1 - \sin^2 x}{\sin x - \sin^2 x}}\ .\ dx \qquad [\because \quad (a+b)(a-b) = a^2 - b^2]$$

$$= \int \frac{\cos x}{\sqrt{\sin x - \sin^2 x}}\ .\ dx \qquad [\because \quad \sin^2 A + \cos^2 A = 1]$$

$$\sin x = z \quad \Rightarrow \quad \cos x\, dx = dz$$

$$I = \int \frac{1}{\sqrt{z - z^2}}\ .\ dz = \int \frac{1}{\sqrt{-(z^2 - z)}}\ .\ dz$$

$$= \int \frac{1}{\sqrt{\frac{1}{4} - \left(z^2 - z + \frac{1}{4}\right)}}\ .\ dz \qquad \left[\begin{array}{l}\text{Add and subtract } \frac{1}{4} \text{ to the denom.} \\ \because \left(\frac{1}{2} \text{ co-eff. of } x\right)^2 = \frac{1}{4}\end{array}\right]$$

$$= \int \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(z - \frac{1}{2}\right)^2}} \, . dz$$

$$\left(z - \frac{1}{2}\right) = y \quad \Rightarrow \quad dz = dy$$

$$I = \int \frac{1}{\sqrt{(1/2)^2 - y^2}} \, . dy \qquad \left[\text{By using} \int \frac{1}{\sqrt{a^2 - x^2}} \, . dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$= \sin^{-1}\left(\frac{y}{1/2}\right) + c$$

$$= \sin^{-1}\left(\frac{z - 1/2}{1/2}\right) + c \qquad [\because \quad y = z - 1/2]$$

See explanation video

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard34

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IIT JEE 2002 (Very interesting) Integral Calculus divide by x multiply by x to the power m

$$\int (x^{3m} + x^{2m} + x^{m})(2x^{2m} + 3x^{m} + 6)\, dx$$

https://archive.org/details/IITJEE2002IntegralCalculusDivideByXMultiplyByXToThePowerM2

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IIT JEE Integral Calculus modifications to play with x power divide and Multiply

https://archive.org/details/IITJEEIntegralCalculusModificationsToPlayWithXPowerDivideAndMultiply

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Integral Calculus Rationalize and proceed much easier if root is in Numerator

https://archive.org/details/IntegralCalculusRationalizeAndProceedMuchEasierIfRootIsInNumerator1

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Integration of e to the power x by x expand as series and then Integrate individual terms

https://archive.org/details/IntegrationOfEToThePowerXByXExpandAsSeriesAndThenIntegrateIndividualTerms

-

Solve another Integral

$$I = \int \sqrt{\frac{1+x}{x}} \, . \, dx$$

$$= \int \sqrt{\frac{1+x}{x} \times \frac{1+x}{1+x}} \, dx \qquad \text{[Multiply and divided by } (1+x)\text{]}$$

$$= \int \sqrt{\frac{(1+x)^2}{x(1+x)}} \, . \, dx = \int \frac{1+x}{\sqrt{x+x^2}} \, . \, dx$$

Let us write :

$$1 + x = \lambda \, . \, \frac{d}{dx}(x + x^2) + \mu$$

$$\Rightarrow \quad 1 + x = \lambda\,(1 + 2x) + \mu \qquad \text{...(1)}$$

$$\Rightarrow \quad 1 + x = 2\lambda x + \lambda + \mu$$

Comparing the coefficients of x and the constant terms, we have

$$1 = 2\lambda \quad \Rightarrow \quad \lambda = \frac{1}{2}$$

and

$$1 = \lambda + \mu \quad \Rightarrow \quad \mu = 1 - \lambda = 1 - \frac{1}{2} = \frac{1}{2}.$$

Putting the values of λ and μ in (1),

$$1 + x = \frac{1}{2}(1+2x) + \frac{1}{2}.$$

$\therefore \qquad I = \int \dfrac{\frac{1}{2}(1+2x)+\frac{1}{2}}{\sqrt{x+x^2}} \, . \, dx$

$$= \frac{1}{2}\int \frac{1+2x}{\sqrt{x+x^2}} \, dx + \frac{1}{2}\int \frac{1}{\sqrt{x+x^2}} \, . \, dx$$

$\Rightarrow \qquad I = \frac{1}{2}\ I_1 + \frac{1}{2}\ I_2 \qquad$ ...(2)

Now $\qquad I_1 = \int \dfrac{1+2x}{\sqrt{x+x^2}} \, dx$

Put $\qquad x + x^2 = z \ \Rightarrow\ (1+2x)\, dx = dz$

$\therefore \qquad I_1 = \int \dfrac{1}{\sqrt{z}} \, . \, dz = \int z^{-1/2} \, . \, dz = \dfrac{z^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c_1 = 2\sqrt{z} + c_1$

$$= 2\sqrt{x+x^2} + c_1 \qquad \text{...(3)}$$

and $\qquad I_2 = \int \dfrac{1}{\sqrt{x+x^2}} \, . \, dx$

$$= \int \frac{1}{\sqrt{\left(x^2+x+\frac{1}{4}\right)-\frac{1}{4}}} \, . \, dx \qquad \left[\begin{array}{l}\text{Add and subtract } \frac{1}{4} \text{ to the denom.} \\ \because \ \left(\frac{1}{2}\text{ co-eff. of } x\right)^2 = \frac{1}{4}\end{array}\right]$$

$$= \int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}} \, . \, dx$$

Put $\qquad x + \frac{1}{2} = z \ \Rightarrow\ dx = dz$

$\therefore \qquad I_2 = \int \dfrac{1}{\sqrt{z^2-\left(\frac{1}{2}\right)^2}} \, . \, dz \qquad \left[\text{By using } \int \dfrac{1}{\sqrt{x^2-a^2}} \, . \, dx = \log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$$= \log\left|z+\sqrt{z^2-\left(\frac{1}{2}\right)^2}\right| + c_2 = \log\left|\left(x+\frac{1}{2}\right)+\sqrt{\left(x+\frac{1}{2}\right)^2-\frac{1}{4}}\right| + c_2$$

$$= \log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x}\right| + c_2 \qquad \text{...(4)}$$

$\therefore$ **From equation (2),**

$I = \frac{1}{2}\ I_1 + \frac{1}{2}\ I_2 \qquad$ [Using (3) and (4)]

8 examples of Integration

https://archive.org/details/IntegrationExpandSin2xAAndTakeCosXCommonWithinTheRoot

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Solve another problem

$$I = \int \frac{ax^3 + bx}{x^4 + c^2}\, dx = \int \frac{ax^3}{x^4 + c^2} \cdot dx + \int \frac{bx}{x^4 + c^2} \cdot dx$$

$$= a \int \frac{x^3}{x^4 + c^2} \cdot dx + b \int \frac{x}{x^4 + c^2} \cdot dx$$

$$\Rightarrow \quad I = a\, I_1 + b\, I_2 \qquad \ldots(1)$$

Now $\quad I_1 = \int \frac{x^3}{x^4 + c^2} \cdot dx$

$$= \frac{1}{4} \int \frac{4x^3}{x^4 + c^2} \cdot dx \qquad \text{[Multiply and divided by 4]}$$

$$= \frac{1}{4} \log\left| x^4 + c^2 \right| + c_1 \qquad \ldots(2) \quad \left[\because \int \frac{f'(x)}{f(x)} \cdot dx = \log|f(x)| + c\right]$$

and $\quad I_2 = \int \frac{x}{x^4 + c^2} \cdot dx$

$$= \frac{1}{2} \int \frac{2x}{(x^2)^2 + c^2}\, dx \qquad \text{[Multiply and divided by 2]}$$

Put $\quad x^2 = z \;\Rightarrow\; 2x\, dx = dz$

$$= \frac{1}{2} \int \frac{1}{z^2 + c^2}\, dz \qquad \left[\text{By using} \int \frac{1}{x^2 + a^2} \cdot dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c\right]$$

Solve Integration root linear plus root linear in denominator

If $I = \int \frac{dx}{\sqrt{2x+3} + \sqrt{x+2}}$, then I equals

(a) $2(u - v) + \log \left|\frac{u-1}{u+1}\right| + \log \left|\frac{v-1}{v+1}\right| + C$

$u = \sqrt{2x+3}$, $v = \sqrt{x+2}$

(b) $\log \left|\frac{\sqrt{x+2} + \sqrt{2x+3}}{\sqrt{x+2} - \sqrt{2x+3}}\right| + C$

(c) $\log \left(\sqrt{x+2} + \sqrt{2x+3}\right) + C$

(d) is transcedental function in u and v, $u = \sqrt{2x+3}$

$v = \sqrt{x+2}$

Ans. (a), (d)

$$I = \int \frac{\sqrt{2x+3} - \sqrt{x+2}}{x+1}\, dx$$

$$= I_1 - I_2$$

where $I_1 = \int \frac{\sqrt{2x+3}}{x+1}\, dx$ and $I_2 = \int \frac{\sqrt{x+2}}{x+1}\, dx$

Put $2x + 3 = t^2$, in I_1, so that

$$I_1 = \int \frac{2t \cdot t}{t^2 - 1}\, dt = 2 \int \left[1 + \frac{1}{t^2 - 1}\right] dt$$

$$= 2 \left[t + \frac{1}{2} \log \left|\frac{t-1}{t+1}\right|\right]$$

In I_2, put $x + 2 = y^2$, so that

$$I_2 = \int \frac{2y^2}{y^2 - 1} dy = 2y + \log \left|\frac{y-1}{y+1}\right|$$

Thus,

$$I = 2\left(\sqrt{2x+3} - \sqrt{x+2}\right) + \log \left|\frac{\sqrt{2x+3} - 1}{\sqrt{2x+3} + 1}\right|$$

$$+ \log \left|\frac{\sqrt{x+2} - 1}{\sqrt{x+2} + 1}\right| + C$$

Solve another Problem

Evaluate $\int \frac{\sin 2x\, dx}{(a + b\cos x)^2}$.

Solution:

We have $I = \int \frac{\sin 2x\, dx}{(a + b\cos x)^2} = 2\int \frac{\sin x \cos x\, dx}{(a + b\cos x)^2}$

Now put $a + b\cos x = t$

so that $-b\sin x\, dx = dt$.

Also $\cos x = \frac{(t-a)}{b}$.

$$\therefore\ I = -\frac{2}{b}\int \frac{(t-a)/b}{t^2}dt = -\frac{2}{b^2}\int\left[\frac{t}{t^2} - \frac{a}{t^2}\right]dt$$

$$= -\frac{2}{b^2}\int\left[\frac{1}{t} - \frac{a}{t^2}\right]dt = -\frac{2}{b^2}\left[\log t + \frac{a}{t}\right]$$

$$= -\frac{2}{b^2}\left[\log(a + b\cos x) + \frac{a}{a + b\cos x}\right].$$

A special Integral

$$\int \frac{(1-\sqrt{1+x+x^2})^2}{x^2\sqrt{(1+x+x^2)}}\,dx$$

Here we set $\sqrt{1+x+x^2} = xt + 1$, so that

$$x = \frac{2t-1}{1-t^2},\ dx = \frac{2t^2-2t+2}{(1-t^2)^2}\,dt \text{ and}$$

$$(1-\sqrt{1+x+x^2}) = \frac{-2t^2+t}{(1-t^2)}$$

Substitution of these values in the given integral transforms the problem in the form

$$\int \frac{(-2t^2+t)^2\,(1-t^2)^2\,(1-t^2)\,(2t^2-2t+2)}{(1-t^2)^2\,(2t-1)^2\,(t^2-t+1)\,(1-t^2)^2}\,dt$$

$$= +2\int \frac{t^2}{1-t^2}\,dt = -\,2t + \ln\left|\frac{1+t}{1-t}\right| + C$$

7 very interesting and strange Integrals

https://archive.org/details/IntegrationRootWithinRootSoPutTheFullExpressionOfTheFirstRootAsT2

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An advanced example

$$I = \int \frac{(x+1)}{x(1+xe^x)^2}\,dx$$

$$I = \int \frac{e^x(x+1)}{x\,e^x(1+xe^x)^2}\,dx$$

put $1 + xc^x = t$, $(xe^x + e^x)\,dx = dt$

$$I = \int \frac{dt}{(t-1)t^2} = \int\left(\frac{1}{1-t}+\frac{1}{t}+\frac{1}{t^2}\right)dt$$

$$= -\log|1-t| + \log|t| - \frac{1}{t} + C = \log\left|\frac{t}{1-t}\right| - \frac{1}{t} + C$$

$$= \log\left|\frac{1+xe^x}{-xe^x}\right| - \frac{1}{1+xe^x} + C = \log\left(\frac{1+xe^x}{xe^x}\right) - \frac{1}{1+xe^x} + C$$

Practice Example

Let $f(x)$ be a function defined by $f(x) = \int_1^x x(x^2-3x+2)\,dx$, $1 \le x \le 3$, then the range of $f(x)$ is

(a) $\left[-\frac{1}{4}, 2\right]$ (b) $\left[-\frac{1}{4}, 4\right]$

(c) $[0, 2]$ (d) none of these

Solution :

(a). We have,

$f'(x) = x(x^2 - 3x + 2) = x(x-1)(x-2)$

Clearly, $f'(x) \le 0$ in $1 \le x \le 2$ and $f'(x) \ge 0$ in $2 \le x \le 3$.

$\therefore$ $f'(x)$ is monotonic decreasing in [1, 2] and monotonic increasing in [2, 3].

$$\therefore \text{Min. } f(x) = f(2) = \int_1^2 x(x^2 - 3x + 2)\,dx$$

$$= \left|\frac{x^4}{4} - x^3 + x^2\right|_1^2 = \frac{-1}{4}$$

Max. $f(x)$ = the greatest among $(f(1), f(3))$

$$\text{Now, } f(1) = \int_1^1 x(x^2 - 3x + 2)\,dx = 0$$

$$f(3) = \int_1^3 x(x^2 - 3x + 2)\,dx$$

$$= \frac{x^4}{4} - x^3 + 2\Big|_1^3 = 2. \quad \therefore \ \text{Max. } f(x) = 2$$

Hence, Range $= \left[\frac{-1}{4}, 2\right]$

Practice Example

If $I = \int_0^\infty \frac{\sqrt{x}\,dx}{(1+x)(2+x)(3+x)}$, then I equals

(a) $\frac{\pi}{2}(2\sqrt{2} - \sqrt{3} - 1)$ (b) $\frac{\pi}{2}(2\sqrt{2} + \sqrt{3} - 1)$

(c) $\frac{\pi}{2}(2\sqrt{2} - \sqrt{3} + 1)$ (d) none of these

Ans. (a)

Solution Put $\sqrt{x} = t$ or $x = t^2$, so that

$$I = 2\int_0^\infty \frac{t^2}{(1+t^2)(2+t^2)(3+t^2)}\,dt$$

$$= \int_0^\infty \left(-\frac{1}{1+t^2} + \frac{4}{2+t^2} - \frac{3}{3+t^2}\right) dt$$

$$= \left(-\tan^{-1} t + \frac{4}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right) - \frac{3}{\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)\right)\Bigg]_0^\infty$$

$$= -\frac{\pi}{2} + 2\sqrt{2}\left(\frac{\pi}{2}\right) - \sqrt{3}\left(\frac{\pi}{2}\right)$$

$$= \frac{\pi}{2}(2\sqrt{2} - \sqrt{3} - 1).$$

Practice Example

The value $\int_0^1 \cot^{-1}(1 + x^2 - x)\, dx$ is

(a) $\pi/2 - \log 2$ (b) $\pi - \log 2$

(c) $\pi/4 - \log 2$ (d) $2\int_0^1 \tan^{-1} x\, dx$

Ans. (a), (d)

Solution $\cot^{-1}(1 + x^2 - x) = \tan^{-1}\left(\dfrac{x + 1 - x}{1 - x(1 - x)}\right)$

$$= \tan^{-1} x + \tan^{-1}(1 - x)$$

$$I = \int_0^1 \cot^{-1}(1 + x^2 - x)\,dx = \int_0^1 \tan^{-1} x\,dx + \int_0^1 \tan^{-1}(1 - x)\,dx$$

$$= \int_0^1 \tan^{-1} x\,dx + \int_0^1 \tan^{-1} x\,dx = 2\int_0^1 \tan^{-1} x\,dx$$

$$= 2x\tan^{-1} x\Big]_0^1 - \int_0^1 \frac{2x}{1 + x^2}\,dx$$

$$= 2\tan^{-1}(1) - \log(1 + x^2)\Big]_0^1$$

$$= 2(\pi/4) - \log 2 = \pi/2 - \log 2$$

Practice Example

$$\int_{\sqrt{(3a^2+b^2)/4}}^{\sqrt{(a^2+b^2)/2}} \frac{x}{\sqrt{(x^2-a^2)(b^2-x^2)}}\,dx =$$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

Solution :

(d). Let $I = \int_{\sqrt{(3a^2+b^2)/4}}^{\sqrt{(a^2+b^2)/2}} \frac{x}{\sqrt{(x^2-a^2)(b^2-x^2)}}\,dx$

Put $x^2 = a^2\cos^2 t + b^2\sin^2 t$

$\Rightarrow \quad 2x\,dx = [2a^2\cos t(-\sin t) + 2b^2\sin t(\cos t)]\,dt$

$\Rightarrow \quad x\,dx = \frac{1}{2}(b^2-a^2)\sin 2t.dt$

For $\quad x^2 = \frac{a^2+b^2}{2} = a^2\cos^2 t + b^2\sin^2 t$

$\Rightarrow \quad a^2+b^2 = 2(1-\sin^2 t)a^2 + 2b^2\sin^2 t$

or, $\quad (a^2+b^2) = 2a^2 + 2(b^2-a^2)\sin^2 t$

$\Rightarrow \quad \sin^2 t = \frac{1}{2} \Rightarrow \cos 2t = 0 \quad \Rightarrow t = \pi/4$

For $\quad x^2 = \frac{3a^2+b^2}{4} = a^2\cos^2 t + b^2\sin^2 t$

$\Rightarrow \quad 3a^2+b^2 = 4a^2 + 4(b^2-a^2)\sin^2 t$

$\Rightarrow \quad \sin^2 t = \frac{1}{4} \Rightarrow \cos 2t = \frac{1}{2} \Rightarrow t = \frac{\pi}{4}$

$\therefore \quad I = \int_{\pi/6}^{\pi/4} \frac{1}{2}\frac{(b^2-a^2)\sin 2t.dt}{\sqrt{(b^2-a^2)\sin^2 t(b^2-a)\cos^2 t}}$

$= \int_{\pi/6}^{\pi/4} dt = (t)_{\pi/6}^{\pi/4} = \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$

Practice Example

If $\int_0^{\pi/2} \frac{x^2 \cos x}{(1+\sin x)^2} dx = A\ \pi - \pi^2$ then A is

Ans. 2

Solution Integrating by parts, we have

$$\int_0^{\pi} \frac{x^2 \cos x}{(1+\sin x)^2}\, \mathrm{d}x$$

$$= -\frac{x^2}{1+\sin x}\Bigg|_0^{\pi} + 2\int_0^{\pi} \frac{x}{1+\sin x}\mathrm{d}x = -\pi^2 + 2I$$

where

$$I = \int_0^{\pi} \frac{x}{1+\sin x} dx = \int_0^{\pi} \frac{\pi - x}{1+\sin x} dx = \pi \int_0^{\pi} \frac{dx}{1+\sin x} - I$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{dx}{1+\sin x} = 2\pi \int_0^{\pi/2} \frac{dx}{1+\sin x}$$

$$\Rightarrow \quad I = \pi \int_0^{\pi/2} \frac{dx}{1+\sin x} = \pi \int_0^{\pi/2} \frac{dx}{1+\sin(\pi/2 - x)}$$

$$= \int_0^{\pi/2} \frac{dx}{1+\cos x}$$

$$= \frac{\pi}{2}\int_0^{\pi/2} \sec^2 (x/2) dx = \pi \tan (x/2)\Big]_0^{\pi/2} = \pi$$

Hence $\int_0^{\pi} \frac{x^2 \cos x}{(1+\sin x)^2} dx = -\pi^2 + 2\pi$

Practice example

Example $\displaystyle\int_0^{\pi} \frac{dx}{(1+a^2)-2a\cos x} = \frac{\pi}{1-a^2}$ or $\dfrac{\pi}{a^2-1}$

according as $a < 1$ or $a > 1$.

The given problem may be re-written in the form

$$\int_0^{\pi} \frac{dx}{(1+a^2)\left(\cos^2\frac{x}{2}+\sin^2\frac{x}{2}\right)-2a\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)}$$

which can be expressed in the forms

$$I = \frac{2}{(1+a^2)^2}\int \frac{dt}{\left(\frac{1-a}{1+a}\right)^2+t^2} \text{ or } \frac{2}{(1+a^2)^2}\int \frac{dt}{\left(\frac{a-1}{a+1}\right)^2+t^2}$$

according as $a < 1$ or > 1, where $t = \tan\dfrac{x}{2}$

Hence

Hence

$$I = \frac{2}{(1-a^2)}\left[\tan^{-1}\frac{t(1+a)}{(1-a)}\right]_0^{\infty} = \frac{\pi}{1-a^2} \text{ if } a < 1$$

Similarly in the other case the answer shall be $\dfrac{\pi}{a^2-1}$, $a > 1$

Practice example

$\displaystyle\int_0^{\sin^2 x} \sin^{-1}\left(\sqrt{t}\right)dt + \int_0^{\cos^2 x} \cos^{-1}\left(\sqrt{t}\right)dt$ is equal to

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{6}$

(c) 0 (d) none of these

Solution :

(a). We have,

$$I = \int_0^{\sin^2 x} \sin^{-1}\left(\sqrt{t}\right) dt + \int_0^{\cos^2 x} \cos^{-1}\left(\sqrt{t}\right) dt$$

$$= \left[t \sin^{-1}\left(\sqrt{t}\right)\right]_0^{\sin^2 x} - \int_0^{\sin^2 x} \frac{\sqrt{t}}{\sqrt[2]{1-t}}\, dt$$

$$+ \left[t \cos^{-1}\left(\sqrt{t}\right)\right]_0^{\cos^2 x} - \int_0^{\cos^2 x} \frac{\sqrt{t}}{\sqrt[2]{1-t}}\, dt$$

$$= x \sin^2 x + \int_{\sin^2 x}^{0} \frac{\sqrt{t}}{\sqrt[2]{1-t}}\, dt + x\cos^2 x + \int_0^{\cos^2 x} \frac{\sqrt{t}}{\sqrt[2]{1-t}}\, dt$$

$$= x\,(\sin^2 x + \cos^2 x) + \int_{\sin^2 x}^{\cos^2 x} \frac{\sqrt{t}}{\sqrt[2]{1-t}}\, dt$$

Putting $t = \sin^2 \theta$ and $dt = 2 \sin \theta \cos \theta\, d\theta$, we get,

$$\int \frac{\sqrt{t}}{\sqrt[2]{1-t}}\, dt = \int \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\, 2\sin\theta \cos\theta\, d\theta$$

$$= \int \sin^2\theta\, d\theta = \int \frac{1-\cos 2\theta}{2}\, d\theta$$

$$= \frac{\theta}{2} - \frac{\sin 2\theta}{4}$$

Also, when $t = \sin^2 x$, $\theta = x$ and when $t = \cos^2 x$, $\theta = \pi/2 - x$

$$\therefore\ I = x + \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_x^{\pi/2 - x}$$

$$= x + \left(\frac{\pi}{4} - \frac{x}{2} - \frac{\sin 2x}{4}\right) - \left(\frac{x}{2} - \frac{\sin 2x}{4}\right)$$

$$= x + \frac{\pi}{4} - x = \frac{\pi}{4}$$

Practice example

$$I = \int_0^{\pi/4} \frac{\sin 2\theta \, d\theta}{\sin^4\theta + \cos^4\theta} = \int_0^{\pi/4} \frac{2\sin\theta\cos\theta}{\sin^4\theta + \cos^4\theta} d\theta$$

$$= \int_0^{\pi/4} \frac{2\tan\theta \sec^2\theta \, d\theta}{1 + \tan^4\theta},$$

dividing the numerator and denominator by $\cos^4\theta$

Put $\tan^2\theta = t$,

so that $2\tan\theta\sec^2\theta\, d\theta = dt$.

When $\theta = 0$,

$t = \tan^2 0 = 0$

and when $\theta = \frac{\pi}{4}$,

$t = \tan^2 \frac{1}{4}\pi = 1.$

$$\therefore \quad I = \int_0^1 \frac{dt}{1+t^2} = \left[\tan^{-1} t\right]_0^1$$

$$= \tan^{-1} 1 - \tan^{-1} 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

Practice example

If $f(x)$ satisfies the relation $\int_{-2}^{x} f(t)\, dt + x f''(3)$

$= \int_1^x x^3 dx + f'(1)\int_2^x x^2 dx + f''(2)\int_3^x x\, dx$, then

(a) $f(x) = x^3 + 5x^2 + 2x - 6$

(b) $f(x) = x^3 - 5x^2 + 2x + 6$

(c) $f(x) = x^3 + 5x^2 + 2x - 6$

(d) $f(x) = x^3 - 5x^2 + 2x - 6$

Solution :

(d). Differentiating the given equation w.r.t. x, we get

$$f(x)+f'''(3)=x^3+x^2f'(1)+xf''(2) \quad ...(1)$$

Differentiating successively w.r.t. x, we get

$$f'(x)=3x^2+2xf'(1)+f''(2) \quad ...(2)$$

$$f''(x)=6x+2f'(1) \quad ...(3)$$

$$f'''(x)=6. \quad ...(4)$$

Putting $x = 1$, 2 and 3 in equations (2), (3) and (4) respectively, we get

$$f'(1)=3+2f'(1)+f''(2), \quad f''(2)=12+2f'(1)$$

and, $f'''(3)=6$

Solving, we have

$$f'(1)=-5, f''(2)=2, f'''(3)=6$$

Putting the values in equation (1), we have

$$f(x)=x^3-5x^2+2x-6.$$

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Very Important Integral Calculus problem IIT JEE problems modification n th root

https://archive.org/details/VeryImportantIntegralCalculusProblemIITJEEProblemsModificationNThRoot

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Practice example

If $I_1 = \int_{1/e}^{\tan x} \frac{t}{1+t^2}\,dt$ and $I_2 = \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)}$ then the value of $I_1 + I_2$ is

(a) 1/2 (b) 1
(c) $e/2$ (d) $(1/2)(e + 1/e)$

Ans. (b)

Solution Putting $t = 1/u$ in I_2 we have

$$I_2 = -\int_{e}^{\tan x} \frac{u\,du}{1+u^2} = -\int_{1/e}^{\tan x} \frac{u\,du}{1+u^2} + \int_{1/e}^{e} \frac{u\,du}{1+u^2}$$

$$= -I_1 + \frac{1}{2}\int_{1/e}^{e} \frac{2u\,du}{1+u^2}$$

So
$$I_1 + I_2 = \frac{1}{2}\log(u^2+1)\Big]_{1/e}^{e} = \frac{1}{2}\left[\log(e^2+1) - \log\left(\frac{e^2+1}{e^2}\right)\right]$$

$$= \frac{1}{2} \times 2 = 1.$$

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Integral dx by (x^2+a^2)^3 by 2 common important for many Physics Numericals

https://archive.org/details/8IntegralDxByx2a23By2CommonImportantForManyPhysicsNumericals

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Practice example

Evaluate $\int_0^a \left(a^2 + x^2\right)^{5/2} dx.$

Solution :

$$I = \int_0^a \left(a^2 + x^2\right)^{5/2} dx \qquad \text{Put} \quad x = a \tan\theta$$

$$\therefore \quad dx = a \sec^2\theta \, d\theta$$

$$= \int_0^{\pi/4} \left(a^2 + a^2 \tan^2\theta\right)^{5/2} \cdot a \sec^2\theta \, d\theta$$

$$= a^6 \int_0^{\pi/4} \sec^7\theta \, d\theta$$

$$= a^6 \left[\left(\frac{\sec^5\theta \tan\theta}{6} \right)_0^{\pi/4} + \frac{5}{6} \int_0^{\pi/4} \sec^5\theta \, d\theta \right]$$

$$= a^6 \left[\frac{2\sqrt{2}}{3} + \frac{5}{6} \int_0^{\pi/4} \sec^5\theta \, d\theta \right]$$

$$= a^6 \left[\frac{2\sqrt{2}}{3} + \frac{5}{6} \left\{ \left(\frac{\sec^3\theta + \tan\theta}{4} \right)_0^{\pi/4} + \frac{3}{4} \int_0^{\pi/4} \sec^3\theta \, d\theta \right\} \right]$$

$$= a^6 \left[\frac{2\sqrt{2}}{3} + \frac{5}{6} \left\{ \frac{2\sqrt{2}}{4} + \frac{3}{4} \int_0^{\pi/4} \sec^3\theta \, d\theta \right\} \right]$$

$$= a^6 \left[\frac{2\sqrt{2}}{3} + \frac{5\sqrt{2}}{12} + \frac{5}{8} \int_0^{\pi/4} \sec^3\theta \, d\theta \right]$$

$$= a^6 \left[\frac{2\sqrt{2}}{3} + \frac{5\sqrt{2}}{12} + \frac{5}{8} \left\{ \left(\frac{\sec\theta \tan\theta}{2} \right)_0^{\pi/4} + \frac{1}{2} \int_0^{\pi/4} \sec\theta \, d\theta \right\} \right]$$

$$= a^6 \left[\frac{2\sqrt{2}}{3} + \frac{5\sqrt{2}}{12} + \frac{5}{8} \left\{ \frac{\sqrt{2}}{2} + \frac{1}{2} \left\{ \log\left(\sec\theta + \tan\theta\right) \right\}_0^{\pi/4} \right\} \right]$$

$$= a^6 \left[\frac{2\sqrt{2}}{3} + \frac{5\sqrt{2}}{12} + \frac{5\sqrt{2}}{16} + \frac{5}{16} \log\left(\sqrt{2} + 1\right) \right]$$

$$= a^6\left[\frac{32\sqrt{2}}{48}+\frac{20\sqrt{2}}{48}+\frac{15\sqrt{2}}{48}+\frac{5}{16}\log\left(\sqrt{2}+1\right)\right]$$

$$= a^6\left[\frac{32\sqrt{2}+20\sqrt{2}+15\sqrt{2}}{48}+\frac{5}{16}\log\left(\sqrt{2}+1\right)\right]$$

$$= a^6\left[\frac{67\sqrt{2}}{48}+\frac{5}{16}\log\left(\sqrt{2}+1\right)\right]$$

$$= \frac{a^6}{48}\left[67\sqrt{2}+15\log\left(\sqrt{2}+1\right)\right]$$

-

Practice example

$\int_0^5 \frac{\tan^{-1}(x-[x])}{1+(x-[x])^2}\,dx$, where [·] denotes the greatest integer function, is equal to

(a) $\frac{\pi^2}{32}$ (b) $\frac{3\pi^2}{32}$

(c) $\frac{5\pi^2}{32}$ (d) none of these

Solution :

(c). $\int_0^5 \frac{\tan^{-1}(x-[x])}{1+(x-[x])^2}\,dx$

$$= \int_0^5 \frac{\tan^{-1}(x-[x])}{1+(x-[x])^2}\,dx$$

$$= \int_0^1 \frac{\tan^{-1}x}{1+x^2}\,dx + \int_1^2 \frac{\tan^{-1}(x-1)}{1+(x-1)^2}\,dx + \ldots$$

$$+\int_4^5 \frac{\tan^{-1}(x-4)}{1+(x-4)^2}\,dx$$

$$= \int_0^1 \frac{\tan^{-1} x}{1+x^2}\,dx + \int_0^1 \frac{\tan^{-1} t}{1+t^2}\,dt + \dots + \int_0^1 \frac{\tan^{-1} t}{1+t^2}\,dt$$

(Putting $x-1=t$) (Putting $x-4=t$)

$$= 5\int_0^1 \frac{\tan^{-1} x}{1+x^2}\,dx = 5\int_0^{\pi/4} u\,du \quad [\text{Putting } \tan^{-1} x = u]$$

$$= 5\left[\frac{u^2}{2}\right]_0^{\pi/2} = \frac{5\pi^2}{32}$$

-

Practice example

Let $I_1 = \int_{\sec^2 z}^{2-\tan^2 z} x\,f(x(3-x))\,dx$

and, $I_2 = \int_{\sec^2 z}^{2-\tan^2 z} f(x(3-x))\,dx$,

where f is a continuous function and z is any real number, then $I_1/I_2 =$

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$

(c) 1 (d) none of these

Solution

(a). We have, $I_1 = \int_{\sec^2 z}^{2-\tan^2 z} x\, f(x(3-x))\, dx$

$$= \int_{\sec^2 z}^{2-\tan^2 z} (3-x)\, f((3-x)\{3-(3-x)\})\, dx$$

$$\left[\because \int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx\right]$$

$$= \int_{\sec^2 z}^{2-\tan^2 z} (3-x)\, f(x(3-x))$$

$$= 3\int_{\sec^2 z}^{2-\tan^2 z} f(x(3-x))\, dx - \int_{\sec^2 z}^{2-\tan^2 z} x f(x(3-x))\, dx$$

$$= 3\, I_2 - I_1$$

$\therefore \quad 2\, I_1 = 3\, I_2$ and so $I_1/I_2 = \dfrac{3}{2}$

-

Practice example

Evaluate $\int_0^{\pi/4} \tan^5 \theta\, d\theta$

$$I = \int_0^{\pi/4} \tan^5 \theta\, d\theta$$

$$= \left(\frac{\tan^4\theta}{4}\right)_0^{\pi/4} - \int_0^{\pi/4} \tan^3\theta\, d\theta$$

$$= \frac{1}{4} - \int_0^{\pi/4} \tan^3\theta\, d\theta$$

$$= \frac{1}{4} - \left[\left(\frac{\tan^2\theta}{2}\right)_0^{\pi/4} - \int_0^{\pi/4} \tan\theta\, d\theta\right]$$

$$= \frac{1}{4} - \left[\frac{1}{2} - (\log\sec\theta)_0^{\pi/4}\right]$$

$$= \frac{1}{4} - \left[\frac{1}{2} - \log\sqrt{2}\right]$$

$$= -\frac{1}{4} + \log\sqrt{2}$$

$$= -\frac{1}{4} + \frac{1}{2}\log 2$$

-

Practice example

If $\varphi(n) = \int_0^{\pi/4} \tan^n x\, dx$, show that $\varphi(n) + \varphi(n-2) = \frac{1}{n-1}$ and deduce the value of $\varphi(5)$.

Solution :

$$\varphi(n) = \int_0^{\pi/4} \tan^n x\, dx$$

$$= \left(\frac{\tan^{n-1} x}{n-1}\right)_0^{\pi/4} - \int_0^{\pi/4} \tan^{n-2} x\, dx$$

$$= \frac{1}{n-1} - \varphi_{n-2}$$

$$\Rightarrow \varphi_n + \varphi_{n-2} = \frac{1}{n-1} \qquad \text{Proved}$$

Now $\quad \varphi(5) = \frac{1}{4} - \varphi_3$

$$= \frac{1}{4} - \left[\frac{1}{2} - \varphi_1\right]$$

$$= -\frac{1}{4} + \varphi_1$$

$$= -\frac{1}{4} + \int_0^{\pi/4} \tan x\, dx$$

Practice Example

Prove that

$$\int_0^{\pi/2} \cos^m x \sin mx\, dx = \frac{1}{2^{m+1}}\left\{2 + \frac{2^2}{2} + \frac{2^3}{3} + \ldots.. + \frac{2^m}{m}\right\}$$

Solution :

We know that

$$\int_0^{\pi/2} \cos^m x \sin mx\, dx$$

$$= \left[-\frac{\cos^m x \cos mx}{m+m}\right]_0^{\pi/2} + \frac{m}{m+m} \int_0^{\pi/2} \cos^{m-1} x \sin(m-1)x\, dx$$

$$\Rightarrow \qquad I_{m,m} = \frac{1}{2m} + \frac{1}{2} I_{m-1,m-1}$$

Put $m - 1$ for m,

$$I_{m-1,m-1} = \frac{1}{2(m-1)} + \frac{1}{2} I_{m-2,m-2}$$

$$I_{m,m} = \frac{1}{2m} + \frac{1}{2}\left[\frac{1}{2(m-1)} + \frac{1}{2} I_{m-2,m-2}\right]$$

$$= \frac{1}{2m} + \frac{1}{2^2(m-1)} + \frac{1}{2^2} I_{m-2,m-2}$$

$$= \frac{1}{2m} + \frac{1}{2^2(m-1)} + \frac{1}{2^3(m-2)} + \frac{1}{2^3} I_{m-3,m-3}$$

| Proceeding similarly

$$= \frac{1}{2m} + \frac{1}{2^2(m-1)} + \frac{1}{2^3(m-2)} + \ldots\ldots + \frac{1}{2^m\{m-(m-1)\}} + \frac{1}{2^m} I_{m-m,m-m}$$

$$= \frac{1}{2m} + \frac{1}{2^2(m-1)} + \frac{1}{2^3(m-2)} + \ldots\ldots + \frac{1}{2^m 1} + \frac{1}{2^m} I_{o,o}$$

$$= \frac{1}{2m} + \frac{1}{2^2(m-1)} + \frac{1}{2^3(m-2)} + \ldots\ldots + \frac{1}{2^m \cdot 1} + \frac{1}{2^m}\int_0^{\pi/2} o\,dx$$

Now $\int_0^{\pi/2} o\,dx \; = \; [c]_0^{\pi/2} = c - c = o$

$$\therefore \quad I_{m,m} = \frac{1}{2m} + \frac{1}{2^2(m-1)} + \frac{1}{2^3(m-2)} + \ldots\ldots + \frac{1}{2^m \cdot 1}$$

Writing the series in the reverse order

$$= \frac{1}{2^m \cdot 1} + \frac{1}{2^{m-1}\cdot 2} + \frac{1}{2^{m-2}\cdot 3} + \ldots\ldots + \frac{1}{2m}$$

$$= \frac{1}{2^{m+1}}\left[\frac{2^{m+1}}{2^m \cdot 1} + \frac{2^{m+1}}{2^{m-1}\cdot 2} + \frac{2^{m+1}}{2^{m-2}\cdot 3} + \ldots\ldots + \frac{2^{m+1}}{2m}\right]$$

Practice Example

Prove that $\int_0^{\pi/2} \cos^{n-2} x \sin nx \, dx = \frac{1}{n-1}$; n being an integer greater than unity.

Solution :

$$
\begin{aligned}
I &= \int_0^{\pi/2} \cos^{n-2} x \sin nx \, dx \\
&= \int_0^{\pi/2} \cos^{n-2} x \sin \{(n-1)x + x\} \, dx \\
&= \int_0^{\pi/2} \cos^{n-2} x \{\sin (n-1) x \cos x \\
&\qquad\qquad + \cos (n-1) x \sin x\} \, dx \\
&= \int_0^{\pi/2} \underset{\mathrm{I}}{\cos^{n-1} x} \, \underset{\mathrm{II}}{\sin (n-1) x} \, dx \\
&\qquad + \int_0^{\pi/2} \cos^{n-2} x \cos (n-1) x \sin x \, dx
\end{aligned}
$$

Integrating the first integral only by parts

$$
\begin{aligned}
&= \left\{\cos^{n-1} x - \frac{\cos (n-1) x}{n-1}\right\}_0^{\pi/2} \\
&\quad - \int_0^{\pi/2} (n-1) \cos^{n-2} x (-\sin x) \cdot \left\{-\frac{\cos (n-1) x}{n-1}\right\} dx \\
&\qquad + \int_0^{\pi/2} \cos^{n-2} x \cos (n-1) x \sin x \, dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{n-1} - \int_0^{\pi/2} \cos^{n-2} x \cos (n-1) x \sin x \, dx \\
&\qquad + \int_0^{\pi/2} \cos^{n-2} x \cos (n-1) x \sin x \, dx
\end{aligned}
$$

$$
= \frac{1}{n-1}
$$

Practice Example

If $I_{1,n} = \int_0^{\pi/2} \frac{\sin(2n-1)x}{\sin x} dx$ and $I_{2,n} = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin^2 x} dx$

$n \in N$, then

(a) $I_{2,n+1} - I_{2,n} = I_{1,n}$

(b) $I_{2,n+1} - I_{2,n} = I_{1,n+1}$

(c) $I_{2,n+1} + I_{1,n} = I_{2,n}$

(d) $I_{2,n+1} + I_{1,n+1} = I_{2,n}$

Solution

(b). $I_{2,n} - I_{2,n-1} = \int_0^{\pi/2} \frac{(\sin^2 nx - \sin^2 (n-1)x)}{\sin^2 x} dx$

$$= \int_0^{\pi/2} \frac{\sin (2n-1)x \sin x}{\sin^2 x} dx$$

$$= \int_0^{\pi/2} \frac{\sin (2n-1)x}{\sin x} dx = I_{1,n}$$

$\therefore \quad I_{2,n+1} - I_{2,n} = I_{1,n+1}$

Reduction forms

Let $I_n = \int \sin^n x\,dx$ or $I_n = \int \sin^{n-1} x \sin x\,dx$.

Integrating by parts regarding sin x as the 2nd function, we have

$$I_n = \sin^{n-1} x.(-\cos x) - \int (n-1)\sin^{n-2} x.\cos x.(-\cos x)dx$$

$$= -\sin^{n-1} x.\cos x + (n-1)\int \sin^{n-2} x.\cos^2 x\,dx$$

$$= -\sin^{n-1} x.\cos x + (n-1)\int \sin^{n-2} x.\left(1-\sin^2 x\right)dx$$

$$= -\sin^{n-1} x.\cos x + (n-1)\int \sin^{n-2} x\,dx - (n-1)\int \sin^n x\,dx$$

$$= -\sin^{n-1} x.\cos x + (n-1)\int \sin^{n-2} x\,dx - (n-1)I_n.$$

Transposing the last term to the left, we have

$$I_n (1 + n - 1) = -\sin^{n-1} x.\cos x + (n-1) I_{n-2},$$

$$\left[\because I_{n-2} = \int \sin^{n-2} x\,dx\right]$$

or $nI_n = -\sin^{n-1} x \cos x + (n-1) I_{n-2}$

or $I_n = -\dfrac{\sin^{n-1} x \cos x}{n} + \dfrac{n-1}{n} I_{n-2}$.

Let $I_n = \int \cos^n x\,dx$ or $I_n = \int \cos^{n-1} x.\cos x\,dx$.

Integrating by parts regarding cos x as the 2nd function, we have

$$I_n = \cos^{n-1} x.\sin x - \int (n-1)\cos^{n-2} x.(\sin x).\sin x\,dx$$

$$= \cos^{n-1} x.\sin x + (n+1)\int \cos^{n-2} x.\sin^2 x\,dx$$

$$= \cos^{n-1} x.\sin x + (n-1)\int \cos^{n-2} x\left(1-\cos^2 x\right)dx$$

$$= \cos^{n-1} x.\sin x + (n-1)\int \cos^{n-2} x\,dx - (n-1)\int \cos^n x\,dx$$

$$= \cos^{n-1} x \sin x + (n-1) I_{n-2} - (n-1) I_n.$$

Transposing the last term to the left, we have

$$I_n (1 + n - 1) = \cos^{n-1} x.\sin x + (n-1) I_{n-2}$$

or $n I_n = \cos^{n-1} x.\sin x + (n-1) I_{n-2}$.

$$\therefore \int \cos^n dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx.$$

We have $\int \tan^n x\,dx = \int \tan^{n-2} x.\tan^2 x\,dx$

$= \int \tan^{n-2} x.\left(\sec^2 x - 1\right)dx$

$= \int \tan^{n-2} x.\sec^2 x\,dx - \int \tan^{n-2} x\,dx$

$= \dfrac{(\tan x)^{n-2+1}}{n-2+1} - \int \tan^{n-2} x\,dx$

or $\int \mathbf{\tan^n x\,dx = \dfrac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx}$,

We have $\int \cot^n x\,dx = \int \cot^{n-2} x.\cot^2 x\,dx$

$= \int \cot^{n-2} x\left(\mathrm{cosec}^2 x - 1\right)dx$

$= \int \cot^{n-2} x.\mathrm{cosec}^2 x\,dx - \int \cot^{n-2} x\,dx$

$= -\dfrac{(\cot x)^{n-1}}{n-1} - \int \cot^{n-2} x\,dx$

or $\cot^n x\,dx = -\dfrac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x\,dx$,

We have $I_n = \int \sec^n x\,dx = \int \sec^{n-2} x.\sec^2 x\,dx$

Integrating by parts regarding $\sec^2 x$ as the 2nd function, we have

$I_n = \sec^{n-2} x \tan x - \int (n-2)\sec^{n-3} x \sec x \tan^2 x\,dx$

$= \sec^{n-2} x \tan x - (n-2)\int \sec^{n-2} x\left(\sec^2 x - 1\right)dx$

$= \sec^{n-2} x \tan x - (n-2)\int \sec^n x\,dx + (n+2)\int \sec^{n-2} x\,dx$.

Transposing the term containing $\int \sec^n x\,dx$ to the left,

we have

$(n-2+1)\int \sec^n x\,dx = \sec^{n-2} x \tan x + (n-2)\int \sec^{n-2} x\,dx$

$$\int \sec^n x \, dx = \sec^{n-2} x \tan x - \int (n-2)\sec^{n-2} x \tan x \tan x \, dx$$
$$= \tan x \sec^{n-2} x - (n-2)\int \sec^{n-2} x \, (\sec^2 x - 1)\, dx$$
$$= \tan x \sec^{n-2} x - (n-2)\left(\sec^n x - \int \sec^{n-2} x \, dx\right)$$
$$[1+(n-2)]\int \sec^n x \, dx = \tan x \sec^{n-2} x + (n-2)\int \sec^{n-2} x \, dx$$
$$\int \sec^n x \, dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x \, dx$$

$$\int \operatorname{cosec}^n x \, dx = \int \operatorname{cosec}^{n-2} x \operatorname{cosec}^2 x \, dx$$

Integrating by parts,

$$\int \operatorname{cosec}^n x \, dx = \operatorname{cosec}^{n-2} x \, (-\cot x) - \int (n-2)\operatorname{cosec}^{n-3} x \, (-\operatorname{cosec} x \cot x)(-\cot x)\, dx$$
$$= -\cot x \operatorname{cosec}^{n-2} x - (n-2)\int \operatorname{cosec}^{n-2} x \, (\operatorname{cosec}^2 x - 1)\, dx$$
$$= -\cot x \operatorname{cosec}^{n-2} x - (n-2)\left(\int \operatorname{cosec}^n x - \int \operatorname{cosec}^{n-2} x \, dx\right)$$
$$[1+(n-2)]\int \operatorname{cosec}^n x \, dx = -\cot x \operatorname{cosec}^{n-2} x + (n-2)\int \operatorname{cosec}^{n-2} x \, dx$$
$$\int \operatorname{cosec}^n x \, dx = \frac{-\cot x \operatorname{cosec}^{n-2} x}{n-1} + \frac{n-2}{n-1}\int \operatorname{cosec}^{n-2} x \, dx$$

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Landmark Integral Problem IIT JEE 2002 x becomes x to the power m inside the bracket

https://archive.org/details/LandmarkIntegralProblemIITJEE2002XBecomesXToThePowerMInsideTheBracket2

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To find the Area of a circle of Radius r

https://archive.org/details/7AreaOfCircleP1IITJEEPhyMath

Area of Circle Surface Area and Volume of Sphere

https://archive.org/details/7AreaOfCircleP2SurfaceAreaVolumeOfSphereIITJEEPhyMath

Very important concept of Edge Modeling Surface Area Volume of Sphere

https://archive.org/details/7EdgeModellingP2SurfaceAreaVolumeOfSphereIITJEEPhyMath

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Very important concept of solving differential equation of d2x by dt square equals f(x)

https://archive.org/details/7Steps2SolveD2xByDtSquarefxSHMIITJEEPhyMaths1

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A few Differential Equation Problems

https://archive.org/details/AIEEEDifferentialEquation2008CanBeSolvedAsHomogeneousAndAlsoLinear

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AIEEE 2009 - (Important) Shortest Distance between Graph and line by finding tangent

https://archive.org/details/AIEEEImportantShortestDistanceBetweenGraphAndLineByFindingTangent2009

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AIEEE-2002 Definite Integral Elimination of x by f(a-x) form even and odd functions

https://archive.org/details/AIEEE2002DefiniteIntegralEliminationOfXByFaXFormEvenAndOddFunctions

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AIEEE - Area by Integration - This one is easier to do by integrating f(y) dy seen from right

https://archive.org/details/AIEEEAreaByIntegrationThisOneIsEasierToDoByIntegratingFyDySeenFromRight

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AIEEE 2007 - Expansion of e to the power -1 - must know the expansions

https://archive.org/details/AIEEEExpansionOfEToThePower1MustKnowTheExpansions2007

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AIEEE 2008 - Inequality of Integrals - Sin x by root x and Cos x by root x

https://archive.org/details/AIEEEInequalityOfIntegralsSinXByRootXAndCosXByRootX2008

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AIEEE 2002 Problems in Calculus now known as IIT JEE main

https://archive.org/details/AIEEE2002MathsNiceIntegralWithTrickSimilarToAnIITProblemOf80s

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Very important recall in a definite integral tan inverse is in the denominator

https://archive.org/details/VeryImportantRecallInADefiniteIntegralTanInverseIsInTheDenominator

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Miscellaneous Integration Problems solved by a very good Teacher, Mr Keshav Agarwal. (from youtube)

https://archive.org/details/INTEGRATIONPROBLEMSANDSOLUTIONS

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Special extra problem in Limits solved by expansion

https://archive.org/details/VeryImportantLimitOfSinxToThePowerSinxLnSinXEtcSolvedByExpansions

AIEEE 2007 - Limit-Infinity- infinity form, expand e^x and do this, instead of L-Hospitals

https://archive.org/details/AIEEELimitInfinityInfinityFormExpandExAndDoThisInsteadOfLHospitals2007

Limits - Is this easy.... (1 + Sin x) to the power Cot x by 2

https://archive.org/details/LimitsIsThisEasy....1SinXToThePowerCotXBy2

-

AIEEE 2009 - Special extra problem in Implicit Differentiation-x to the power x and Cot y and x^2x

https://archive.org/details/AIEEEImplicitDifferentiationXToThePowerXAndCotYAndX2x2009

-

Solve by First Principle $\int_a^b x\,dx$

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\ldots+f\left(a+(n-1)h\right)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=a$, $b=b$, and $f(x)=x$

$$\therefore \int_a^b x\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[a+(a+h)\ldots(a+2h)\ldots a+(n-1)h\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{a+a+a+\ldots+a}_{n\text{ times}}\right)+\left(h+2h+3h+\ldots+(n-1)h\right)\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+h\left(1+2+3+\ldots+(n-1)\right)\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+h\left\{\frac{(n-1)(n)}{2}\right\}\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+\frac{n(n-1)h}{2}\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{n}{n}\left[a+\frac{(n-1)h}{2}\right]$$

$$= (b-a)\lim_{n\to\infty}\left[a+\frac{(n-1)h}{2}\right]$$

$$= (b-a)\lim_{n\to\infty}\left[a+\frac{(n-1)(b-a)}{2n}\right]$$

$$= (b-a)\lim_{n\to\infty}\left[a+\frac{\left(1-\frac{1}{n}\right)(b-a)}{2}\right]$$

$$= (b-a)\left[a+\frac{(b-a)}{2}\right]$$

$$= (b-a)\left[\frac{2a+b-a}{2}\right]$$

$$= \frac{(b-a)(b+a)}{2}$$

$$= \frac{1}{2}\left(b^2-a^2\right)$$

-

Find $\int_0^5 (x+1)\,dx$

Let $I = \int_0^5 (x+1)\,dx$

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)\ldots f\left(a+(n-1)h\right)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=0$, $b=5$, and $f(x)=(x+1)$

$$\Rightarrow h=\frac{5-0}{n}=\frac{5}{n}$$

$$\therefore \int_0^5 (x+1)\,dx = (5-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+\ldots+f\left((n-1)\frac{5}{n}\right)\right]$$

$$=5\lim_{n\to\infty}\frac{1}{n}\left[1+\left(\frac{5}{n}+1\right)+\ldots\left\{1+\left(\frac{5(n-1)}{n}\right)\right\}\right]$$

$$=5\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1+1+1\ldots1}_{n\text{ times}}\right)+\left[\frac{5}{n}+2\cdot\frac{5}{n}+3\cdot\frac{5}{n}+\ldots(n-1)\frac{5}{n}\right]\right]$$

$$=5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\{1+2+3\ldots(n-1)\}\right]$$

$$=5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\cdot\frac{(n-1)n}{2}\right]$$

$$=5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5(n-1)}{2}\right]$$

$$=5\lim_{n\to\infty}\left[1+\frac{5}{2}\left(1-\frac{1}{n}\right)\right]$$

$$=5\left[1+\frac{5}{2}\right]$$

$$=5\left[\frac{7}{2}\right]$$

$$=\frac{35}{2}$$

-

Find $\int_2^3 x^2 dx$

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+f(a+2h)\ldots f\{a+(n-1)h\}\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=2$, $b=3$, and $f(x)=x^2$

$$\Rightarrow h=\frac{3-2}{n}=\frac{1}{n}$$

$$\therefore \int_2^3 x^2 dx = (3-2)\lim_{n\to\infty}\frac{1}{n}\left[f(2)+f\left(2+\frac{1}{n}\right)+f\left(2+\frac{2}{n}\right)\ldots f\left\{2+(n-1)\frac{1}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[(2)^2+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+\ldots\left(2+\frac{(n-1)}{n}\right)^2\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[2^2+\left\{2^2+\left(\frac{1}{n}\right)^2+2\cdot 2\cdot\frac{1}{n}\right\}+\ldots+\left\{(2)^2+\frac{(n-1)^2}{n^2}+2\cdot 2\cdot\frac{(n-1)}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{2^2+\ldots+2^2}_{n\text{ times}}\right)+\left\{\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\ldots+\left(\frac{n-1}{n}\right)^2\right\}+2\cdot 2\cdot\left\{\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots+\frac{(n-1)}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{1^2+2^2+3^2\ldots+(n-1)^2\right\}+\frac{4}{n}\left\{1+2+\ldots+(n-1)\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{\frac{n(n-1)(2n-1)}{6}\right\}+\frac{4}{n}\left\{\frac{n(n-1)}{2}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{n\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}+\frac{4n-4}{2}\right]$$

$$=\lim_{n\to\infty}\left[4+\frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+2-\frac{2}{n}\right]$$

$$=4+\frac{2}{6}+2$$

$$=\frac{19}{3}$$

Find $\int_1^4 \left(x^2 - x\right) dx$

$$\text{Let } I = \int_1^4 \left(x^2 - x\right) dx$$
$$= \int_1^4 x^2\, dx - \int_1^4 x\, dx$$
$$\text{Let } I = I_1 - I_2, \text{ where } I_1 = \int_1^4 x^2 dx \text{ and } I_2 = \int_1^4 x\, dx \qquad ...(1)$$

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a) + f(a+h) + f\left(a + (n-1)h\right)\right], \text{ where } h = \frac{b-a}{n}$$

$$\text{For } I_1 = \int_1^4 x^2 dx,$$
$$a = 1, b = 4, \text{ and } f(x) = x^2$$
$$\therefore h = \frac{4-1}{n} = \frac{3}{n}$$

$$I_1 = \int_1^4 x^2 dx = (4-1)\lim_{n\to\infty}\frac{1}{n}\left[f(1) + f(1+h) + \ldots + f\left(1 + (n-1)h\right)\right]$$
$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1^2 + \left(1 + \frac{3}{n}\right)^2 + \left(1 + 2\cdot\frac{3}{n}\right)^2 + \ldots\left(1 + \frac{(n-1)3}{n}\right)^2\right]$$
$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1^2 + \left\{1^2 + \left(\frac{3}{n}\right)^2 + 2\cdot\frac{3}{n}\right\} + \ldots + \left\{1^2 + \left(\frac{(n-1)3}{n}\right)^2 + \frac{2\cdot(n-1)\cdot 3}{n}\right\}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1^2 + \ldots + 1^2}_{n \text{ times}}\right) + \left(\frac{3}{n}\right)^2 \left\{1^2 + 2^2 + \ldots + (n-1)^2\right\} + 2\cdot\frac{3}{n}\left\{1 + 2 + \ldots + (n-1)\right\}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[n + \frac{9}{n^2}\left\{\frac{(n-1)(n)(2n-1)}{6}\right\} + \frac{6}{n}\left\{\frac{(n-1)(n)}{2}\right\}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[n + \frac{9n}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right) + \frac{6n-6}{2}\right]$$

$$= 3\lim_{n\to\infty}\left[1 + \frac{9}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right) + 3 - \frac{3}{n}\right]$$

$$= 3[1+3+3]$$

$$= 3[7]$$

$$I_1 = 21 \qquad \ldots(2)$$

For $I_2 = \int_1^4 x dx$,

$a = 1, b = 4,$ and $f(x) = x$

$$\Rightarrow h = \frac{4-1}{n} = \frac{3}{n}$$

$$\therefore I_2 = (4-1)\lim_{n\to\infty}\frac{1}{n}\left[f(1) + f(1+h) + \ldots f\left(a + (n-1)h\right)\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1+(1+h)+\ldots+(1+(n-1)h)\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1+\left(1+\frac{3}{n}\right)+\ldots+\left\{1+(n-1)\frac{3}{n}\right\}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1+1+\ldots+1}_{n \text{ times}}\right)+\frac{3}{n}\left(1+2+\ldots+(n-1)\right)\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{3}{n}\left\{\frac{(n-1)n}{2}\right\}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1+\frac{3}{2}\left(1-\frac{1}{n}\right)\right]$$

$$= 3\left[1+\frac{3}{2}\right]$$

$$= 3\left[\frac{5}{2}\right]$$

$$I_2 = \frac{15}{2} \qquad \ldots(3)$$

From equations (2) and (3), we obtain

$$I = I_1 + I_2 = 21 - \frac{15}{2} = \frac{27}{2}$$

-

Find $\int_{-1}^{1} e^x dx$

$$\text{Let } I = \int_{-1}^{1} e^x dx \qquad ...(1)$$

It is known that,

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)...f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=-1$, $b=1$, and $f(x)=e^x$

$$\therefore h=\frac{1+1}{n}=\frac{2}{n}$$

$$\therefore I=(1+1)\lim_{n\to\infty}\frac{1}{n}\left[f(-1)+f\left(-1+\frac{2}{n}\right)+f\left(-1+2\cdot\frac{2}{n}\right)+...+f\left(-1+\frac{(n-1)2}{n}\right)\right]$$

$$=2\lim_{n\to\infty}\frac{1}{n}\left[e^{-1}+e^{\left(-1+\frac{2}{n}\right)}+e^{\left(-1+2\frac{2}{n}\right)}+...e^{\left(-1+(n-1)\frac{2}{n}\right)}\right]$$

$$=2\lim_{n\to\infty}\frac{1}{n}\left[e^{-1}\left\{1+e^{\frac{2}{n}}+e^{\frac{4}{n}}+e^{\frac{6}{n}}+e^{(n-1)\frac{2}{n}}\right\}\right]$$

$$=2\lim_{n\to\infty}\frac{e^{-1}}{n}\left[\frac{e^{\frac{2n}{n}}-1}{e^{\frac{2}{n}}-1}\right]$$

$$=e^{-1}\times 2\lim_{n\to\infty}\frac{1}{n}\left[\frac{e^2-1}{e^{\frac{2}{n}}-1}\right]$$

$$=\frac{e^{-1}\times 2(e^2-1)}{\lim\limits_{\frac{2}{n}\to 0}\left(\frac{e^{\frac{2}{n}}-1}{\frac{2}{n}}\right)\times 2}$$

$$=e^{-1}\left[\frac{2(e^2-1)}{2}\right] \qquad \left[\lim_{h\to 0}\left(\frac{e^h-1}{h}\right)=1\right]$$

$$=\frac{e^2-1}{e}$$

$$=\left(e-\frac{1}{e}\right)$$

Find $\int_0^4 \left(x+e^{2x}\right)dx$

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\ldots+f\left(a+(n-1)h\right)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=0$, $b=4$, and $f(x)=x+e^{2x}$

$$\therefore h=\frac{4-0}{n}=\frac{4}{n}$$

$$\Rightarrow \int_0^4\left(x+e^{2x}\right)dx=(4-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f(h)+f(2h)+\ldots+f\left((n-1)h\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\left(0+e^{0}\right)+\left(h+e^{2h}\right)+\left(2h+e^{2\cdot 2h}\right)+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[1+\left(h+e^{2h}\right)+\left(2h+e^{4h}\right)+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\left\{h+2h+3h+\ldots+(n-1)h\right\}+\left(1+e^{2h}+e^{4h}+\ldots+e^{2(n-1)h}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[h\left\{1+2+\ldots(n-1)\right\}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\frac{\left(h(n-1)n\right)}{2}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\frac{4}{n}\cdot\frac{(n-1)n}{2}+\left(\frac{e^{8}-1}{e^{\frac{8}{n}}-1}\right)\right]$$

$$=4(2)+4\lim_{n\to\infty}\frac{\left(e^{8}-1\right)}{\left(\dfrac{e^{\frac{8}{n}}-1}{\frac{8}{n}}\right)8}$$

$$=8+\frac{4\cdot\left(e^{8}-1\right)}{8} \qquad \left(\lim_{x\to 0}\frac{e^{x}-1}{x}=1\right)$$

$$=8+\frac{e^{8}-1}{2}$$

$$=\frac{15+e^{8}}{2}$$

$$\int_{-1}^{1}(x+1)dx$$

Answer :

Let $I=\int_{-1}^{1}(x+1)dx$

$$\int(x+1)\,dx=\frac{x^2}{2}+x=\mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}I&=\mathrm{F}(1)-\mathrm{F}(-1)\\&=\left(\frac{1}{2}+1\right)-\left(\frac{1}{2}-1\right)\\&=\frac{1}{2}+1-\frac{1}{2}+1\\&=2\end{aligned}$$

-

$$\int_{2}^{3}\frac{1}{x}dx$$

Answer :

Let $I=\int_{2}^{3}\frac{1}{x}\,dx$

$$\int\frac{1}{x}\,dx=\log|x|=\mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}I&=\mathrm{F}(3)-\mathrm{F}(2)\\&=\log|3|-\log|2|=\log\frac{3}{2}\end{aligned}$$

-

$$\int_1^2 \left(4x^3 - 5x^2 + 6x + 9\right) dx$$

Answer :

Let $I = \int_1^2 \left(4x^3 - 5x^2 + 6x + 9\right) dx$

$$\int \left(4x^3 - 5x^2 + 6x + 9\right) dx = 4\left(\frac{x^4}{4}\right) - 5\left(\frac{x^3}{3}\right) + 6\left(\frac{x^2}{2}\right) + 9(x)$$

$$= x^4 - \frac{5x^3}{3} + 3x^2 + 9x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(2) - \mathrm{F}(1)$$

$$I = \left\{2^4 - \frac{5\cdot(2)^3}{3} + 3(2)^2 + 9(2)\right\} - \left\{(1)^4 - \frac{5(1)^3}{3} + 3(1)^2 + 9(1)\right\}$$

$$= \left(16 - \frac{40}{3} + 12 + 18\right) - \left(1 - \frac{5}{3} + 3 + 9\right)$$

$$= 16 - \frac{40}{3} + 12 + 18 - 1 + \frac{5}{3} - 3 - 9$$

$$= 33 - \frac{35}{3}$$

$$= \frac{99 - 35}{3}$$

$$= \frac{64}{3}$$

-

$$\int_0^{\frac{\pi}{4}} \sin 2x dx$$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \sin 2x \, dx$$

$$\int \sin 2x \, dx = \left(\frac{-\cos 2x}{2}\right) = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}\left(\frac{\pi}{4}\right) - \mathrm{F}(0)$$

$$= -\frac{1}{2}\left[\cos 2\left(\frac{\pi}{4}\right) - \cos 0\right]$$

$$= -\frac{1}{2}\left[\cos\left(\frac{\pi}{2}\right) - \cos 0\right]$$

$$= -\frac{1}{2}[0-1]$$

$$= \frac{1}{2}$$

-

$$\int_0^{\frac{\pi}{2}} \cos 2x\, dx$$

Answer :

Let $I = \int_0^{\frac{\pi}{2}} \cos 2x\, dx$

$\int \cos 2x\, dx = \left(\frac{\sin 2x}{2}\right) = \mathrm{F}(x)$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \mathrm{F}\left(\frac{\pi}{2}\right) - \mathrm{F}(0) \\ &= \frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right) - \sin 0\right] \\ &= \frac{1}{2}[\sin\pi - \sin 0] \\ &= \frac{1}{2}[0-0] = 0 \end{aligned}$$

-

$$\int_4^5 e^x\, dx$$

Answer :

Let $I = \int_4^5 e^x\, dx$

$\int e^x\, dx = e^x = \mathrm{F}(x)$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \mathrm{F}(5) - \mathrm{F}(4) \\ &= e^5 - e^4 \\ &= e^4(e-1) \end{aligned}$$

-

$$\int_0^{\frac{\pi}{4}} \tan x \, dx$$

Answer :

Let $I = \int_0^{\frac{\pi}{4}} \tan x \, dx$

$$\int \tan x \, dx = -\log|\cos x| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F(0)$$

$$= -\log\left|\cos\frac{\pi}{4}\right| + \log|\cos 0|$$

$$= -\log\left|\frac{1}{\sqrt{2}}\right| + \log|1|$$

$$= -\log(2)^{-\frac{1}{2}}$$

$$= \frac{1}{2}\log 2$$

-

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$$

Answer :

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$

$$\int \operatorname{cosec} x \, dx = \log\left|\operatorname{cosec} x - \cot x\right| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F\left(\frac{\pi}{6}\right)$$

$$= \log\left|\operatorname{cosec}\frac{\pi}{4} - \cot\frac{\pi}{4}\right| - \log\left|\operatorname{cosec}\frac{\pi}{6} - \cot\frac{\pi}{6}\right|$$

$$= \log\left|\sqrt{2} - 1\right| - \log\left|2 - \sqrt{3}\right|$$

$$= \log\left(\frac{\sqrt{2} - 1}{2 - \sqrt{3}}\right)$$

-

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

Answer :

Let $I = \int_0^1 \frac{dx}{\sqrt{1-x^2}}$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \mathrm{F}(1) - \mathrm{F}(0) \\ &= \sin^{-1}(1) - \sin^{-1}(0) \\ &= \frac{\pi}{2} - 0 \\ &= \frac{\pi}{2} \end{aligned}$$

-

$$\int_0^1 \frac{dx}{1+x^2}$$

Answer :

Let $I = \int_0^1 \frac{dx}{1+x^2}$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \mathrm{F}(1) - \mathrm{F}(0) \\ &= \tan^{-1}(1) - \tan^{-1}(0) \\ &= \frac{\pi}{4} \end{aligned}$$

-

$$\int_2^3 \frac{dx}{x^2-1}$$

Answer :

$$\text{Let } I = \int_2^3 \frac{dx}{x^2-1}$$

$$\int \frac{dx}{x^2-1} = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(3) - \mathrm{F}(2)$$

$$= \frac{1}{2}\left[\log\left|\frac{3-1}{3+1}\right| - \log\left|\frac{2-1}{2+1}\right|\right]$$

$$= \frac{1}{2}\left[\log\left|\frac{2}{4}\right| - \log\left|\frac{1}{3}\right|\right]$$

$$= \frac{1}{2}\left[\log\frac{1}{2} - \log\frac{1}{3}\right]$$

$$= \frac{1}{2}\left[\log\frac{3}{2}\right]$$

-

$$\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

Answer :

Let $I = \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$

$$\int \cos^2 x \, dx = \int\left(\frac{1+\cos 2x}{2}\right)dx = \frac{x}{2}+\frac{\sin 2x}{4} = \frac{1}{2}\left(x+\frac{\sin 2x}{2}\right) = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \left[\mathrm{F}\left(\frac{\pi}{2}\right)-\mathrm{F}(0)\right]$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{2}-\frac{\sin \pi}{2}\right)-\left(0+\frac{\sin 0}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2}+0-0-0\right]$$

$$= \frac{\pi}{4}$$

-

$$\int_2^3 \frac{x\,dx}{x^2+1}$$

Answer :

$$\text{Let } I = \int_2^3 \frac{x}{x^2+1}dx$$

$$\int \frac{x}{x^2+1}dx = \frac{1}{2}\int \frac{2x}{x^2+1}dx = \frac{1}{2}\log\left(1+x^2\right) = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \mathrm{F}(3) - \mathrm{F}(2) \\ &= \frac{1}{2}\left[\log\left(1+(3)^2\right) - \log\left(1+(2)^2\right)\right] \\ &= \frac{1}{2}\left[\log(10) - \log(5)\right] \\ &= \frac{1}{2}\log\left(\frac{10}{5}\right) = \frac{1}{2}\log 2 \end{aligned}$$

-

$$\int_0^1 \frac{2x+3}{5x^2+1}dx$$

Answer :

$$\text{Let } I = \int_0^1 \frac{2x+3}{5x^2+1}dx$$

$$\int \frac{2x+3}{5x^2+1}dx = \frac{1}{5}\int \frac{5(2x+3)}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x+15}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5\left(x^2+\frac{1}{5}\right)}dx$$

$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{5}\cdot\frac{1}{\frac{1}{\sqrt{5}}}\tan^{-1}\frac{x}{\frac{1}{\sqrt{5}}}$$

$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}x\right)$$

= F(x)

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(0)$$

$$= \left\{\frac{1}{5}\log(5+1) + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}\right)\right\} - \left\{\frac{1}{5}\log(1) + \frac{3}{\sqrt{5}}\tan^{-1}(0)\right\}$$

$$= \frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$$

-

$$\int_0^1 xe^{x^2}\,dx$$

Answer :

Let $I = \int_0^1 xe^{x^2}\,dx$

Put $x^2 = t \Rightarrow 2x\ dx = dt$

As $x \to 0, t \to 0$ and as $x \to 1, t \to 1,$

$\therefore I = \frac{1}{2}\int_0^1 e^t\,dt$

$\frac{1}{2}\int e^t\,dt = \frac{1}{2}e^t = \mathrm{F}(t)$

By second fundamental theorem of calculus, we obtain

$I = \mathrm{F}(1) - \mathrm{F}(0)$

$= \frac{1}{2}e - \frac{1}{2}e^0$

$= \frac{1}{2}(e-1)$

-

$$\int \frac{5x^2}{x^2+4x+3}$$

Answer :

Let $I = \int_1^2 \frac{5x^2}{x^2+4x+3}dx$

Dividing $5x^2$ by x^2+4x+3, we obtain

$$I = \int_1^2 \left\{5 - \frac{20x+15}{x^2+4x+3}\right\}dx$$

$$= \int_1^2 5dx - \int_1^2 \frac{20x+15}{x^2+4x+3}dx$$

$$= [5x]_1^2 - \int_1^2 \frac{20x+15}{x^2+4x+3}dx$$

$$I = 5 - I_1, \text{ where } I = \int_1^2 \frac{20x+15}{x^2+4x+3}dx \qquad ...(1)$$

Consider $I_1 = \int_1^2 \frac{20x+15}{x^2+4x+8}dx$

$$\text{Let } 20x+15 = \mathrm{A}\frac{d}{dx}\left(x^2+4x+3\right)+\mathrm{B}$$
$$= 2\mathrm{A}x+\left(4\mathrm{A}+\mathrm{B}\right)$$

Equating the coefficients of x and constant term, we obtain

A = 10 and B = - 25

$$\Rightarrow I_1 = 10\int_1^2 \frac{2x+4}{x^2+4x+3}dx - 25\int_1^2 \frac{dx}{x^2+4x+3}$$

$$\text{Let } x^2+4x+3 = t$$
$$\Rightarrow (2x+4)\,dx = dt$$

$$\Rightarrow I_1 = 10\int\frac{dt}{t} - 25\int\frac{dx}{(x+2)^2-1^2}$$
$$= 10\log t - 25\left[\frac{1}{2}\log\left(\frac{x+2-1}{x+2+1}\right)\right]$$
$$= \left[10\log\left(x^2+4x+3\right)\right]_1^2 - 25\left[\frac{1}{2}\log\left(\frac{x+1}{x+3}\right)\right]_1^2$$
$$= \left[10\log 15 - 10\log 8\right] - 25\left[\frac{1}{2}\log\frac{3}{5} - \frac{1}{2}\log\frac{2}{4}\right]$$

-

$$=\left[10\log(5\times 3)-10\log(4\times 2)\right]-\frac{25}{2}\left[\log 3-\log 5-\log 2+\log 4\right]$$

$$\begin{aligned}
&=\left[10\log 5+10\log 3-10\log 4-10\log 2\right]-\frac{25}{2}\left[\log 3-\log 5-\log 2+\log 4\right]\\
&=\left[10+\frac{25}{2}\right]\log 5+\left[-10-\frac{25}{2}\right]\log 4+\left[10-\frac{25}{2}\right]\log 3+\left[-10+\frac{25}{2}\right]\log 2\\
&=\frac{45}{2}\log 5-\frac{45}{2}\log 4-\frac{5}{2}\log 3+\frac{5}{2}\log 2\\
&=\frac{45}{2}\log\frac{5}{4}-\frac{5}{2}\log\frac{3}{2}
\end{aligned}$$

Substituting the value of I_1 in (1), we obtain

$$\begin{aligned}
I&=5-\left[\frac{45}{2}\log\frac{5}{4}-\frac{5}{2}\log\frac{3}{2}\right]\\
&=5-\frac{5}{2}\left[9\log\frac{5}{4}-\log\frac{3}{2}\right]
\end{aligned}$$

-

$$\int_0^{\frac{\pi}{4}}\left(2\sec^2 x + x^3 + 2\right)dx$$

Answer :

Let $I = \int_0^{\frac{\pi}{4}}\left(2\sec^2 x + x^3 + 2\right)dx$

$$\int\left(2\sec^2 x + x^3 + 2\right)dx = 2\tan x + \frac{x^4}{4} + 2x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}\left(\frac{\pi}{4}\right) - \mathrm{F}(0)$$

$$= \left\{\left(2\tan\frac{\pi}{4} + \frac{1}{4}\left(\frac{\pi}{4}\right)^4 + 2\left(\frac{\pi}{4}\right)\right) - (2\tan 0 + 0 + 0)\right\}$$

$$= 2\tan\frac{\pi}{4} + \frac{\pi^4}{4^5} + \frac{\pi}{2}$$

$$= 2 + \frac{\pi}{2} + \frac{\pi^4}{1024}$$

-

$$\int_0^{\pi}\left(\sin^2\frac{x}{2}-\cos^2\frac{x}{2}\right)dx$$

Answer :

Let $I = \int_0^{\pi}\left(\sin^2\frac{x}{2}-\cos^2\frac{x}{2}\right)dx$

$= -\int_0^{\pi}\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)dx$

$= -\int_0^{\pi}\cos x\, dx$

$\int\cos x\, dx = \sin x = \mathrm{F}(x)$

By second fundamental theorem of calculus, we obtain

$I = \mathrm{F}(\pi) - \mathrm{F}(0)$

$= \sin\pi - \sin 0$

$= 0$

-

$$\int_0^2 \frac{6x+3}{x^2+4}dx$$

Answer :

$$\text{Let } I = \int_0^2 \frac{6x+3}{x^2+4}dx$$

$$\int \frac{6x+3}{x^2+4}dx = 3\int \frac{2x+1}{x^2+4}dx$$

$$= 3\int \frac{2x}{x^2+4}dx + 3\int \frac{1}{x^2+4}dx$$

$$= 3\log\left(x^2+4\right) + \frac{3}{2}\tan^{-1}\frac{x}{2} = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(2) - \mathrm{F}(0)$$

$$= \left\{3\log\left(2^2+4\right) + \frac{3}{2}\tan^{-1}\left(\frac{2}{2}\right)\right\} - \left\{3\log\left(0+4\right) + \frac{3}{2}\tan^{-1}\left(\frac{0}{2}\right)\right\}$$

$$= 3\log 8 + \frac{3}{2}\tan^{-1}1 - 3\log 4 - \frac{3}{2}\tan^{-1}0$$

$$= 3\log 8 + \frac{3}{2}\left(\frac{\pi}{4}\right) - 3\log 4 - 0$$

$$= 3\log\left(\frac{8}{4}\right) + \frac{3\pi}{8}$$

$$= 3\log 2 + \frac{3\pi}{8}$$

-

$$\int_0^1 \left(xe^x + \sin\frac{\pi x}{4} \right) dx$$

Answer :

$$\text{Let } I = \int_0^1 \left(xe^x + \sin\frac{\pi x}{4} \right) dx$$

$$\int \left(xe^x + \sin\frac{\pi x}{4} \right) dx = x\int e^x dx - \int \left\{ \left(\frac{d}{dx} x \right) \int e^x dx \right\} dx + \left\{ \frac{-\cos\frac{\pi x}{4}}{\frac{\pi}{4}} \right\}$$

$$= xe^x - \int e^x dx - \frac{4\pi}{\pi}\cos\frac{x}{4}$$

$$= xe^x - e^x - \frac{4\pi}{\pi}\cos\frac{x}{4}$$

$$= \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(1) - \mathrm{F}(0)$$

$$= \left(1.e^1 - e^1 - \frac{4}{\pi}\cos\frac{\pi}{4} \right) - \left(0.e^0 - e^0 - \frac{4}{\pi}\cos 0 \right)$$

$$= e - e - \frac{4}{\pi}\left(\frac{1}{\sqrt{2}} \right) + 1 + \frac{4}{\pi}$$

$$= 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$$

-

$\int^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{12}$

Answer :

$$\int \frac{dx}{1+x^2} = \tan^{-1} x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\int^{\sqrt{3}} \frac{dx}{1+x^2} = F\left(\sqrt{3}\right) - F(1)$$

$$= \tan^{-1} \sqrt{3} - \tan^{-1} 1$$
$$= \frac{\pi}{3} - \frac{\pi}{4}$$
$$= \frac{\pi}{12}$$

Hence, the correct answer is D.

-

$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$ equals

A. $\frac{\pi}{6}$

B. $\frac{\pi}{12}$

C. $\frac{\pi}{24}$

D. $\frac{\pi}{4}$

Answer :

$$\int \frac{dx}{4+9x^2} = \int \frac{dx}{(2)^2+(3x)^2}$$

Put $3x = t \Rightarrow 3dx = dt$

$$\therefore \int \frac{dx}{(2)^2+(3x)^2} = \frac{1}{3}\int \frac{dt}{(2)^2+t^2}$$

$$= \frac{1}{3}\left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]$$
$$= \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)$$
$$= F(x)$$

By second fundamental theorem of calculus, we obtain

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} = F\left(\frac{2}{3}\right) - F(0)$$
$$= \frac{1}{6}\tan^{-1}\left(\frac{3}{2}\cdot\frac{2}{3}\right) - \frac{1}{6}\tan^{-1} 0$$
$$= \frac{1}{6}\tan^{-1} 1 - 0$$
$$= \frac{1}{6}\times\frac{\pi}{4}$$
$$= \frac{\pi}{24}$$

Hence, the correct answer is C.

-

$$\int_0^1 \frac{x}{x^2+1}dx$$

Answer :

$$\int_0^1 \frac{x}{x^2+1}dx$$

Let $x^2+1=t \Rightarrow 2x\,dx=dt$

When $x=0$, $t=1$ and when $x=1$, $t=2$

$$\begin{aligned}\therefore \int_0^1 \frac{x}{x^2+1}dx &= \frac{1}{2}\int_1^2 \frac{dt}{t} \\ &= \frac{1}{2}\left[\log|t|\right]_1^2 \\ &= \frac{1}{2}\left[\log 2-\log 1\right] \\ &= \frac{1}{2}\log 2\end{aligned}$$

-

$$\int_0^1 \frac{x}{x^2+1}dx$$

Answer :

$$\int_0^1 \frac{x}{x^2+1}dx$$

Let $x^2+1=t \Rightarrow 2x\,dx = dt$

When $x = 0$, $t = 1$ and when $x = 1$, $t = 2$

$$\therefore \int_0^1 \frac{x}{x^2+1}dx = \frac{1}{2}\int_1^2 \frac{dt}{t}$$
$$= \frac{1}{2}\left[\log|t|\right]_1^2$$
$$= \frac{1}{2}\left[\log 2 - \log 1\right]$$
$$= \frac{1}{2}\log 2$$

-

$$\int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^5\phi\, d\phi$$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^5\phi\, d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^4\phi\cos\phi\, d\phi$$

Also, let $\sin\phi = t \Rightarrow \cos\phi\, d\phi = dt$

When $\phi = 0$, $t = 0$ and when $\phi = \frac{\pi}{2}$, $t = 1$

$$\therefore I = \int_0^1 \sqrt{t}\left(1-t^2\right)^2 dt$$
$$= \int_0^1 t^{\frac{1}{2}}\left(1+t^4-2t^2\right)dt$$
$$= \int_0^1 \left[t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}}\right]dt$$

$$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{11}{2}}}{\frac{11}{2}} - \frac{2t^{\frac{7}{2}}}{\frac{7}{2}} \right]_0^1$$

$$= \frac{2}{3} + \frac{2}{11} - \frac{4}{7}$$

$$= \frac{154 + 42 - 132}{231}$$

$$= \frac{64}{231}$$

-

- $\int_0^{\frac{\pi}{2}} \sqrt{\sin\phi} \cos^5\phi d\phi$

Answer :

Let $I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi} \cos^5\phi \, d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi} \cos^4\phi \cos\phi \, d\phi$

Also, let $\sin\phi = t \Rightarrow \cos\phi \, d\phi = dt$

When $\phi = 0$, $t = 0$ and when $\phi = \frac{\pi}{2}$, $t = 1$

$$\therefore I = \int_0^1 \sqrt{t} \left(1 - t^2\right)^2 dt$$

$$= \int_0^1 t^{\frac{1}{2}} \left(1 + t^4 - 2t^2\right) dt$$

$$= \int_0^1 \left[t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}} \right] dt$$

$$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{11}{2}}}{\frac{11}{2}} - \frac{2t^{\frac{7}{2}}}{\frac{7}{2}} \right]_0^1$$

$$= \frac{2}{3} + \frac{2}{11} - \frac{4}{7}$$
$$= \frac{154 + 42 - 132}{231}$$
$$= \frac{64}{231}$$

-

$$\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$$

Answer :

Let $I = \int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

Also, let $x = \tan\theta \Rightarrow dx = \sec^2\theta \, d\theta$

When $x = 0$, $\theta = 0$ and when $x = 1$, $\theta = \frac{\pi}{4}$

$$I = \int_0^{\frac{\pi}{4}} \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) \sec^2\theta \, d\theta$$
$$= \int_0^{\frac{\pi}{4}} \sin^{-1}(\sin 2\theta) \sec^2\theta \, d\theta$$
$$= \int_0^{\frac{\pi}{4}} 2\theta \cdot \sec^2\theta \, d\theta$$
$$= 2\int_0^{\frac{\pi}{4}} \theta \cdot \sec^2\theta \, d\theta$$

Taking θ as first function and $\sec^2\theta$ as second function and integrating by parts, we obtain

$$I = 2\left[\theta \int \sec^2\theta \, d\theta - \int \left\{ \left(\frac{d}{dx}\theta \right) \int \sec^2\theta \, d\theta \right\} d\theta \right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\theta \tan\theta - \int \tan\theta \, d\theta\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\theta \tan\theta + \log|\cos\theta|\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\frac{\pi}{4}\tan\frac{\pi}{4} + \log\left|\cos\frac{\pi}{4}\right| - \log|\cos 0|\right]$$

$$= 2\left[\frac{\pi}{4} + \log\left(\frac{1}{\sqrt{2}}\right) - \log 1\right]$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2\right]$$

$$= \frac{\pi}{2} - \log 2$$

-

$$\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$$

Answer :

Let $I = \int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

Also, let $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

When $x = 0$, $\theta = 0$ and when $x = 1$, $\theta = \frac{\pi}{4}$

$$I = \int_0^{\frac{\pi}{4}} \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)\sec^2\theta\, d\theta$$
$$= \int_0^{\frac{\pi}{4}} \sin^{-1}(\sin 2\theta)\sec^2\theta\, d\theta$$
$$= \int_0^{\frac{\pi}{4}} 2\theta\cdot\sec^2\theta\, d\theta$$
$$= 2\int_0^{\frac{\pi}{4}} \theta\cdot\sec^2\theta\, d\theta$$

Takingθas first function and $\sec^2\theta$ as second function and integrating by parts, we obtain

$$I = 2\left[\theta\int\sec^2\theta\, d\theta - \int\left\{\left(\frac{d}{dx}\theta\right)\int\sec^2\theta\, d\theta\right\}d\theta\right]_0^{\frac{\pi}{4}}$$
$$= 2\left[\theta\tan\theta - \int\tan\theta\, d\theta\right]_0^{\frac{\pi}{4}}$$
$$= 2\left[\theta\tan\theta + \log|\cos\theta|\right]_0^{\frac{\pi}{4}}$$
$$= 2\left[\frac{\pi}{4}\tan\frac{\pi}{4} + \log\left|\cos\frac{\pi}{4}\right| - \log|\cos 0|\right]$$
$$= 2\left[\frac{\pi}{4} + \log\left(\frac{1}{\sqrt{2}}\right) - \log 1\right]$$
$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2\right]$$
$$= \frac{\pi}{2} - \log 2$$

-

$$\int_0^2 x\sqrt{x+2}\ \left(\text{Put } x+2=t^2\right)$$

Answer :

$$\int_0^2 x\sqrt{x+2}dx$$

Let $x+2=t^2 \Rightarrow dx=2tdt$

When $x=0$, $t=\sqrt{2}$ and when $x=2$, $t=2$

$$\therefore \int_0^2 x\sqrt{x+2}dx = \int_{\sqrt{2}}^2 \left(t^2-2\right)\sqrt{t^2}\,2tdt$$

$$= 2\int_{\sqrt{2}}^2 \left(t^2-2\right)t^2 dt$$

$$= 2\int_{\sqrt{2}}^2 \left(t^4-2t^2\right)dt$$

$$= 2\left[\frac{t^5}{5}-\frac{2t^3}{3}\right]_{\sqrt{2}}^2$$

$$= 2\left[\frac{32}{5}-\frac{16}{3}-\frac{4\sqrt{2}}{5}+\frac{4\sqrt{2}}{3}\right]$$

$$= 2\left[\frac{96-80-12\sqrt{2}+20\sqrt{2}}{15}\right]$$

$$= 2\left[\frac{16+8\sqrt{2}}{15}\right]$$

$$= \frac{16\left(2+\sqrt{2}\right)}{15}$$

$$= \frac{16\sqrt{2}\left(\sqrt{2}+1\right)}{15}$$

-

$$\int_0^2 x\sqrt{x+2} \ \left(\text{Put } x+2=t^2\right)$$

Answer :

$$\int_0^2 x\sqrt{x+2}dx$$

Let $x + 2 = t^2 \Rightarrow dx = 2tdt$

When $x = 0$, $t=\sqrt{2}$ and when $x = 2$, $t = 2$

$$\begin{aligned}
\therefore \int_0^2 x\sqrt{x+2}dx &= \int_{\sqrt{2}}^2 \left(t^2-2\right)\sqrt{t^2}\,2tdt \\
&= 2\int_{\sqrt{2}}^2 \left(t^2-2\right)t^2 dt \\
&= 2\int_{\sqrt{2}}^2 \left(t^4-2t^2\right)dt \\
&= 2\left[\frac{t^5}{5}-\frac{2t^3}{3}\right]_{\sqrt{2}}^2 \\
&= 2\left[\frac{32}{5}-\frac{16}{3}-\frac{4\sqrt{2}}{5}+\frac{4\sqrt{2}}{3}\right]
\end{aligned}$$

$$= 2\left[\frac{96-80-12\sqrt{2}+20\sqrt{2}}{15}\right]$$

$$= 2\left[\frac{16+8\sqrt{2}}{15}\right]$$

$$= \frac{16\left(2+\sqrt{2}\right)}{15}$$

$$= \frac{16\sqrt{2}\left(\sqrt{2}+1\right)}{15}$$

-

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$$

Answer :

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$$

Let cos $x = t \Rightarrow$ - sin$x$ $dx = dt$

When $x = 0$, $t = 1$ and when $x = \frac{\pi}{2}, t = 0$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx = -\int_1^0 \frac{dt}{1+t^2}$$

$$= -\left[\tan^{-1} t\right]_1^0$$

$$= -\left[\tan^{-1} 0 - \tan^{-1} 1\right]$$

$$= -\left[-\frac{\pi}{4}\right]$$

$$= \frac{\pi}{4}$$

-

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$$

Answer :

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$$

Let cos $x = t \Rightarrow$ - sin$x$ $dx = dt$

When $x = 0$, $t = 1$ and when $x = \frac{\pi}{2}, t = 0$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx = -\int_1^0 \frac{dt}{1+t^2}$$

$$= -\left[\tan^{-1} t\right]_1^0$$

$$= -\left[\tan^{-1} 0 - \tan^{-1} 1\right]$$

$$= -\left[-\frac{\pi}{4}\right]$$

$$= \frac{\pi}{4}$$

-

$$\int_0^2 \frac{dx}{x+4-x^2}$$

Answer :

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-\left(x^2-x-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]}$$

$$= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}$$

Let $x-\frac{1}{2}=t \Rightarrow dx = dt$

When $x=0$, $t=-\frac{1}{2}$ and when $x=2$, $t=\frac{3}{2}$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}=\int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2-t^2}$$

$$=\left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2}+t}{\frac{\sqrt{17}}{2}-t}\right]_{-\frac{1}{2}}^{\frac{3}{2}}$$

$$=\frac{1}{\sqrt{17}}\left[\log \frac{\frac{\sqrt{17}}{2}+\frac{3}{2}}{\frac{\sqrt{17}}{2}-\frac{3}{2}}-\frac{\log \frac{\sqrt{17}}{2}-\frac{1}{2}}{\log \frac{\sqrt{17}}{2}+\frac{1}{2}}\right]$$

$$=\frac{1}{\sqrt{17}}\left[\log \frac{\sqrt{17}+3}{\sqrt{17}-3}-\log \frac{\sqrt{17}-1}{\sqrt{17}+1}\right]$$

$$= \frac{1}{\sqrt{17}} \log \frac{\sqrt{17}+3}{\sqrt{17}-3} \times \frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{5+\sqrt{17}}{5-\sqrt{17}} \right)$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{\left(5+\sqrt{17}\right)\left(5+\sqrt{17}\right)}{25-17} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{25+17+10\sqrt{17}}{8} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{42+10\sqrt{17}}{8} \right)$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{21+5\sqrt{17}}{4} \right)$$

-

$$\int_0^2 \frac{dx}{x+4-x^2}$$

Answer :

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-\left(x^2-x-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]}$$

$$= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}$$

Let $x-\frac{1}{2}=t \Rightarrow dx = dt$

When $x=0$, $t=-\frac{1}{2}$ and when $x=2$, $t=\frac{3}{2}$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}=\int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2-t^2}$$

$$=\left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2}+t}{\frac{\sqrt{17}}{2}-t}\right]_{-\frac{1}{2}}^{\frac{3}{2}}$$

$$=\frac{1}{\sqrt{17}}\left[\log \frac{\frac{\sqrt{17}}{2}+\frac{3}{2}}{\frac{\sqrt{17}}{2}-\frac{3}{2}}-\frac{\log \frac{\sqrt{17}}{2}-\frac{1}{2}}{\log \frac{\sqrt{17}}{2}+\frac{1}{2}}\right]$$

$$=\frac{1}{\sqrt{17}}\left[\log \frac{\sqrt{17}+3}{\sqrt{17}-3}-\log \frac{\sqrt{17}-1}{\sqrt{17}+1}\right]$$

$$=\frac{1}{\sqrt{17}} \log \frac{\sqrt{17}+3}{\sqrt{17}-3} \times \frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$= \frac{1}{\sqrt{17}} \log\left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log\left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log\left(\frac{5+\sqrt{17}}{5-\sqrt{17}}\right)$$

$$= \frac{1}{\sqrt{17}} \log\left[\frac{\left(5+\sqrt{17}\right)\left(5+\sqrt{17}\right)}{25-17}\right]$$

$$= \frac{1}{\sqrt{17}} \log\left[\frac{25+17+10\sqrt{17}}{8}\right]$$

$$= \frac{1}{\sqrt{17}} \log\left(\frac{42+10\sqrt{17}}{8}\right)$$

$$= \frac{1}{\sqrt{17}} \log\left(\frac{21+5\sqrt{17}}{4}\right)$$

-

$$\int_{-1}^{1} \frac{dx}{x^2+2x+5}$$

Answer :

$$\int_{-1}^{1} \frac{dx}{x^2+2x+5} = \int_{-1}^{1} \frac{dx}{\left(x^2+2x+1\right)+4} = \int_{-1}^{1} \frac{dx}{(x+1)^2+(2)^2}$$

Let $x + 1 = t \Rightarrow dx = dt$

When $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

$$\therefore \int_{-1}^{1} \frac{dx}{(x+1)^2+(2)^2} = \int_{0}^{2} \frac{dt}{t^2+2^2}$$

$$= \left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]_0^2$$

$$= \frac{1}{2}\tan^{-1}1 - \frac{1}{2}\tan^{-1}0$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$$

-

$$\int_{-1}^{1} \frac{dx}{x^2 + 2x + 5}$$

Answer :

$$\int_{-1}^{1} \frac{dx}{x^2 + 2x + 5} = \int_{-1}^{1} \frac{dx}{\left(x^2 + 2x + 1\right) + 4} = \int_{-1}^{1} \frac{dx}{\left(x+1\right)^2 + \left(2\right)^2}$$

Let $x + 1 = t \Rightarrow dx = dt$

When $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

$$\therefore \int_{-1}^{1} \frac{dx}{\left(x+1\right)^2 + \left(2\right)^2} = \int_{0}^{2} \frac{dt}{t^2 + 2^2}$$

$$= \left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]_0^2$$

$$= \frac{1}{2}\tan^{-1}1 - \frac{1}{2}\tan^{-1}0$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$$

-

$$\int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx$$

Answer :

$$\int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx$$

Let $2x = t \Rightarrow 2dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$$\therefore \int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx = \frac{1}{2}\int_2^4 \left(\frac{2}{t}-\frac{2}{t^2}\right)e^t dt$$

$$= \int_2^4 \left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt$$

Let $\frac{1}{t} = f(t)$

Then, $f'(t) = -\frac{1}{t^2}$

$$\Rightarrow \int_2^4 \left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt = \int_2^4 e^t\left[f(t)+f'(t)\right]dt$$

$$=\left[e^t f(t)\right]_2^4$$

$$=\left[e^t \cdot \frac{2}{t}\right]_2^4$$

$$=\left[\frac{e^t}{t}\right]_2^4$$

$$=\frac{e^4}{4}-\frac{e^2}{2}$$

$$=\frac{e^2\left(e^2-2\right)}{4}$$

-

$$\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$$

Answer :

$$\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$$

Let $2x = t \Rightarrow 2dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$$\therefore \int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx = \frac{1}{2} \int_2^4 \left(\frac{2}{t} - \frac{2}{t^2}\right) e^t dt$$

$$= \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt$$

Let $\frac{1}{t} = f(t)$

Then, $f'(t) = -\frac{1}{t^2}$

$$\Rightarrow \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt = \int_2^4 e^t \left[f(t) + f'(t)\right] dt$$

$$= \left[e^t f(t)\right]_2^4$$

$$= \left[e^t \cdot \frac{2}{t}\right]_2^4$$

$$= \left[\frac{e^t}{t}\right]_2^4$$

$$= \frac{e^4}{4} - \frac{e^2}{2}$$

$$= \frac{e^2\left(e^2 - 2\right)}{4}$$

-

The value of the integral $\int_{\frac{1}{3}}^{1} \frac{\left(x-x^3\right)^{\frac{1}{3}}}{x^4} dx$ is

A. 6

B. 0

C. 3

D. 4

Answer :

Let $I = \int_{\frac{1}{3}}^{1} \frac{\left(x-x^3\right)^{\frac{1}{3}}}{x^4} dx$

Also, let $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

When $x = \frac{1}{3}$, $\theta = \sin^{-1}\left(\frac{1}{3}\right)$ and when $x = 1$, $\theta = \frac{\pi}{2}$

$$\Rightarrow I = \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{\left(\sin\theta - \sin^3\theta\right)^{\frac{1}{3}}}{\sin^4\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{\left(\sin\theta\right)^{\frac{1}{3}}\left(1-\sin^2\theta\right)^{\frac{1}{3}}}{\sin^4\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{\left(\sin\theta\right)^{\frac{1}{3}}\left(\cos\theta\right)^{\frac{2}{3}}}{\sin^4\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{\left(\sin\theta\right)^{\frac{1}{3}}\left(\cos\theta\right)^{\frac{2}{3}}}{\sin^2\theta\sin^2\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\cos\theta)^{\frac{5}{3}}}{(\sin\theta)^{\frac{5}{3}}} \operatorname{cosec}^2\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} (\cot\theta)^{\frac{5}{3}} \operatorname{cosec}^2\theta\, d\theta$$

Let cotθ = $t \Rightarrow$ - cosec2θ $d\theta$= dt

When $\theta = \sin^{-1}\left(\frac{1}{3}\right)$, $t = 2\sqrt{2}$ and when $\theta = \frac{\pi}{2}$, $t = 0$

$$\therefore I = -\int_{2\sqrt{2}}^{0} (t)^{\frac{5}{3}} dt$$

$$= -\left[\frac{3}{8}(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$$

$$= -\frac{3}{8}\left[(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$$

$$= -\frac{3}{8}\left[-\left(2\sqrt{2}\right)^{\frac{8}{3}}\right]$$

$$= \frac{3}{8}\left[\left(\sqrt{8}\right)^{\frac{8}{3}}\right]$$

$$= \frac{3}{8}\left[(8)^{\frac{4}{3}}\right]$$

$$= \frac{3}{8}[16]$$

$$= 3 \times 2$$

$$= 6$$

Hence, the correct answer is A.

-

If $f(x)=\int_0^x t\sin t\,dt$, then $f'(x)$ is

A. $\cos x + x\sin x$

B. $x\sin x$

C. $x\cos x$

D. $\sin x + x\cos x$

$f(x)=\int_0^x t\sin t\,dt$

Integrating by parts, we obtain

$$f(x)=t\int_0^x \sin t\,dt-\int_0^x\left\{\left(\frac{d}{dt}t\right)\int\sin t\,dt\right\}dt$$
$$=\left[t(-\cos t)\right]_0^x-\int_0^x(-\cos t)\,dt$$
$$=\left[-t\cos t+\sin t\right]_0^x$$
$$=-x\cos x+\sin x$$

$$\Rightarrow f'(x)=-\left[\{x(-\sin x)\}+\cos x\right]+\cos x$$
$$=x\sin x-\cos x+\cos x$$
$$=x\sin x$$

Hence, the correct answer is B.

-

If $f(x)=\int_0^x t\sin t\,dt$, then $f'(x)$ is

A. $\cos x + x\sin x$

B. $x\sin x$

C. $x\cos x$

D. $\sin x + x\cos x$

$$f(x) = \int_0^x t \sin t dt$$

Integrating by parts, we obtain

$$f(x) = t\int_0^x \sin t\, dt - \int_0^x \left\{\left(\frac{d}{dt}t\right)\int \sin t\, dt\right\} dt$$
$$= \left[t(-\cos t)\right]_0^x - \int_0^x (-\cos t)\, dt$$
$$= \left[-t\cos t + \sin t\right]_0^x$$
$$= -x\cos x + \sin x$$

$$\Rightarrow f'(x) = -\left[\left\{x(-\sin x)\right\} + \cos x\right] + \cos x$$
$$= x\sin x - \cos x + \cos x$$
$$= x\sin x$$

Hence, the correct answer is B.

-

$$\int_0^{\frac{\pi}{2}} \cos^2 x\,dx$$

Answer :

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x\,dx \qquad ...(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2}-x\right)dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^2 x\right)dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

-

$$\int_0^{\frac{\pi}{2}} \cos^2 x\, dx$$

Answer :

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x\, dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2} - x\right) dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^2 x\right) dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

-

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$$

Answer :

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos}}{\sqrt{\cos}+\sqrt{\sin x}} dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1\, dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

-

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$$

Answer :

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx \quad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}} dx \quad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos}}{\sqrt{\cos}+\sqrt{\sin x}} dx \quad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1\,dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

-

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)+\cos^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

-

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \, dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

-

$$\int_0^{\frac{\pi}{2}} \frac{\cos^5 x dx}{\sin^5 x + \cos^5 x}$$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^5\left(\frac{\pi}{2} - x\right)}{\sin^5\left(\frac{\pi}{2} - x\right) + \cos^5\left(\frac{\pi}{2} - x\right)} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

-

$$\int_{-5}^{5} |x+2|\, dx$$

Answer :

Let $I = \int_{-5}^{5} |x+2| dx$

It can be seen that $(x + 2) \leq 0$ on $[-5, -2]$ and $(x + 2) \geq 0$ on $[-2, 5]$.

$$\therefore I = \int_{-5}^{-2} -(x+2)dx + \int_{-2}^{5} (x+2)\, dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$I = -\left[\frac{x^2}{2} + 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{5}$$

$$= -\left[\frac{(-2)^2}{2} + 2(-2) - \frac{(-5)^2}{2} - 2(-5)\right] + \left[\frac{(5)^2}{2} + 2(5) - \frac{(-2)^2}{2} - 2(-2)\right]$$

$$= -\left[2 - 4 - \frac{25}{2} + 10\right] + \left[\frac{25}{2} + 10 - 2 + 4\right]$$

$$= -2 + 4 + \frac{25}{2} - 10 + \frac{25}{2} + 10 - 2 + 4$$

$$= 29$$

-

$$\int_2^8 |x-5|\,dx$$

Answer :

Let $I = \int_2^8 |x-5|\,dx$

It can be seen that $(x - 5) \le 0$ on [2, 5] and $(x - 5) \ge 0$ on [5, 8].

$$I = \int_2^5 -(x-5)\,dx + \int_2^8 (x-5)\,dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$= -\left[\frac{x^2}{2} - 5x\right]_2^5 + \left[\frac{x^2}{2} - 5x\right]_5^8$$

$$= -\left[\frac{25}{2} - 25 - 2 + 10\right] + \left[32 - 40 - \frac{25}{2} + 25\right]$$

$$= 9$$

-

$$\int_0^1 x(1-x)^n\,dx$$

Answer :

Let $I = \int_0^1 x(1-x)^n\,dx$

$$\therefore I = \int_0^1 (1-x)\left(1-(1-x)\right)^n\,dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$= \int_0^1 (1-x)(x)^n\,dx$$

$$= \int_0^1 \left(x^n - x^{n+1}\right)dx$$

$$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$$

$$= \left[\frac{1}{n+1} - \frac{1}{n+2}\right]$$

$$= \frac{(n+2)-(n+1)}{(n+1)(n+2)}$$

$$= \frac{1}{(n+1)(n+2)}$$

-

$$\int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right)dx$$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right)dx \qquad \ldots(1)$$

$$\therefore I = \int_0^{\frac{\pi}{4}} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right\}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{1-\tan x}{1+\tan}\right\}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\frac{2}{\left(1+\tan x\right)}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2\,dx - \int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right)dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2dx - I \qquad \left[\text{From } (1)\right]$$

$$\Rightarrow 2I = \left[x\log 2\right]_0^{\frac{\pi}{4}}$$

$$\Rightarrow 2I = \frac{\pi}{4}\log 2$$

$$\Rightarrow I = \frac{\pi}{8}\log 2$$

-

$$\int_0^2 x\sqrt{2-x}\,dx$$

Answer :

$$\text{Let } I = \int_0^2 x\sqrt{2-x}\,dx$$

$$I = \int_0^2 (2-x)\sqrt{x}\,dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$= \int_0^2 \left\{2x^{\frac{1}{2}} - x^{\frac{3}{2}}\right\}dx$$

$$= \left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$$

$$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^2$$

$$= \frac{4}{3}(2)^{\frac{3}{2}} - \frac{2}{5}(2)^{\frac{5}{2}}$$

$$= \frac{4\times 2\sqrt{2}}{3} - \frac{2}{5}\times 4\sqrt{2}$$

$$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$$

$$= \frac{40\sqrt{2} - 24\sqrt{2}}{15}$$

$$= \frac{16\sqrt{2}}{15}$$

-

$$\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)\,dx$$

Answer :

Let $I=\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)\,dx$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{2\log\sin x-\log(2\sin x\cos x)\}\,dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{2\log\sin x-\log\sin x-\log\cos x-\log 2\}\,dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{\log\sin x-\log\cos x-\log 2\}\,dx \qquad ...(1)$$

It is known that, $\left(\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx\right)$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{\log\cos x-\log\sin x-\log 2\}\,dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}(-\log 2-\log 2)\,dx$$

$$\Rightarrow 2I=-2\log 2\int_0^{\frac{\pi}{2}}1\,dx$$

$$\Rightarrow I=-\log 2\left[\frac{\pi}{2}\right]$$

$$\Rightarrow I=\frac{\pi}{2}(-\log 2)$$

$$\Rightarrow I=\frac{\pi}{2}\left[\log\frac{1}{2}\right]$$

$$\Rightarrow I=\frac{\pi}{2}\log\frac{1}{2}$$

-

$$\int_0^\pi \frac{x\,dx}{1+\sin x}$$

Answer :

$$\text{Let } I = \int_0^\pi \frac{x\,dx}{1+\sin x} \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^\pi \frac{(\pi - x)}{1+\sin(\pi - x)}dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^\pi \frac{(\pi - x)}{1+\sin x}dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^\pi \frac{\pi}{1+\sin x}dx$$

$$\Rightarrow 2I = \pi\int_0^\pi \frac{(1-\sin x)}{(1+\sin x)(1-\sin x)}dx$$

$$\Rightarrow 2I = \pi\int_0^\pi \frac{1-\sin x}{\cos^2 x}dx$$

$$\Rightarrow 2I = \pi\int_0^\pi \left\{\sec^2 x - \tan x \sec x\right\}dx$$

$$\Rightarrow 2I = \pi\left[\tan x - \sec x\right]_0^\pi$$

$$\Rightarrow 2I = \pi[2]$$

$$\Rightarrow I = \pi$$

-

Series

F(x) in general can be expanded around a value. Often the Value around which it is expanded is chosen as x = 0

$$\begin{aligned} f(x) &= f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 \\ &\quad + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \frac{f''''(x_0)}{4!}(x-x_0)^4 + \cdots \\ &= \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n. \end{aligned}$$

Taylor series: $f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$

Taylor polynomial: $f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$

Remainder: $R_n(x) = \frac{f^{(n+1)}(a)}{(n+1)!}(x-a)^{n+1} + \frac{f^{(n+2)}(a)}{(n+2)!}(x-a)^{n+2} + \frac{f^{(n+3)}(a)}{(n+3)!}(x-a)^{n+3} + \cdots$

Derivative form of remainder: $R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}$ where z is a number between a and x

Integral form of remainder: $R_n(x) = \frac{1}{n!}\int_a^x f^{(n+1)}(t)(x-t)^n \, dt$

Function	Maclaurin series
e^x	$\sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$
$\sin x$	$\sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$
$\cos x$	$\sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$
$\frac{1}{1-x}$	$\sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \cdots$ (if $-1 < x < 1$)
$\ln(1+x)$	$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ (if $-1 < x \le 1$)

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \qquad R = \infty$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \qquad R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \qquad R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!} x^2 + \frac{k(k-1)(k-2)}{3!} x^3 + \dots \qquad R = 1$$

We start by supposing that f is any function that can be represented by a power series:

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + c_4(x-a)^4 + \dots \qquad |x-a| < R \tag{1}$$

Let's try to determine what the coefficients c_n must be in terms of f. To begin, notice that if we put $x = a$ in Equation 1, then all terms after the first one are 0 and we get

$$f(a) = c_0$$

By Theorem 8.6.2, we can differentiate the series in Equation 1 term by term:

$$f'(x) = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + 4c_4(x-a)^3 + \dots \qquad |x-a| < R \tag{2}$$

and substitution of $x = a$ in Equation 2 gives $f'(a) = c_1$ Now we differentiate both sides of Equation 2 and obtain

$$f''(x) = 2c_2 + 2 \cdot 3c_3(x-a) + 3 \cdot 4c_4(x-a)^2 + \dots \qquad |x-a| < R \tag{3}$$

Again we put $x = a$ in Equation 3. The result is

$$f''(a) = 2c_2$$

Let's apply the procedure one more time. Differentiation of the series in Equation 3 gives

$$f'''(x) = 2\cdot 3c_3 + 2\cdot 3\cdot 4c_4(x-a) + 3\cdot 4\cdot 5c_5(x-a)^2 + \dots \qquad |x-a| < R \tag{4}$$

and substitution of $x = a$ in Equation 4 gives

$$f'''(a) = 2\cdot 3c_3 = 3!c_3$$

By now you can see the pattern. If we continue to differentiate and substitute $x = a$, we obtain

$$f^{(n)}(a) = 2\cdot 3\cdot 4\cdot \dots \cdot nc_n = n!c_n$$

Solving this equation for the nth coefficient c_n, we get

$$c_n = \frac{f^{(n)}(a)}{n!}$$

This formula remains valid even for $n = 0$ if we adopt the conventions that $0! = 1$ and $f^{(0)} = f$.

Thus we have proved the following theorem.

THEOREM: If f has a power series representation (expansion) at a, that is, if

$$\boxed{f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n \quad \text{and} \quad |x-a| < R, \quad \text{then} \quad c_n = \frac{f^{(n)}(a)}{n!}} \tag{5}$$

Substituting this formula for c_n back into the series, we see that *if* f has a power series expansion at a, then it must be of the following form.

$$\boxed{\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \\ &= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots \end{aligned}} \tag{6}$$

The series in Equation 6 is called the **Taylor series of the function** f at a (or about a **or centered at** a). For the special case $a = 0$ the Taylor series becomes

$$\boxed{f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots} \tag{7}$$

This case arises frequently enough that is is given the special name **Maclaurin series.**

NOTE: We have shown that *if* f can be represented as a power series about a, then f is equal to the sum of its Taylor series. But there exist functions that are not equal to the sum of their Taylor series. For example, one can show that the function defined by

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ \\ 0 & \text{if } x = 0 \end{cases}$$

is not equal to its Maclaurin series.

Let's investigate the more general question: Under what circumstances is a function equal to the sum of its Taylor series? In other words, if f has derivatives of all orders, when is it true that

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

As with any convergent series, this means that $f(x)$ is the limit of the sequence of partial sums. In the case of the Taylor series, the partial sums are

$$T_n(x) \quad = \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Notice that T_n is a polynomial of degree n called the **nth-degree Taylor polynomial of f at** a. For instance, for the exponential function $f(x) = e^x$, the result of Example 1 shows that the Taylor polynomials at 0 (or Maclaurin polynomials) with $n = 1, 2$, and 3 are

$$T_1(x) = 1 + x, \qquad T_2(x) = 1 + x + \frac{x^2}{2!}, \qquad T_3(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

In general, $f(x)$ is the sum of its Taylor series if

$$f(x) = \lim_{n\to\infty} T_n(x)$$

If we let

$$R_n(x) = f(x) - T_n(x) \quad \text{so that} \quad f(x) = T_n(x) + R_n(x)$$

then $R_n(x)$ is called the **remainder** of the Taylor series. If we can somehow show that $\lim\limits_{n\to\infty} R_n(x) = 0$, then it follows that

$$\lim_{n\to\infty} T_n(x) = \lim_{n\to\infty} [f(x) - R_n(x)] = f(x) - \lim_{n\to\infty} R_n(x) = f(x)$$

We have therefore proved the following.

THEOREM: If $f(x) = T_n(x) + R_n(x)$, where T_n is the nth-degree Taylor polynomial of f at a and

$$\lim_{n\to\infty} R_n(x) = 0 \quad (8)$$

for $|x-a| < R$, then f is equal to the sum of its Taylor series on the interval $|x-a| < R$.

In trying to show that $\lim_{n\to\infty} R_n(x) = 0$ for a specific function f, we usually use the expression in the next theorem.

THEOREM (TAYLOR'S FORMULA): If f has $n+1$ derivatives in an interval I that contains the number a, then for x in I there is a number z strictly between x and a such that the remainder term in the Taylor series can be expressed as

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}$$

NOTE 1: For the special case $n = 0$, if we put $x = b$ and $z = c$ in Taylor's Formula, we get

$$f(b) = f(a) + f'(c)(b-a) \quad (9)$$

which is the Mean Value Theorem. In fact, Theorem 9 can be proved by a method similar to the proof of the Mean Value Theorem.

NOTE 2: Notice that the remainder term

$$\boxed{R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}} \quad (10)$$

is very similar to the terms in the Taylor series except that $f^{(n+1)}$ is evaluated at z instead of at a. All we say about the number z is that it lies somewhere between x and a. The expression for $R_n(x)$ in Equation 10 is known as **Lagrange's form of the remainder term**.

NOTE 3: In Section 8.8 we will explore the use of Taylor's Formula in approximating functions. Our immediate use of it is in conjunction with Theorem 8. In applying Theorems 8 and 9 it is often helpful to make use of the following fact:

$$\boxed{\lim_{n\to\infty} \frac{x^n}{n!} = 0 \quad \text{for every real number } x} \tag{11}$$

This is true because we know from Example 1 that the series $\sum \frac{x^n}{n!}$ converges for all x and so its n th term approaches 0.

EXAMPLE 2: Prove that e^x is equal to the sum of its Taylor series with $a = 0$ (Maclaurin series).

From Example 2 it follows that

$$\boxed{e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \text{for all } x} \tag{12}$$

In particular, if we put $x = 1$ in Equation 12, we obtain

$$\boxed{e = \sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots} \tag{13}$$

EXAMPLE 3: Find the Taylor series for $f(x) = e^x$ at $a = 2$.

Solution: We have $f^{(n)}(2) = e^2$ and so, putting $a = 2$ in the definition of a Taylor series (6), we get

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(2)}{n!}(x-2)^n = \sum_{n=0}^{\infty} \frac{e^2}{n!}(x-2)^n$$

Again it can be verified, as in Example 1, that the radius of convergence is $R = \infty$. As in Example 2 we can verify that $\lim_{n\to\infty} R_n(x) = 0$, so

$$e^x = \sum_{n=0}^{\infty} \frac{e^2}{n!}(x-2)^n \quad \text{for all } x \tag{14}$$

We have two power series expansions for e^x, the Maclaurin series in Equation 12 and the Taylor series in Equation 14. The first is better if we are interested in values of x near 0 and the second is better if x is near 2.

EXAMPLE 4: Find the Maclaurin series for $\sin x$ and prove that it represents $\sin x$ for all x.

Solution: We arrange our computation in two columns as follows:

$$\begin{aligned} f(x) &= \sin x & f(0) &= 0 \\ f'(x) &= \cos x & f'(0) &= 1 \\ f''(x) &= -\sin x & f''(0) &= 0 \\ f'''(x) &= -\cos x & f'''(0) &= -1 \\ f^{(4)}(x) &= \sin x & f^{(4)}(0) &= 0 \end{aligned}$$

Since the derivatives repeat in a cycle of four, we can write the Maclaurin series as follows:

$$\begin{aligned} f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \dots &= 0 + \frac{1}{1!}x + \frac{0}{2!}x^2 + \frac{-1}{3!}x^3 + \frac{0}{4!}x^4 + \dots \\ &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \\ &= \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} \end{aligned}$$

Using the remainder term (10) with $a = 0$, we have

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}x^{n+1}$$

where $f(x) = \sin x$ and z lies between 0 and x. But $f^{(n+1)}(z)$ is $\pm \sin z$ or $\pm \cos z$. In any case, $|f^{(n+1)}(z)| \leq 1$ and so

$$0 \leq |R_n(x)| = \frac{|f^{(n+1)}(z)|}{(n+1)!}|x^{n+1}| \leq \frac{1}{(n+1)!}|x^{n+1}| = \frac{|x|^{n+1}}{(n+1)!} \quad (15)$$

By Equation 11 the right side of this inequality approaches 0 as $n \to \infty$, so $R_n(x) \to 0$ by the Squeeze Theorem. It follows that $R_n(x) \to 0$ as $n \to \infty$, so $\sin x$ is equal to the sum of its Maclaurin series by Theorem 8. Thus

$$\boxed{\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} \quad \text{for all } x} \quad (16)$$

EXAMPLE 5: Find the Maclaurin series for $\cos x$.

Solution 1: We arrange our computation in two columns as follows:

$$\begin{aligned} f(x) &= \cos x & f(0) &= 1 \\ f'(x) &= -\sin x & f'(0) &= 0 \\ f''(x) &= -\cos x & f''(0) &= -1 \\ f'''(x) &= \sin x & f'''(0) &= 0 \\ f^{(4)}(x) &= \cos x & f^{(4)}(0) &= 1 \end{aligned}$$

Since the derivatives repeat in a cycle of four, we can write the Maclaurin series as follows:

$$\begin{aligned} f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \ldots &= 1 + \frac{0}{1!}x + \frac{-1}{2!}x^2 + \frac{0}{3!}x^3 + \frac{1}{4!}x^4 + \ldots \\ &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots \\ &= \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!} \end{aligned}$$

Solution 2: We differentiate the Maclaurin series for $\sin x$ given by Equation 16:

$$\begin{aligned} \cos x = \frac{d}{dx}(\sin x) &= \frac{d}{dx}\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\right) \\ &= 1 - \frac{3x^2}{3!} + \frac{5x^4}{5!} - \frac{7x^6}{7!} + \ldots \\ &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots \end{aligned}$$

Since the Maclaurin series for $\sin x$ converges for all x, Theorem tells us that the differentiated series for $\cos x$ also converges for all x. Thus

$$\boxed{\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!} \quad \text{for all } x} \quad (17)$$

The power series that we obtained by indirect methods in Examples 5 and 6 and in Section 8.6 are indeed the Taylor or Maclaurin series of the given functions because Theorem 5 asserts that, no matter how we obtain a power series representation

$$f(x) = \sum c_n (x-a)^n$$

it is always true that $c_n = f^{(n)}(a)/n!$. In other words, the coefficients are uniquely determined.

EXAMPLE 7: Find the Maclaurin series for $f(x) = (1+x)^k$, where k is any real number.

Solution: Arranging our work in columns, we have

$$\begin{aligned}
&f(x) = (1+x)^k && f(0) = 1 \\
&f'(x) = k(1+x)^{k-1} && f'(0) = k \\
&f''(x) = k(k-1)(1+x)^{k-2} && f''(0) = k(k-1) \\
&f'''(x) = k(k-1)(k-2)(1+x)^{k-3} && f'''(0) = k(k-1)(k-2) \\
&\vdots && \vdots \\
&f^{(n)}(x) = k(k-1)\ldots(k-n+1)(1+x)^{k-n} && f^{(n)}(0) = k(k-1)\ldots(k-n+1)
\end{aligned}$$

Therefore, the Maclaurin series of $f(x) = (1+x)^k$ is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = 1 + \sum_{n=1}^{\infty} \frac{k(k-1)\ldots(k-n+1)}{n!} x^n$$

This series is called the **binomial series**. If its nth term is a_n, then

$$\begin{aligned}
\left|\frac{a_{n+1}}{a_n}\right| &= \left|\frac{k(k-1)\ldots(k-n+1)(k-n)x^{n+1}}{(n+1)!} \cdot \frac{n!}{k(k-1)\ldots(k-n+1)x^n}\right| \\
&= \frac{|k-n|}{n+1}|x| = \frac{\left|\frac{k}{n}-1\right|}{1+\frac{1}{n}}|x| \to |x| \quad \text{as } n \to \infty
\end{aligned}$$

Thus by the Ratio Test the binomial series converges if $|x| < 1$ and diverges if $|x| > 1$.

The traditional notation for the coefficients in the binomial series is

$$\binom{k}{n} = \frac{k!}{n!(k-n)!} = \frac{(k-n)!(k-n+1)\ldots(k-2)(k-1)k}{n!(k-n)!} = \frac{k(k-1)(k-2)\ldots(k-n+1)}{n!}$$

and these numbers are called the **binomial coefficients**. The following theorem states that $(1+x)^k$ is equal to the sum of its Maclaurin series. It is possible to prove this by showing that the remainder term $R_n(x)$ approaches 0, but that turns out to be quite difficult.

THEOREM (THE BINOMIAL SERIES): If k is any real number and $|x|<1$, then

$$\boxed{(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \dots} \quad (18)$$

Although the binomial series always converges when $|x|<1$, the question of whether or not it converges at the endpoints, ± 1, depends on the value of k. It turns out that the series converges at 1 if $-1<k<0$ and at both endpoints if $k \geq 0$. Notice that if k is a positive integer and $n>k$, then the expression for $\binom{k}{n}$ contains a factor $(k-k)$, so

$$\binom{k}{n} = 0$$

for $n>k$. This means that the series terminates and reduces to the ordinary Binomial Theorem when k is a positive integer.

Say we have to expand x Cos x then write general expansion of Cos x and multiply with x

$$x\cos x = x\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!} = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n)!}$$

Find the Maclaurin series for $f(x) = \dfrac{1}{\sqrt{4-x}}$ and its radius of convergence.

We write $f(x)$ in a form where we can use the binomial series:

$$\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{4\left(1-\frac{x}{4}\right)}} = \frac{1}{2\sqrt{1-\frac{x}{4}}} = \frac{1}{2}\left(1-\frac{x}{4}\right)^{-1/2}$$

Using the binomial series with $k = -\dfrac{1}{2}$ and with x replaced by $-\dfrac{x}{4}$, we have

$$\frac{1}{\sqrt{4-x}} = \frac{1}{2}\left(1-\frac{x}{4}\right)^{-1/2} = \tfrac{1}{2}\sum_{n=0}^{\infty}\binom{-\frac{1}{2}}{n}\left(-\frac{x}{4}\right)^n$$

$$= \frac{1}{2}\left[1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}\left(-\frac{x}{4}\right)^2+\frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}\left(-\frac{x}{4}\right)^3\right.$$

$$\left.+\ldots+\frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})\ldots(-\frac{1}{2}-n+1)}{n!}\left(-\frac{x}{4}\right)^n+\ldots\right]$$

$$= \frac{1}{2}\left[1+\frac{1}{8}x+\frac{1\cdot 3}{2!8^2}x^2+\frac{1\cdot 3\cdot 5}{3!8^3}x^3+\ldots+\frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)}{n!8^n}x^n+\ldots\right]$$

We know from (18) that this series converges when $|-x/4|<1$, that is, $|x|<4$, so the radius of convergence is $R=4$.

(a) Evaluate $\int e^{-x^2}dx$ as an infinite series.

(b) Evaluate $\int_0^1 e^{-x^2}dx$ correct to within an error of 0.001.

(a) First we find the Maclaurin series for $f(x)=e^{-x^2}$. Although it's possible to use the direct method, let's find it simply by replacing x with $-x^2$ in the series for e^x given in the table of Maclaurin series. Thus, for all values of x,

$$e^{-x^2} = \sum_{n=0}^{\infty}\frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{n!} = 1-\frac{x^2}{1!}+\frac{x^4}{2!}-\frac{x^6}{3!}+\ldots$$

Now we integrate term by term:

$$\int e^{-x^2}dx = \int\left(1-\frac{x^2}{1!}+\frac{x^4}{2!}-\frac{x^6}{3!}+\ldots+(-1)^n\frac{x^{2n}}{n!}+\ldots\right)dx$$

$$= C+x-\frac{x^3}{3\cdot 1!}+\frac{x^5}{5\cdot 2!}-\frac{x^7}{7\cdot 3!}+\ldots+(-1)^n\frac{x^{2n+1}}{(2n+1)n!}+\ldots$$

This series converges for all x because the original series for e^{-x^2} converges for all x.

(b) The Fundamental Theorem of Calculus gives

$$\int_0^1 e^{-x^2}dx = \left[x - \frac{x^3}{3\cdot 1!} + \frac{x^5}{5\cdot 2!} - \frac{x^7}{7\cdot 3!} + \frac{x^9}{9\cdot 4!} - \dots\right]_0^1$$

$$= 1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42} + \frac{1}{216} - \dots$$

$$\approx 1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42} + \frac{1}{216} \approx 0.7475$$

The Alternating Series Estimation Theorem shows that the error involved in this approximation is less than

$$\frac{1}{11\cdot 5!} = \frac{1}{1320} < 0.001$$

Evaluate $\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$

Using the Maclaurin series for e^x, we have

$$\lim_{x\to 0}\frac{e^x - 1 - x}{x^2} = \lim_{x\to 0}\frac{\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\right) - 1 - x}{x^2}$$

$$= \lim_{x\to 0}\frac{\frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots}{x^2}$$

$$= \lim_{x\to 0}\left(\frac{1}{2} + \frac{x}{3!} + \frac{x^2}{4!} + \frac{x^3}{5!} + \dots\right) = \frac{1}{2}$$

because power series are continuous functions.

Thus the conclusive Remark :

Any random function which cannot be integrated by simple methods, can be expanded into power series and then integrated by individual terms. The number of terms to be integrated depends on the accuracy we require.

Practically we rarely need integration of indefinite functions. Definite integrals can be evaluated upto any degree of accuracy by numerical techniques.

—

Solve

$$\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{(x)} + \cos^{-1}\sqrt{(x)}}\, dx$$

$$\text{Evaluate} \int \frac{(x-1)\, dx}{(x+1)\sqrt{(x^3+x^2+x)}}$$

$$\int \frac{(x^2-1)\, dx}{(x^4+3x^2+1)\tan^{-1}\left(x+\frac{1}{x}\right)}$$

$$\int \frac{x^2-2}{(x^4+5x^2+4)\tan^{-1}\left(\frac{x^2+2}{x}\right)}\, dx$$

$$\int \frac{x(x-1)}{(x^2+1)(x+1)\sqrt{x^3+x^2+x}}\, dx$$

$$\int \left(\frac{1}{1-x^8}\right)\left[\cos^{-1}\frac{2x}{1+x^2} + \tan^{-1}\frac{2x}{1-x^2}\right] dx$$

$$\int \frac{dx}{x+\sqrt{x^2-x+1}}$$

$$\{x + \sqrt{x^2 - x + 1}\}\{x - \sqrt{x^2 - x + 1}\} = x - 1 \quad ...(A)$$

Now let $z = x + \sqrt{x^2 - x + 1}$

$$\frac{x-1}{z} = x - \sqrt{x^2 + 1} \text{ by (A)}$$

$$\therefore \quad z + \frac{x-1}{z} = 2x \text{ or } z^2 + (x-1) = 2xz$$

$$\text{or} \quad z^2 - 1 = x(2z - 1)$$

$$\therefore \quad x = \frac{z^2 - 1}{2z - 1} \text{ or } \frac{dx}{dz} = \frac{2z^2 - 2z + 2}{(2z-1)^2}$$

$$\therefore \quad I = \int \frac{1}{z} \frac{2z^2 - 2z + 2}{(2z-1)^2} dz = 2\int \frac{z^2 - z + 2}{z(2z-1)^2} dz$$

Split into partial fractions

$$\therefore \quad I = 2\int \left[\frac{1}{z} - \frac{3}{2(2z-1)} + \frac{3}{2(2z-1)^2}\right] dz$$

$$= 2\log z - \frac{3}{2}\log(2z-1) - \frac{3}{4}\frac{1}{(2z-1)}$$

re $z = x + \sqrt{x^2 - x + 1}$.

—

Solve

$$\text{Evaluate} \int \frac{1 + x^{-2/3}}{1 + x} dx$$

$$u = \int_0^\infty \frac{dx}{x^4 + 7x^2 + 1} \text{ and } v = \int_0^\infty \frac{x^2 dx}{x^4 + 7x^2 + 1}$$

$$I = \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\,dx$$

$$= \int \frac{\sin^{-1}\sqrt{x} - (\pi/2 - \sin^{-1}\sqrt{x})}{\pi/2}\,dx$$

$\because \quad [\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}] = \pi/2$

$$= \frac{4}{\pi}\int \sin^{-1}\sqrt{x}\,dx - \int dx$$

$$\int \sin^{-1}\sqrt{x}\,dx$$

Now put $x = \sin^2\theta$.

Then $\quad dx = 2\sin\theta\cos\theta\,d\theta$

Then $\int \sin^{-1}\sqrt{x}\,dx = \int \theta . 2\sin\theta\cos\theta\,d\theta$

$$I = \theta\sin^2\theta - \int \sin^2\theta\,d\theta$$

$$= \theta\sin^2\theta - \frac{1}{2}\int (1 - \cos 2\theta)\,d\theta$$

$$= \theta\sin^2\theta - \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]$$

$$= \frac{\theta}{2}(2\sin^2\theta - 1) + \frac{1}{4}\cdot 2\sin\theta\cos\theta$$

$$= \frac{1}{2}\sin^{-1}\sqrt{x}\cdot(2x - 1) + \frac{1}{2}\sqrt{x}\sqrt{(1 - x)}$$

(b) $I = \int \frac{(1 + 1/x^2)\, dx}{(1/x - x)\sqrt{x^2 + 1/x^2 + 1}}$

Put $x - \frac{1}{x} = z$

$\therefore \quad I = -\int \frac{dz}{z\sqrt{z^2+3}} = -$ Now put $z^2 + 3 = t^2$

or $\left(x - \frac{1}{x}\right)^2 + 3 = t^2$ or $x^2 + \frac{1}{x^2} + 1 = t^2$

$\therefore \quad I = -\int \frac{1}{t^2 - 3}\, dt$ etc.

$$I = \int \frac{x^2 - 1}{(x+1)^2} \cdot \frac{dx}{\sqrt{(x^3 + x^2 + x)}}$$

$$= \int \frac{(x^2 - 1)}{(x^2 + 2x + 1)} \cdot \frac{dx}{\sqrt{(x^3 + x^2 + x)}}$$

$$= \int \frac{(1 - 1/x^2)}{(x + 2 + 1/x)} \cdot \frac{dx}{\sqrt{(x + 1 + 1/x)}}$$

on dividing both N^r and D^r by x^2

Putting $x + 1 + \frac{1}{x} = t^2 \quad \therefore \quad \left(1 - \frac{1}{x^2}\right)dx = 2t\, dt$

$$\therefore \quad I = \int \frac{2t\, dt}{(t^2+1)\,.\,t} = \int \frac{2\, dt}{(t^2+1)}$$

$$= 2\tan^{-1} t = 2\tan^{-1}\sqrt{\left(x + 1 + \frac{1}{x}\right)}$$

$$I = \int \frac{x^2 - 1}{(x+1)^2} \cdot \frac{dx}{\sqrt{(x^3 + x^2 + x)}}$$
$$= \int \frac{(x^2 - 1)}{(x^2 + 2x + 1)} \cdot \frac{dx}{\sqrt{(x^3 + x^2 + x)}}$$
$$= \int \frac{(1 - 1/x^2)}{(x + 2 + 1/x)} \cdot \frac{dx}{\sqrt{(x + 1 + 1/x)}}$$

on dividing both N^r and D^r by x^2.

Putting $x + 1 + \frac{1}{x} = t^2 \quad \therefore \quad \left(1 - \frac{1}{x^2}\right) dx = 2t\, dt$

$$\therefore \quad I = \int \frac{2t\, dt}{(t^2 + 1)\,.\,t} = \int \frac{2\, dt}{(t^2 + 1)}$$
$$= 2 \tan^{-1} t = 2 \tan^{-1} \sqrt{\left(x + 1 + \frac{1}{x}\right)}$$

$$\int_0^{\pi} \frac{dx}{(a - \cos x)} = \frac{\pi}{\sqrt{a^2 - 1}}$$

Differentiating both sides with respect to a, we get

$$-\int_0^{\pi} \frac{dx}{(a - \cos x)^2} = \frac{-\pi a}{(a^2 - 1)^{3/2}}$$

Again differentiating with respect to a, we get

$$2\int_0^{\pi} \frac{dx}{(a - \cos x)^3} = \frac{\pi(1 + 2a^2)}{(a^2 - 1)^{5/2}}$$

Putting $a = \sqrt{10}$, we get $\int_0^{\pi} \frac{dx}{(\sqrt{10} - \cos x)^3} = \frac{7\pi}{81}$

$$\text{Let } I(a) = \int_0^{\pi/2} \log\left(\frac{1+a\sin x}{1-a\sin x}\right)\frac{dx}{\sin x}$$

$$\frac{dI}{da} = \int_0^{\pi/2} \frac{2\sin x}{1-a^2\sin^2 x}\frac{dx}{\sin x}$$

$$= \int_0^{\pi/2} \frac{2\sec^2 x dx}{1+\tan^2 x - a^2\tan^2 x}$$

$$= \int_0^{\pi/2} \frac{2\sec^2 x dx}{1+(1-a^2)\tan^2 x}$$

$$= \int_0^{\infty} \frac{2dt}{1+(1-a^2)t^2}$$

$$= \frac{2}{\sqrt{1-a^2}}\left[\tan^{-1}\left(t\sqrt{1-a^2}\right)\right]_0^{\infty}$$

$$= \frac{\pi}{\sqrt{1-a^2}}$$

$$\therefore\ I = \pi\sin^{-1} a$$

Let $I(a) = \int_0^1 \frac{x^a - 1}{\log x} dx$

Differentiating w.r.t. a keeping x as constant, we get

$$\frac{dI(a)}{da} = \int_0^1 \frac{d}{da}\left(\frac{x^a - 1}{\log x}\right) dx$$

$$= \int_0^1 \frac{x^a \log x}{\log x} dx$$

$$= \int_0^1 x^a \, dx$$

$$= \left.\frac{x^{a+1}}{a+1}\right|_0^1$$

$$= \frac{1}{(a+1)}$$

Integrating both sides w.r.t. a, we get

$I(a) = \log(a + 1) + c$

For $a = 0$, $I(0) = \log 1 + c$

$0 = 0 + c$

$\therefore I = \log(a + 1)$

$$\text{Let } F(k) = \int_0^{\pi/2} \ln(\sin^2\theta + k^2\cos^2\theta)\,d\theta$$

$$\frac{\partial F(k)}{\partial k} = F'(k) = \int_0^{\pi/2} \frac{1}{\sin^2\theta + k^2\cos^2\theta}\,2k\cos^2\theta\,d\theta$$

$$= 2k\int_0^{\pi/2} \frac{\cos^2\theta}{\sin^2\theta + k^2\cos^2\theta}\,d\theta$$

$$= 2k\int_0^{\pi/2} \frac{d\theta}{\tan^2\theta + k^2}$$

$$= 2k\int_0^{\pi/2} \frac{\sec^2\theta - \tan^2\theta}{\tan^2\theta + k^2}\,d\theta$$

$$= 2k\int_0^{\infty} \frac{dt}{t^2+k^2} - 2k\int_0^{\pi/2} d\theta + 2k^3\int_0^{\pi/2} \frac{d\theta}{\tan^2\theta + k^2}$$

(Putting $t = \tan\theta$)

$$= 2k\,\frac{1}{k}\tan^{-1}\frac{1}{k}\Big|_0^{\infty} - 2k\,\frac{\pi}{2} + k^2F'(k)$$

or $(1-k^2)\,F'(k) = \pi - k\pi = \pi(1-k)$

$$f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t\cos x)\,f(t)\,dt$$

The range of $f(x)$ is

a. $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$ b. $\left[-\frac{\sqrt{5}}{3}, \frac{\sqrt{5}}{3}\right]$

c. $\left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$. d. none of these

$f(x)$ is not invertible for

a. $x \in \left[-\frac{\pi}{2} - \tan^{-1} 2, \frac{\pi}{2} - \tan^{-1} 2\right]$

b. $x \in \left[\tan^{-1} \frac{1}{2}, \pi + \tan^{-1} \frac{1}{2}\right]$

c. $x \in \left[\pi + \cot^{-1} 2, 2\pi + \cot^{-1} 2\right]$

d. none of these

$$f(x) = \sin x + \sin x \int_{-\pi/2}^{\pi/2} f(t)\,dt + \cos x \int_{-\pi/2}^{\pi/2} tf(t)\,dt$$

$$= \sin x \left(1 + \int_{-\pi/2}^{\pi/2} f(t)\,dt\right) + \cos x \int_{-\pi/2}^{\pi/2} tf(t)\,dt$$

$$= A \sin x + B \cos x$$

Thus, $A = 1 + \int_{-\pi/2}^{\pi/2} f(t)\,dt$

$$= 1 + \int_{-\pi/2}^{\pi/2} (A \sin t + B \cos t)\,dt$$

$$= 1 + 2B\int_0^{\pi/2} \cos t\,dt$$

$\therefore A = 1 + 2B$

$$B = \int_{-\pi/2}^{\pi/2} tf(t)\,dt$$

$$= \int_{-\pi/2}^{\pi/2} t(A\sin t + B\cos t)\,dt$$

$$= 2A\int_0^{\pi/2} t\sin t\,dt$$

$$= 2A[-t\cos t + \sin t]_0^{\pi/2}$$

$\therefore B = 2A$

From equations (1) and (2), we get

$A = -1/3$, $B = -2/3$

$\therefore f(x) = -\frac{1}{3}(\sin x + 2\cos x)$

Thus, the range of $f(x)$ is $\left[-\frac{\sqrt{5}}{3}, \frac{\sqrt{5}}{3}\right]$

$$f(x) = -\frac{1}{3}(\sin x + 2\cos x)$$

$$= -\frac{\sqrt{5}}{3}\sin\left(x + \tan^{-1} 2\right)$$

$$= -\frac{\sqrt{5}}{3}\cos\left(x - \tan^{-1}\frac{1}{2}\right)$$

$f(x)$ is invertible if $-\frac{\pi}{2} \le x + \tan^{-1} 2 \le \frac{\pi}{2}$

or $-\frac{\pi}{2} - \tan^{-1} 2 \le x \le \frac{\pi}{2} - \tan^{-1} 2$

or $0 \le x - \tan^{-1}\frac{1}{2} \le \pi$

$$\int_0^{\pi/2} f(x)\,dx = -\frac{1}{3}\int_0^{\pi/2} (\sin x + 2\cos x)\,dx$$
$$= -\frac{1}{3}\left[-\cos x + 2\sin x\right]_0^{\pi/2}$$
$$= -1$$

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Appendix :

The word Appendix is from mid 16 th century Latin word Appendere meaning hang upon. Apart from the hanging body part; which is not needed by us now; We all know; it also means, supplementary material at the end of a book, article, document, or other text, usually of an explanatory, statistical, or bibliographic nature.

[in simple words Appendix is extra, and may not exactly be needed].

Almost all authors, including me, feel, that something more can be here. Not everything was supposed to be at the beginning. It is not possible to put everything at the beginning, nor that should be done.

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http://www.celebatheists.com/wiki/Main_Page gives names of Hundreds of Atheists.

Douglas Adams, Ayaan Hirsi Ali, Woody Allen, Fred Armisen, Lance Armstrong, Darren Aronofsky, Isaac Asimov, Julian Assange, Dan Barker, Dave Barry, Ingmar Bergman, Pierre Berton, Niels Bohr, Richard Branson, Derren Brown, Kari Byron, James Cameron, Asia Carrera, George Carlin, John Carmack, Adam Carolla, John Carpenter, Asia Carrera, Fidel Castro, Noam Chomsky, Jeremy Clarkson, Billy Connolly, Francis Crick, David Cronenberg, David Cross, Alan Cumming, Rodney Dangerfield, Richard Dawkins, Daniel Dennett, Ani DiFranco, Micky Dolenz, Albert Einstein, Harlan Ellison, Paul Erdős, Richard Feynman, Harvey Fierstein, Reginald Finley, Barney Frank, Morgan Freeman, Larry Flynt, Dave Foley, Arian Foster, Jodie Foster, Janeane Garofalo, Bill Gates, Bob Geldof, Ricky Gervais, Ira Glass, James Gleick, Robert Heinlein, Ernest Hemingway, Katharine Hepburn, Christopher Hitchens, Jamie Hyneman, Eddie Izzard, Penn Jillette, Billy Joel, Ana Kasparian, Diane Keaton, Skandar Keynes Michael Kinsley, Keira Knightley, Kramer, John Landis, Hugh Laurie, Artie Lange, Richard Leakey, Bruce Lee, Tom Lehrer, John Lennon, Tom Leykis, James Lipton, H.P. Lovecraft, Ernst Mach, Seth MacFarlane, Bill Maher, John Malkovich, Barry Manilow, Todd McFarlane, Sir Ian McKellen, Arthur Miller, Frank Miller, Claude Monet, Julianne Moore, Rafael Nadal, Randy Newman, Mike Nichols, Jack Nicholson, Gary Numan, Bob Odenkirk, Patton Oswalt, Camille Paglia, Trey Parker, PewDiePie, Steven Pinker, Brad Pitt, Joaquin Phoenix, Paula Poundstone, Terry Pratchett, Robin Quivers, Daniel Radcliffe, James Randi, Ron Reagan Jr., Rob Reiner, Keanu Reeves, Rick Reynolds, Gene Roddenberry, Henry Rollins, Andy Rooney, Salman Rushdie, Adam Savage, Brian Sapient, Erwin Schrödinger, Bob Simon, Steven Soderbergh, Annika Sorenstam, George Soros, Richard Stallman, Howard Stern, Matt Stone, Julia Sweeney, Teller, Studs Terkel, Pat Tillman, Tool, Alan Turing, Eddie Vedder, Jesse Ventura, Gore Vidal, Vincent van Gogh, Kurt Vonnegut Jr., Steven Weinberg, Joss Whedon, Ted Williams, Steve Wozniak, HUNDREDS MORE...

World's Greatest Scientists are all Atheists

See https://www.youtube.com/watch?v=UKbslSOfrRo

https://www.youtube.com/watch?v=GdqC2bVLesQ

https://www.youtube.com/watch?v=BCUmeE8sIVo

https://www.youtube.com/watch?v=YUe0_4rdj0U

https://www.youtube.com/watch?v=eY1pDkP9Qxk

https://www.youtube.com/watch?v=XYohZRivNhI

https://www.youtube.com/watch?v=f4tbDI3K1ZU

Since Many years there are too many articles on Women Sex Predators, and aggressive women

Sex education: Why ARE so many female teachers having affairs with their teenage students... and is the 'cougar effect' to blame?

DAILYMAIL.CO.UK

Motherly Love Redefined …

http://crimeblog.dallasnews.com/2016/05/prosper-woman-who-had-sex-with-sons-teenage-friend-headed-to-prison.html/

http://www.wtol.com/story/6975375/mother-sentenced-for-having-sex-with-son

http://www.dailymail.co.uk/news/article-2716412/Mother-jailed-having-sex-12-year-old-SON-partner-watched-told-webcam.html

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http://www.nhregister.com/article/NH/20120921/NEWS/309219751

http://articles.orlandosentinel.com/1996-10-27/news/9610260994_1_extorting-endangerment-elementary-school-principal

http://www.independent.ie/irish-news/incest-mother-is-convicted-of-sex-assault-on-her-two-sons-26462211.html

http://canadiancrc.com/newspaper_articles/Tor_Star_Mother_confesses_sex_with_sons_03OCT04.aspx

SEX WITH MY STUDENTS: THE TEACHERS WHO HAVE BEEN ARRESTED, CHARGED OR JAILED OVER THE PAST SCHOOL YEAR

Angela New, 39, from Gladewater, Texas, was arrested last week after school chiefs at Union Grove High School, where she taught English received a tip off about an alleged affair between her and an 18-year-old student.

Had the offence taken place a month later - after the teen graduated - she might not have been charged as he was no longer a full-time student and at the age of consent.

April Alexander, 26, from Irving, Texas, was last week arrested after being accused of having sex with a 16-year old student on more than 25 occasions.

The teen, now 18, told police he and the biology teacher had sex on more than two dozen occasions at MacArthur High School in Irving and in Alexander's car.

Brittni Colleps, 27, from Arlington, Texas, was arrested last week after being accused of having sex with five of her teenage students during an orgy at her home.

The English teacher and girl's basketball coach at Kennedale High School allegedly invited the boys to her home while her husband was away with the military and the sex romp was allegedly filmed on the students' cell phones.
The married mother-of- three faces up to ten years in jail.

Michelle McCutchan, 38, was jailed in Checotah, Oklahoma, after admitting making a sex tape with her daughter's 16-year-old boyfriend.

The mother-of-one confessed to having sex with the teen on at least five occasions and setting up a video camera to film two of the romps.

http://www.thedailybeast.com/articles/2014/06/10/canada-s-newest-refugee-a-florida-mom-convicted-of-unlawful-sex-with-a-minor.html

http://www.ibtimes.co.uk/us-idaho-lawsuit-reveals-sexual-assault-by-staff-male-teens-juvenile-detention-centers-1494582

http://www.mirror.co.uk/news/world-news/biology-teacher-who-sex-five-8850667

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http://www.gympietimes.com.au/news/roma-mother-guilty-of-bruising-sons-genitals/3089847/

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SEX WITH MY STUDENTS: THE TEACHERS WHO HAVE BEEN ARRESTED, CHARGED OR JAILED OVER THE PAST SCHOOL YEAR

Nicole Chapman, 28, was this month jailed for 10 to 12 months earlier this month for having sex with a 19-year-old special needs student in North Carolina.

Bizarrely the teenager's mother approved of the relationship between her son and the ex North Shelby teacher and told local TV: 'I ain't no victim. If it is love, man, it's love. Nobody can stop this.'

Marie L Fisher, 21, was last month charged with having a sexual relationship with a 15-year-old boy after sending him explicit text messages.

Fisher, who worked in the Special Education Department at Reeds High School in Sparks, Nevada, is alleged to have sent him a photo of her 'half naked breast' and later slept with him.

Bethyl Shepherd, 34, who worked in the same department as Fisher, was arrested last month after officials found out about an alleged threesome with two 17-year-old students.

Shepherd, who had taught at the school for 10 years, claimed one of the teens forced her to have sex while the other watched.

Barbara Anderson, 37, a teacher at a Washington State school, was arrested in March after allegedly having sex with a 17-year-old student.

The pupil in question told his uncle he was 'getting laid by a teacher,' according to court documents. She sent almost 800 text messages to the boy between January 15 and February 21, including more than 100 texts in one 24-hour period.

http://equalitycanada.com/why-are-so-many-women-raping-boys-research-into-female-perpetrated-sexual-violence/

http://www.theindychannel.com/news/local-news/teacher-accused-of-sex-with-student-10-times-reaches-plea-deal-for-1-count-of-child-seduction

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http://www.express.co.uk/news/world/656971/Bullies-bikinis-attacked-sunbathing-victim-filmed-assault

http://www.bustle.com/articles/123975-6-signs-you-have-a-toxic-mother

http://txktoday.com/news/new-boston-woman-pleads-guilty-to-sexually-assaulting-13-year-old-boy/

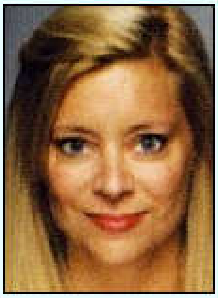
SEX WITH MY STUDENTS: THE TEACHERS WHO HAVE BEEN ARRESTED, CHARGED OR JAILED OVER THE PAST SCHOOL YEAR

Jamie Waite, 35, a swimming instructor at a school in Utah, was arrested in March for allegedly having sexual relations with a 17-year-old student.

Police in Utah arrested the teacher after a tip off from friends of the student who claimed the pair were having a relationship.

Deborah J Cox, 58, of Naperville became involved with the boy while he was at Neuqua Valley High School, police said.

The teacher's aide, had been the boy's personal academic tutor for several years, before she was arrested in March for having an alleged sexual relationship with him.



Carrie Shafer, 38, a biology teacher, was caught in March by police partially naked in a car with one of her students.

An arrest report revealed the married mother-of-two, from Kentucky, was in a compromising position with a 17-year-old student from her school in Louisville. Police said the windows of the car were 'steamed up' and both occupants were partially clothed.

Gail Gagne, 29, a weight room supervisor at Cretin-Derham Hall High School, Minnesota, was sentenced to two years probation in February after having sex with a then 16-year-old student - both at her home in Bloomington and at a nearby hotel.

http://www.ibtimes.co.uk/married-teacher-who-had-affair-14-year-old-pupil-sent-him-video-online-charged-rape-1579807

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http://www.telegraph.co.uk/news/uknews/law-and-order/5251042/Rise-of-ladette-culture-as-241-women-arrested-each-day-for-violence.html

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http://www.huffingtonpost.com/2013/11/09/nicole-kurowski-teacher-sex_n_4241276.html

SEX WITH MY STUDENTS: THE TEACHERS WHO HAVE BEEN ARRESTED, CHARGED OR JAILED OVER THE PAST SCHOOL YEAR

Chanda Frank, 34, a physical education coach at Haywood High in Brownsville, was charged with sexual battery in February, after allegedly fondling a 14-year-old female student on her softball team.

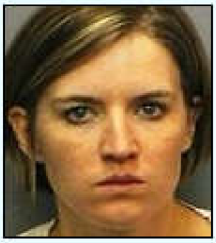
It happened at least twice between the dates of August 2010 and February 2011, according to court documents.

Ashley Blumenshine, 27, a PE and dance teacher was arrested in January behind a department store in Plainfield, Illinois, after police caught her in a car with a 16-year-old boy student.

Police believe they had had sex shortly before the officers arrived and say the relationship may have gone on for more than a month.

Stacy Schuler, 32, of Mason, Ohio, was charged with committing sex acts with students from her high school football team in February.

She faces 19 felony counts of sexual battery involving five male students, three misdemeanor charges of serving alcohol to underage youths and more than 96 years in prison if convicted.



Courtney Bowles, 31, a teacher who advised colleagues on how to avoid affairs with students, was caught having sex with a teenager in her car.

Bowles was found by a police officer lying naked on top of the boy, who was also naked, from her school in Colorado. A partly consumed bottle of vodka was also found in the car with the couple.

http://www.craveonline.com/site/1062074-uk-teacher-had-sex-with-15-year-old-more-than-50-times-claimed

http://www.sun-sentinel.com/local/broward/fl-female-bank-robbers-20160722-story.html

http://www.heraldsun.com.au/news/law-order/fight-to-extradite-ultraorthodox-jewish-school-principal-accused-of-molesting-and-raping-students-dropped/news-story/194cad2f934cca5858500ffebf4858c4

https://www.theguardian.com/society/2009/oct/04/uk-female-child-sex-offenders

http://www.adelaidenow.com.au/news/south-australia/woman-who-tried-to-kill-exs-new-girlfriend-unable-to-show-empathy-for-her-victim-court-told/news-story/7b22da29e8cc18e413269a0be58800b0

https://www.youtube.com/watch?v=syWtUykS7L0

https://www.youtube.com/watch?v=3g_OPKvDgpU

https://www.youtube.com/watch?v=VPl5PkjVs3A

https://www.youtube.com/watch?v=AFk1FyKDYec

https://www.youtube.com/watch?v=oIn5OfNFa5I

SEX WITH MY STUDENTS: THE TEACHERS WHO HAVE BEEN ARRESTED, CHARGED OR JAILED OVER THE PAST SCHOOL YEAR

Marcie Lynn Rousseau, 34, had sex with a 16-year-old student at least 100 times.

The Michigan English teacher was sent to prison last December to serve a minimum of four years having pleaded guilty to all the charges.

But her defence lawyer said of the student: 'He took part in this also ... He courted her, bought her lunch, brought her flowers.'

Megan Baumann, 28, is currently serving three years in prison after pleading guilty to various sex charges involving three male students.

The social sciences teacher had sex with one and sent him naked pictures of herself by text. The former Tennessee teacher sent texts of her breasts and pubic region to another while she fondled the third while he was clothed.

Jennifer Riojas, 27, a science teacher in Fort Worth, Texas, was pregnant when arrested for sexual assault in November after allegedly having sex with a 16-year-old student.

The pupil told police they met in motels and even had sex when he was in a hospital bed recovering from a football injury.

Carlie Rose Attebury, 31, a former California marching band teacher, denied having sex with a 15-year-old student but admits having sex with two former students when they were 18. Attebury said that she hugged the 15-year-old boy as she did any 'band kid'. She was sentenced to 16 months in prison in March.

http://www.dailymail.co.uk/news/article-3274956/Disturbing-rise-women-child-sex-predators-s-punished-leniently-men.html?ito=social-facebook

https://www.youtube.com/watch?v=L5gWMO2JPa4

https://www.youtube.com/watch?v=76rAn4JZfiA

https://www.youtube.com/watch?v=W5RJBcsQq7Q

https://www.youtube.com/watch?v=yXAM83Lq8d0

https://www.youtube.com/watch?v=XfxkVjawYYg

https://www.youtube.com/watch?v=4_Uum7tEUqg

https://www.youtube.com/watch?v=D3ILPAUmPrw&list=PLfqvIEGoZYGzaCWw7VPYrY6sCtkbxOat8

https://www.youtube.com/watch?v=H6a9Szp8FwY

https://www.youtube.com/watch?v=p-GLJUPrtNU

https://www.youtube.com/watch?v=8uDEB2KG9XU

SEX WITH MY STUDENTS: THE TEACHERS WHO HAVE BEEN ARRESTED, CHARGED OR JAILED OVER THE PAST SCHOOL YEAR

Elizabeth Colleen Wallis, 34, was sentenced in March to four months in the Yuba County Jail for having sex with a former underage special needs student now 17.

Wallis a teacher's assistant at Yuba Gardens Intermediate School in Olivehurst, California, came to the attention of police when they were notified of the alleged inappropriate relationship by a concerned family member of Wallis.

The boy's mother had indicated that her son was in love with Wallis.

Kimme Woolf, was a 29-year-old math teacher at Perrin High School in Perrin, Texas, when she was arrested for allegedly committing sexual assault on a male student, 16, and having an improper relationship with another, 18, last November.

Woolf is alleged to have slept with both boys after they repeatedly asked for sex. She is also thought to have performed oral sex on both.

Felecia Killings, 27, an English teacher at Rodriguez High School in Fairfield, California was arrested in November on allegations of having sex with a 16-year-old student.

Police believe the alleged relations took place at Killings' home.

Sara Leann Dwiggins, 24, a math teacher at South Creek Middle School in Williamston, North Carolina, was arrested on allegations of having sex with a 14-year-old former student. She has been charged with two counts of statutory rape and two counts of sex offence with a student.

Gina Watring, 40, was jailed for five years last September after admitting having sex with a boy, 10, at the primary school in Durham, North Carolina. The mother-of-two had faced up to 70 years after being charged with dozens of offences.

A Psychologist has an explanation …

http://www.dailymail.co.uk/news/article-1391626/Whats-wrong-female-teachers-America-As-schools-summer-young-teacher-arrested-sex-16-year-old-student--latest-dozens-cases-school-year.html

https://www.youtube.com/watch?v=vWikSl0j_wA

A Mother Who Killed Her 5 Children

https://www.youtube.com/watch?v=Mp-zuabUeXU

https://www.youtube.com/watch?v=tz7DCorxLbo

https://www.youtube.com/watch?v=jf6VU5meuho

https://www.youtube.com/watch?v=gEP0k4ZMFfk

https://www.youtube.com/watch?v=vfVFklqG0NM

Why are Modern Women so aggressive ?

Why are modern women so aggressive? The dark side of equality

Gemma Dawson, 27, from Wakefield, (left), Jo Scott, 51, from Sussex, (centre) and Annmarie Fisher, 32, form Iver, (right) reveal what makes them angry and why.

DAILYMAIL.CO.UK

https://www.theguardian.com/education/2006/jan/23/pupilbehaviour.schools

Female Sex Predators: A Crime Epidemic
13 hrs ·

More perverts . . . .

Model walks topless through New York in support of Free The Nipple

Model Emily Bloom, 23, has bravely walked topless through New York City in support of the Free The Nipple campaign for gender equality, leaving passers-by...

DAILYMAIL.CO.UK

See https://www.facebook.com/WomenCriminals/

North Carolina woman, 45, arrested for having sex with adult son

A 45-year-old North Carolina woman and her 25-year-old son have been arrested for having sex with each other.

NYDAILYNEWS.COM

See https://www.facebook.com/groups/499811210056249/

Published on Sep 14, 2016 | Updated 3 days ago | By Susmita Pathak Mishra | In Featured, World News

Police have charged a North Carolina mom-son pair with incest after an August report claimed that they had sex with each other.

Forty-four-year-old Melissa Nell Kitchens shared a sexual relationship with 25-year-old son Shaun Thomas Pfieffer. As soon as the matter became known to the police, Buncombe County Police started investigating, after which the duo was arrested. Both suspects are due to appear in court later in September.

"Can't get over how handsome you are and I'm about to cry," one of the Facebook posts of Kitchens stated. The post was accompanied by the picture of her son. "Things are very stressful and I love you and I respect any decision — as long as you're happy and safe … I miss you and wish I had more time with you."

The arrest warrants stated the counts of charges on the suspects. Mother and son have both been charged with one count of incest. Where the mother had sex with her son, who is already married to Shannon Roman and has a young son, the son is also due to face charges of indecent liberties with a child. The latter incident took place on August 13 when Pfieffer communicated threats and behaved disruptively.

http://www.australianetworknews.com/melissa-kitchens-incest-american-mom-sex-son-25-gets-arrested/

http://www.irishtimes.com/news/crime-and-law/waterford-mother-convicted-of-child-cruelty-following-seven-week-trial-1.2657598

http://www.fox19.com/story/32236822/convicted-sex-offender-asks-mother-of-14-year-old-i-want-her-what-do-you-want-for-it

http://www.insideedition.com/headlines/16733-mom-and-female-partner-convicted-of-torturing-murdering-2-year-old-son-who-fell-off

https://www.rt.com/uk/354212-wales-mother-porn-court/

http://q13fox.com/2016/02/04/marysville-mother-convicted-of-sex-crimes-involving-daughter/

http://www.charlotteobserver.com/news/local/crime/article77122242.html

http://www.omaha.com/bellevue-mom-convicted-of-sexually-abusing-son-gets--/article_1393b0df-a383-58c3-a8e7-e5af713cc630.html

http://www.huffingtonpost.in/entry/wisconsin-mom-sentenced-sex-crimes-toddler_n_6237550

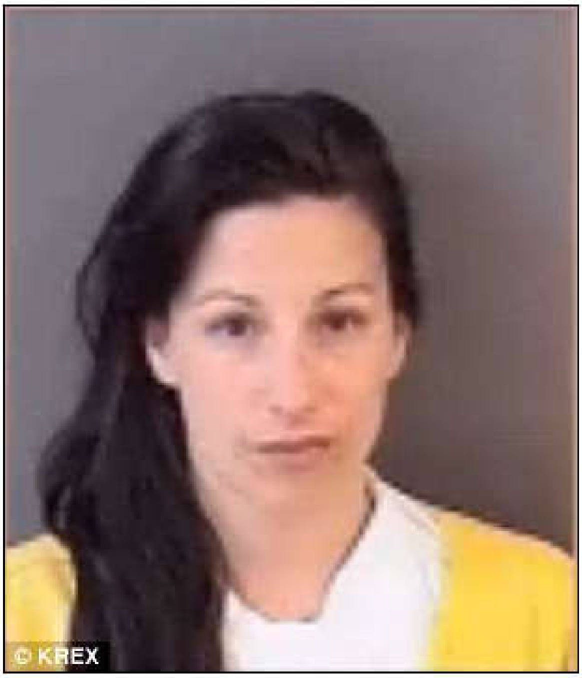
A mother accused of sexually seducing her underage son's friend during a sleepover, telling him he could 'pretend to be 18 for the night,' was arrested March 1.

36-year-old Wendy Crowell now faces seven criminal charges, six of which are felonies, tied to sexual assault on a minor between 15 and 16 years old.

SCROLL DOWN FOR VIDEO

When a police detective questioned Crowell at her Grand Junction, Colorado home, she claimed she exchanged texts with all her son's friends.

A gut feeling led the boy's mother to suspect a possible relationship between her son and Crowell.



Naughty mom: Wendy Cowell of Grand Junction, Colorado is accused of having sex with her son's underage friend multiple times

http://www.dailymail.co.uk/news/article-2287494/Grand-Junction-mom-Wendy-Crowell-sex-sons-underage-best-friend.html

http://www.vindy.com/news/2011/oct/06/pa-mom-sentenced-for-sex-with-son8217s-t/

http://www.murfreesboropost.com/mother-convicted-of-raping-son-years-ago-cms-41753

http://www.digitaljournal.com/article/294597

http://cnews.canoe.com/CNEWS/Crime/2014/08/05/21854361.html

http://www.politicsforum.org/forum/viewtopic.php?t=121028

http://www.complex.com/pop-culture/2012/08/orange-county-mother-convicted-for-crossing-line-with-sons-friend

http://www.usatoday.com/story/sports/nfl/2015/08/21/molly-shattuck-ravens-cheerleader-sentenced-rape-boy/32108039/

POLICE: DELCO MOM HAD SEX WITH SON'S TEEN FRIEND

Police: Delco mom had sex with son's teen friend
none

Share G+1 Tweet

February 23, 2012 6:58:14 AM PST

Action News

UPPER CHICHESTER, Pa. - February 22, 2012 -- A Delaware County mother, accused of having sex with her son's 15-year-old friend, is out of jail and awaiting an arraignment.
A judge has agreed to place 34-year-old Teri Mezzatesta on an electronic monitoring system so she can take care of her disabled grandmother at her Upper Chichester home.

Mezzatesta faces charges of statutory sexual assault, a second-degree felony, and false swearing in official matters by falsely incriminating another.

Mezzatesta claims she was sexually assaulted.

Mezzatesta was arrested on January 27 after, according to the affidavit, her son went to school officials about what he allegedly saw on the night of November 15.

In the affidavit, the 14-year-old son tells police he witnessed his mother having sexual intercourse with his 15-year-old friend who was

http://6abc.com/archive/8554005/

http://bossip.com/920633/hide-ya-kids-cali-mom-sentenced-6-years-in-prison-for-sexing-sons-12-year-old-friend-43081/

http://wncn.com/2016/07/02/ga-mom-sentenced-after-teen-naked-twister-party-with-sex-and-drugs/

http://www.chron.com/news/houston-texas/houston/article/Mother-who-supplied-drugs-in-prom-death-pleads-7431853.php

http://www.starherald.com/news/local_news/sidney-mom-sentenced-in-molestation-of-son/article_09b03185-47a1-51a2-a6b6-98adc270d8ed.html

http://www.twcnews.com/archives/nys/central-ny/2007/12/14/mom-sentenced-in-sex-abuse-case-NY_38392.old.html

http://wtvr.com/2015/08/23/molly-shattuck-oldest-ravens-cheerleader-rapes-sons-15-year-old-friend/

http://maddad0467.newsvine.com/_news/2011/10/07/8203176-mom-sentenced-for-threesome-with-sons-friends

http://www.dispatch.com/content/stories/local/2012/09/05/mother-sentenced-for-raping-her-baby.html

news24 archives

Breaking News. First.

Mom 'had sex with son'

2010-01-28 10:02

Omaha - A 41-year-old US woman is accused of having sex nightly with her teenage son when he was in seventh and eighth grades, officials said on Wednesday.

Omaha Police said the now 15-year-old boy reported the alleged abuse last week to a counsellor, who notified authorities. The boy told police his mother was addicted to prescription drugs when the alleged abuse took place in 2008 and 2009 while he lived with her in Omaha.

The woman, who lives in the state of Nebraska, was arrested on Monday, according to Officer Michael Pecha. She made an initial appearance on Wednesday in court and her bond was set at $30 000.

The Associated Press is not identifying the woman to protect her son's identity as a possible victim of sexual assault.

The teen has a younger brother, but authorities do not suspect the younger boy suffered any abuse, Douglas County Attorney Don Kleine said.

The boy's father told Omaha television station WOWT this week that he had previously had a feeling something was wrong, but didn't learn about the alleged abuse until a few weeks ago. He said his son is receiving counselling.

The woman will be represented by a public defender's office, but an attorney wasn't to be assigned to her case until Thursday. A preliminary hearing to discuss details of the charge against her was scheduled for February 8.

http://www.news24.com/world/news/mom-had-sex-with-son-20100128

http://nypost.com/2016/04/09/mom-and-son-admit-to-incest-go-into-hiding-to-avoid-jail/

http://www.cbc.ca/news/canada/windsor/mom-gets-1-year-for-sex-with-foster-son-1.1121822

http://www.mcall.com/news/breaking/mc-allentown-verdict-woman-accused-molesting-boy-20160309-story.html

http://www.nytimes.com/2015/10/25/magazine/the-strange-case-of-anna-stubblefield.html?_r=0

http://www.norwalkreflector.com/Local/2015/09/21/Sex-offender-039-s-mom-talks-about-2009-juvenile-court-case

http://abcnews.go.com/US/hummer-mom-christine-hubbs-force-sex-teen-boys/story?id=13541399

OC mom had sex with son's underage teammates,

by AP September 20 2011

SHARE

A 44-year-old Orange County woman had sex with at least two boys on her son's hockey team, investigators say. Both team members were under 18.

Orange County sheriff's spokesman Jim Amormino says one of the boys is under 16 years old and the other is under 14.

Amormino tells a local wire service that Kathia Maria Davis of Laguna Niguel was arrested last week and she was booked for investigation of unlawful sex and lewd acts with a minor. She was released after posting $25,000 bail.

Davis was initially suspected of having sex with one boy. Amormino said Monday that a second boy has now surfaced and there may be a third.

http://www.scpr.org/news/2011/09/19/28941/oc-mom-had-sex-sons-underage-teammates-authorities/

http://gasmicgore.com/forum/archive/index.php/t-3786.html

http://lancasteronline.com/news/mom-sentenced-for-prostituting-son/article_d035429a-9354-5d37-8d17-0815efd0a3c2.html

http://www.mercurynews.com/2009/12/15/north-carolina-mom-sentenced-for-putting-son-in-boiling-water/

http://us.geosnews.com/p/us/oh/cuyahoga-county/cleveland/appellate-court-again-rules-mom-convicted-of-helping-son-in-madison-township-murder-should-get-new-trial_4970914

https://www.propublica.org/article/false-rape-accusations-an-unbelievable-story

http://world.sports--news.com/news/lacey-spears-a-mother-accused

http://archive.decaturdaily.com/decaturdaily/news/070621/mom.shtml

http://www.tdcaa.com/node/3056

http://www.pravdareport.com/news/world/americas/04-11-2005/69955-0/

Idaho mom had sex with son's friends

POSTREPLY Search this topic... Search

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Idaho mom had sex with son's friends

by **Abcshanghai** » Sat Jan 05, 2013 3:16 pm

http://www.cbsnews.com/2300-504083_162-10014846.html

I don't know about you guys but when was 15 I wanted to bang everything that moves.

Come to think about it I still want to bang everything that moves, but it still has to pass the ugly meter unless I'm drunk.

I don't feel the 15 year old boys are the victims. Its like branging rights. Hey I banged your mom, no seriously I literally banged your mom.

The victim is the mother in jail and her son having to live with out his mother and being stigmatized by his mother in jail.

The boys with so much testosterone in them thats what they do.

What's the big deal? Its most boys fantasy to have sex with an older women.

I remember in 6th grade my teachers name was Mrs. Rucker and oh boy did I want to...we'll you know. and she had a big old rack.

http://www.shanghaiexpat.com/phpbbforum/idaho-mom-had-sex-with-son-s-friends-t151005.html

http://www.newsgrio.com/articles/248052-mom-drunkenly-let-a-convicted-sex-offender-who-exposed-himself-to-girls-under-13-give-her-three-children-permanent-tattoos.html

https://traffickalerts.wordpress.com/2015/01/15/incest-mom-sentenced-to-219-year-in-prison-over-alabama-sex-ring/

http://www.dailymail.co.uk/femail/article-2081674/Poppy-Burge-gets-liposuction-voucher-Human-Barbie-mum-Sarah-Christmas.html

http://www.nydailynews.com/news/national/florida-mom-charged-setting-fight-daughter-amp-classmate-article-1.1012931#ixzz1kx3LwRVQ

https://uk.style.yahoo.com/blogs/yahoo-lifestyles/mother-gives-botox-injections-her-eight-old-daughter-184941192.html

About Fallacies and Logic

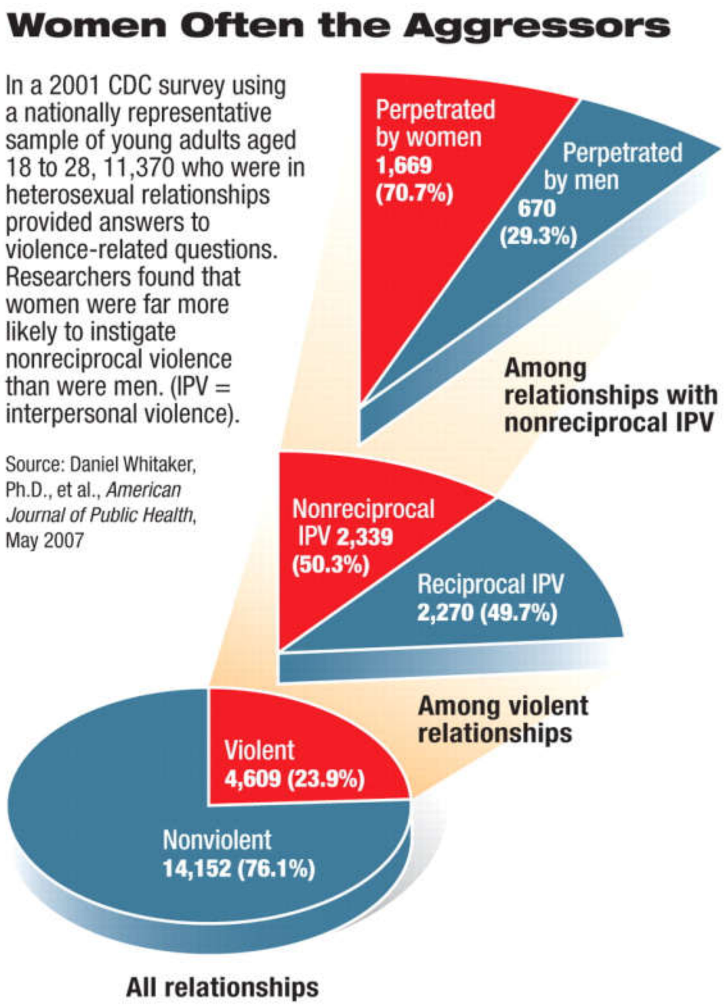
Hasty Generalisation is one of the most common Fallacies practiced by Human Beings. This is (often) the case; because 2 of the important " theorems " of Statistics are NOT appreciated.

Two of these theorems of Statistics being –

S1 – Larger the sample size better the observation. As the sample size approaches the " Total Population " the reality is manifested better.

S2 – The sample types should vary widely. Wider is the variation the random noise is eliminated the most.

It is fallacious to generalize with a very few observations or by personal experience / perception. [Before seeing the Statistics below, try to answer from your perception … "who amongst Men and Women instigate violence ?"]



Also it is known from study of Psychology that (often) people tend to justify their perception and actions ‘ more than required ‘ to avoid being seen as foolish.

People give asymmetrical importance to their opinions and emphasize it too much.

Daniel Kahneman got Nobel Prize in Economics for his work on “ Behavioral Finance “. He had shown that people are NOT “ equi-proportionate “ in their choices, actions and decisions.

There have been interesting developments in “ Game theory “ also giving insights on “ sub-optimal “ choices that people make in their decisions.

An unbiased statistical experiment with sample size larger than the minimum required, and varying widely can throw light on the REALITY.

There are many types of Fallacies, namely –

1) Post Hoc

2) Poisoning the Well

3) Bandwagon

Etc ...

18 types of Logical Fallacies are described at http://kspope.com/fallacies/fallacies.php

42 types of Fallacies are described at http://www.nizkor.org/features/fallacies/

One of the ways of classifying the fallacies is –

1) Formal Fallacies

2) Informal Fallacies

3) Aristotelian Fallacies

3.1 – Material Fallacies

3.2 – Verbal Fallacies

3.3 – Logical Fallacies

A nice list of Fallacies is given at http://en.wikipedia.org/wiki/List_of_fallacies

The following Cognitive Traps we succumb to –

1) Availability Bias – This causes us to base our decisions on information that are more readily available than doing an exhaustive search. If someone asks you the question … In English do we have more words starting with R or more words where R is in the 3rd place ? …. The

correct way to answer this is …. I do not know, we have to search / analyze and see. But as we tend to remember words by their first alphabet we tend to recall words starting with R but hardly can remember words such as FoRt, MaRt, FeRtilizer etc. (It seems after an exhaustive search it is found that we have more WoRds where R is in the 3rd place than in 1st place !)

2) Hindsight Bias – (ex post) – This causes us to attach higher probability to events after they have happened than we did before they happened. This bias also lasts for only a small amount of time such as few days or weeks. In 1970s at Howrah station (Calcutta / Kolkata) a passenger train could not brake in time and dashed at the end of the line (Platform) to stop. [Similar to Chennai / Madras the rail ends one way at Howrah station. The trains do not cross through the station but comes and returns the same way.] This crash caused the first bogie to get mutilated very badly and a few people died. Now this first bogie generally is very crowded, as people want to rush out and run a smaller distance to reach the office / Business. For next few days the first bogie was almost empty in local trains, and slowly was forgotten. In history of Howrah station this type of accident may have happened only 3 – 4 times. Except the one mentioned above the other crashes were minor in nature. So the " Hindsight Bias " explains why people were too cautious for a few days to keep the first bogie empty and then slowly forget.

3) The problem of Induction – This causes us to formulate general rules on the basis of insufficient information. (Hasty Generalization). CPI / CPM parties have been ruling West Bengal for decades so often many outsiders term all Bengalis as communists. I have even seen the following type of conversation sequence … In a training program the trainer gave me a Red pen and jokingly said you will like this colour ! As I asked why do you think so ? He said : You are a Bengali, so you are a communist. Red is the colour of communists ! So you should like it !

4) The fallacy of Conjunction – (or Disjunction) – This causes us to overestimate the probability of 10 events each with 90% probability, will ALL occur, while underestimating the probability that at least 1 of the 10 events with just 10% probability is quite likely to occur. In fact human beings in general are not good as estimating probability or estimating the occurrence frequency of an event.

5) Confirmation Bias – This inclines us to look for confirming evidence of an initial hypothesis, rather than falsifying evidence that would disprove it. Often when the Media / Press wants to malign someone (Character Assassination) then keeps giving biased Negative evidences to paint the character. The readers / TV viewers refer to only this propaganda rather than search opposite evidences of their own.

6) Contamination Effects – This causes us to allow irrelevant but proximate information to influence a decision.

7) The Affect Heuristic – This causes us preconceived value-judgments interfere with our assessment of costs and benefits.

8) Scope Neglect – This prevents us from proportionately adjusting what we should be willing to sacrifice to avoid harms of different orders of magnitude. As the stock market rises, a prudent investor should switch part of equity systematically to Debt funds (say MIPs) and at the peak day should exit all equity to put all her investments into Liquid / Debt funds. But in practice how many people does this ? The peak of Equity market is peak because majority are buying more equity than are selling !

9) Overconfidence in Calibration – This leads us to underestimate the confidence intervals within which our estimates will be robust. (to mixup best case scenario with most probable scenario).

10) Bystander Apathy – This inclines us to abdicate individual responsibility when in a crowd. John Darley & Bibb Latane – Bad Samaritan explanation. Victims chance of being helped within 45 secs was 50% in case of 1 bystander while 0% in case of 5 or more bystander. In the industry, “

Group email “ is great for Information propagation but not for seeking help. Only handful people are active in Discussion groups. Individual emails evoke better response.

Some of the psychological traps that affect way people make business decisions …

- The anchoring trap – Leads to give disproportionate weight to first information or a few first information. Can be avoided by circulating the agenda beforehand.

- The status quo trap – Momentum , culture , heritage problem.

- Sunk-Cost trap – This inclines us to perpetuate the mistakes of the past.

- The confirming evidence trap – This leeds us to seek out information supporting an existing predilection and to discount opposing information

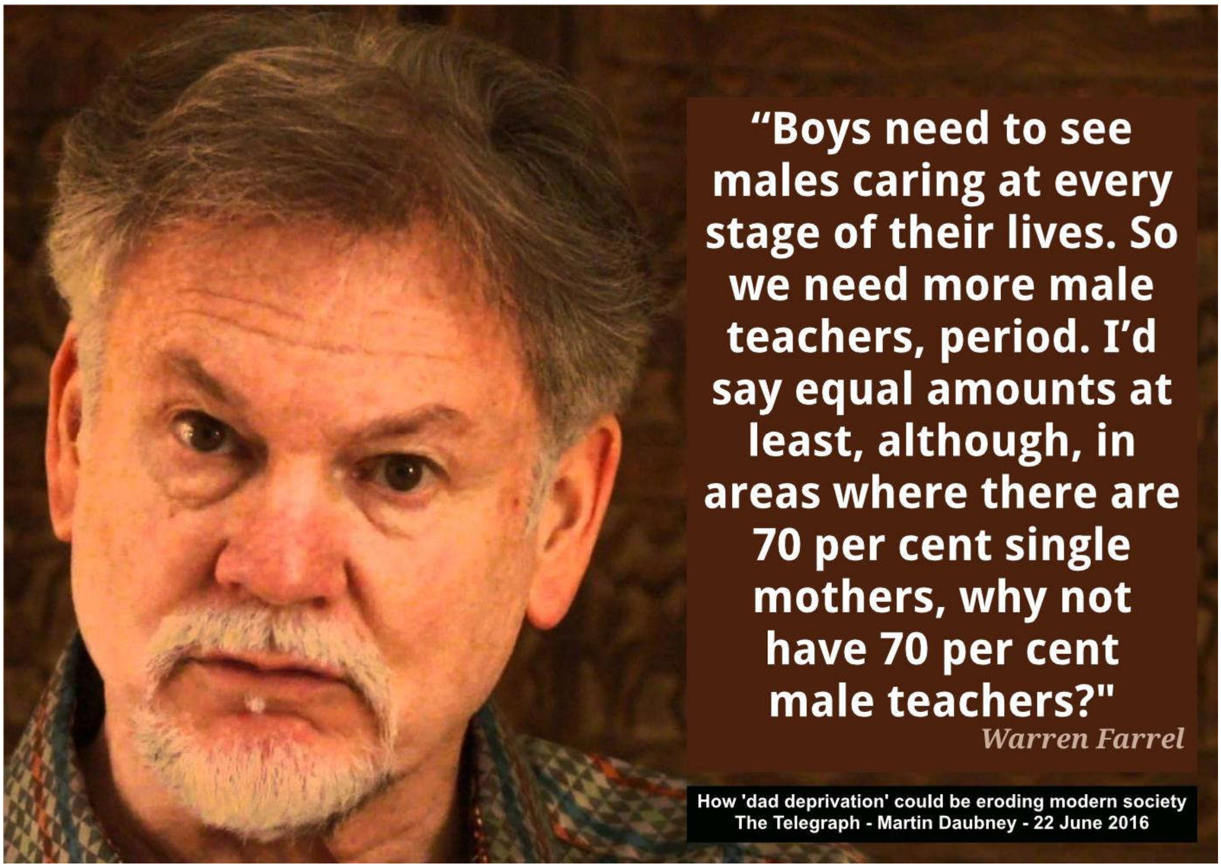
- The framing Trap – This occurs when we misstate a problem, undermining the entire decision – making process.

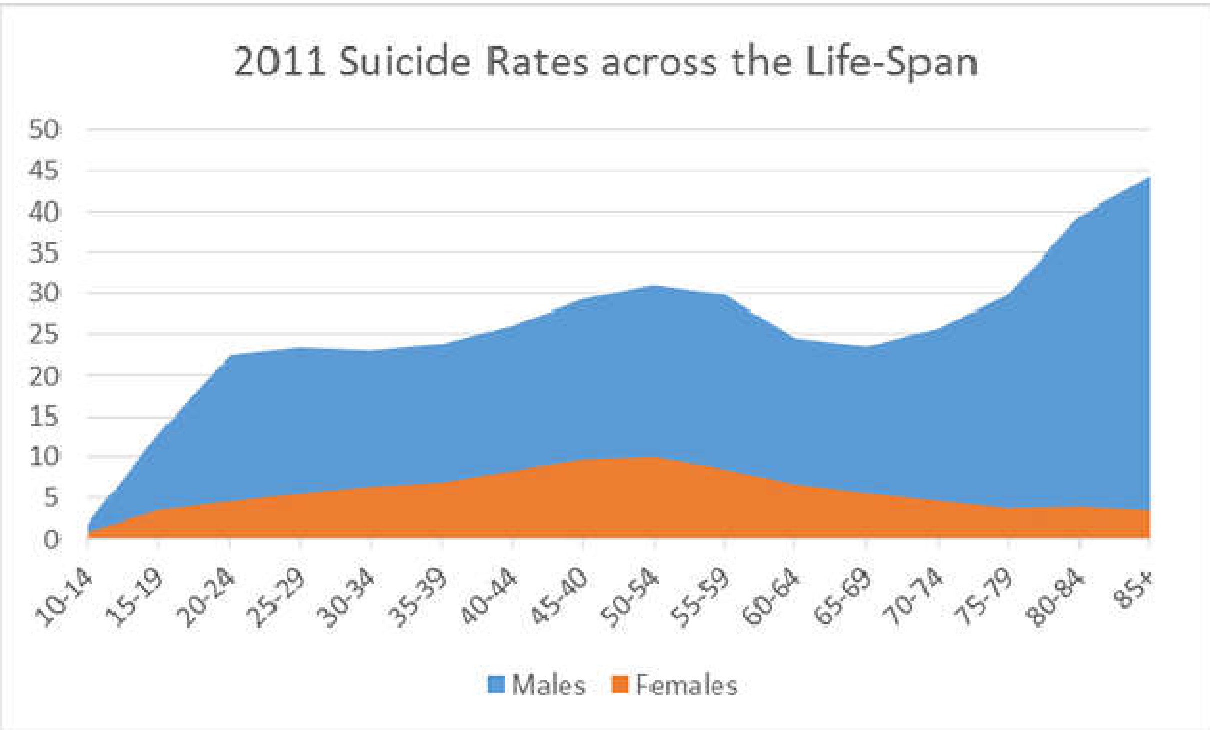
- The prudence tap – This leads us to be overcautious when we make estimates about uncertain events.

- The recallability trap – This leads us to give undue weight to recent, dramatic events.

[For example Dr Warren Farrell is not limited or trapped with traditional Biases]

See … what he says …


“Boys need to see males caring at every stage of their lives. So we need more male teachers, period. I’d say equal amounts at least, although, in areas where there are 70 per cent single mothers, why not have 70 per cent male teachers?"
Warren Farrel
How 'dad deprivation' could be eroding modern society
The Telegraph - Martin Daubney - 22 June 2016


2011 Suicide Rates across the Life-Span
50
45
40
35
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25
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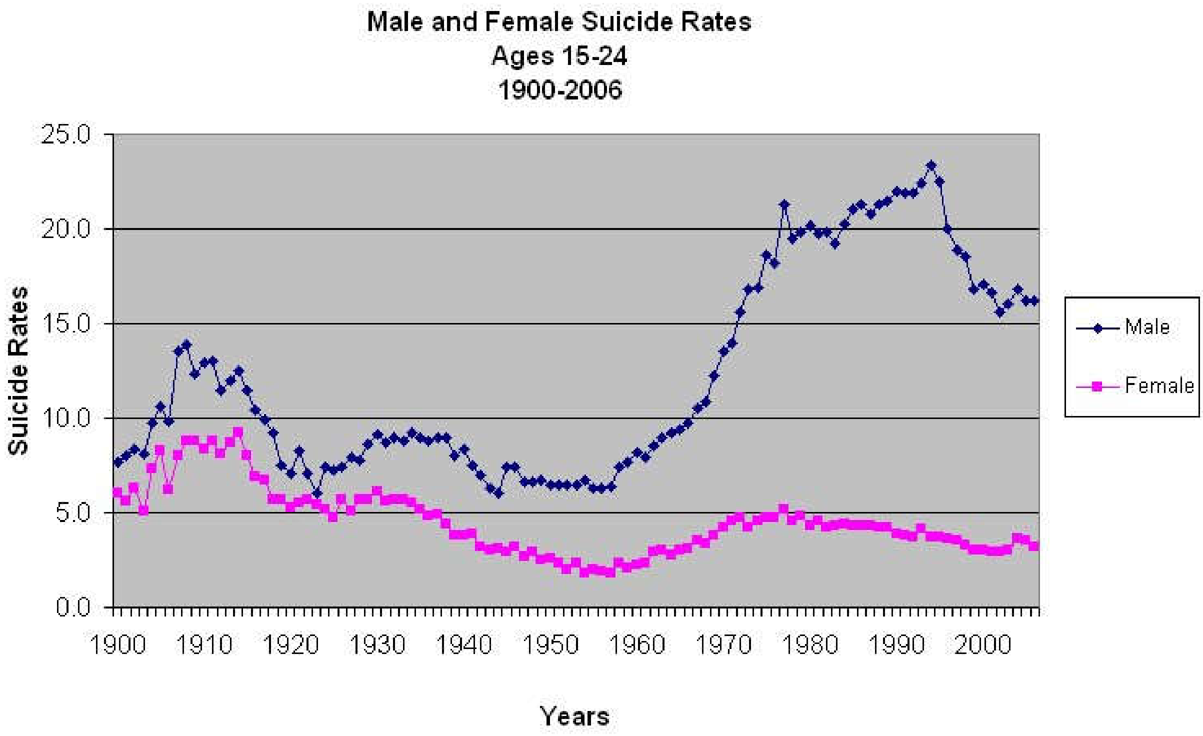
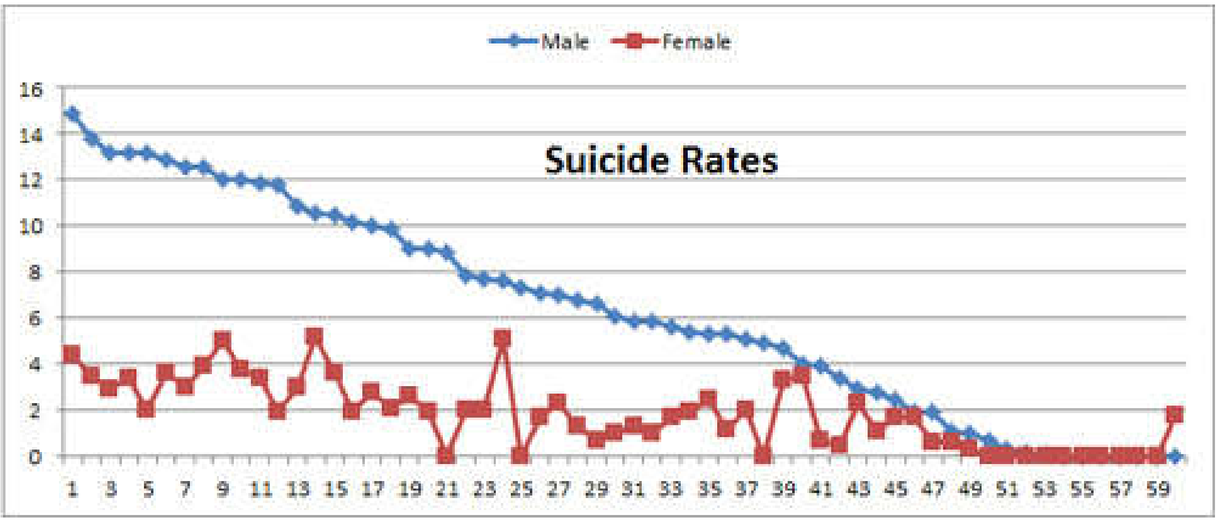
10-14
15-19
20-24
25-29
30-34
35-39
40-44
45-40
50-54
55-59
60-64
65-69
70-74
75-79
80-84
85+
Males
Females

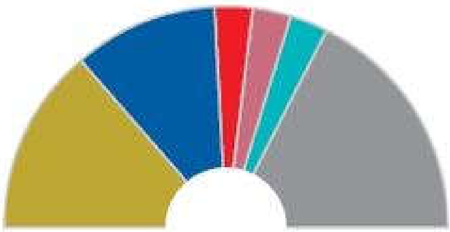


How many people are bothered about Male suicide rate being so high ? Worldwide average rate (of all ages combined) of Male suicide rate is 4 times higher that of Women. Does anyone care ? Most people are Biased to assume Men are **Disposable**.



VIOLENT END: THE CAUSES OF MORTALITY

TOP 5 CAUSES OF UNNATURAL DEATH AMONG MEN



Road accidents	1,17,055
Suicide	90,543
Murder	24,721
Railway accidents	23,639
Drowning	23,111
Other causes	1,48,865
Total	4,27,934

TOP 5 CAUSES OF UNNATURAL DEATH AMONG WOMEN

Suicide	44,256
Road accidents	20,368
Fire	11,488
Poisoning	10,182
Murder	9,180
Other causes	45,809
Total	1,41,283

SOURCE: NATIONAL CRIME RECORDS BUREAU

STATS ON SUICIDES

SUICIDES ACCORDING TO SOCIAL STATUS

Status	Male	Female	Transgender	TOTAL	% Share
Un-Married	17,999	9,820	6	27,825	21.1
Married	59,744	27,064	0	86,808	65.9
Widowed/ Widower	1,410	1,304	–	2,714	2.1
Divorcee	551	417	–	968	0.7
Separated	599	316	1	916	0.7
Total (including other & no status)	89,129	42,521	16	1,31,666	100.0

To recall standard integrals

$f(x)$	$\int f(x)dx$	$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1} \quad (n \neq -1)$	$[g(x)]^n g'(x)$	$\frac{[g(x)]^{n+1}}{n+1} \quad (n \neq -1)$
$\frac{1}{x}$	$\ln\|x\|$	$\frac{g'(x)}{g(x)}$	$\ln\|g(x)\|$
e^x	e^x	a^x	$\frac{a^x}{\ln a} \quad (a > 0)$
$\sin x$	$-\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$-\ln\|\cos x\|$	$\tanh x$	$\ln\cosh x$
$\operatorname{cosec} x$	$\ln\left\|\tan\frac{x}{2}\right\|$	$\operatorname{cosech} x$	$\ln\left\|\tanh\frac{x}{2}\right\|$
$\sec x$	$\ln\|\sec x + \tan x\|$	$\operatorname{sech} x$	$2\tan^{-1} e^x$
$\sec^2 x$	$\tan x$	$\operatorname{sech}^2 x$	$\tanh x$
$\cot x$	$\ln\|\sin x\|$	$\coth x$	$\ln\|\sinh x\|$
$\sin^2 x$	$\frac{x}{2} - \frac{\sin 2x}{4}$	$\sinh^2 x$	$\frac{\sinh 2x}{4} - \frac{x}{2}$
$\cos^2 x$	$\frac{x}{2} + \frac{\sin 2x}{4}$	$\cosh^2 x$	$\frac{\sinh 2x}{4} + \frac{x}{2}$

$f(x)$	$\int f(x)\,dx$	$f(x)$	$\int f(x)\,dx$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$ $(a > 0)$	$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| \quad (0<\|x\|<a)$
		$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| \quad (\|x\| > a>0)$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a}$ $(-a < x < a)$	$\frac{1}{\sqrt{a^2+x^2}}$	$\ln\left\|\frac{x+\sqrt{a^2+x^2}}{a}\right\| \quad (a > 0)$
		$\frac{1}{\sqrt{x^2-a^2}}$	$\ln\left\|\frac{x+\sqrt{x^2-a^2}}{a}\right\| \quad (x>a>0)$
$\sqrt{a^2-x^2}$	$\frac{a^2}{2}\left[\sin^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{a^2-x^2}}{a^2}\right]$	$\sqrt{a^2+x^2}$	$\frac{a^2}{2}\left[\sinh^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{a^2+x^2}}{a^2}\right]$
		$\sqrt{x^2-a^2}$	$\frac{a^2}{2}\left[-\cosh^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{x^2-a^2}}{a^2}\right]$

Some series Expansions -

$$\frac{\pi}{2} = \left(\frac{2}{1}\frac{2}{3}\right)\left(\frac{4}{3}\frac{4}{5}\right)\left(\frac{6}{5}\frac{6}{7}\right)\left(\frac{8}{7}\frac{8}{9}\right)\ldots$$

$$\pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \cdots$$

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots.$$

$$\pi = \sqrt{12}\left(1 - \frac{1}{3\cdot 3} + \frac{1}{5\cdot 3^2} - \frac{1}{7\cdot 3^3} + \cdots\right)$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots. = \sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2}\log 2 = \frac{\pi}{2}\log\frac{1}{2}$$

Solve a series problem

If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ upto $\infty = \frac{\pi^2}{6}$, then value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ up to ∞ is

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^2}{12}$

Ans. (c)

Solution We have $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ upto ∞

$$= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} \cdots \text{ upto } \infty$$

$$- \frac{1}{2^2}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

$$= \frac{\pi^2}{6} - \frac{1}{4}\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{8}$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \ldots. \infty = \frac{\pi^2}{12}$$

$$\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \ldots \infty = \frac{\pi^2}{24}$$

$$\frac{\sin\sqrt{x}}{\sqrt{x}} = 1 - \frac{x}{3!} + \frac{x^2}{5!} - \frac{x^3}{7!} + \frac{x^4}{9!} - \frac{x^5}{11!} + \ldots.$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_{k=0}^{n} \frac{(-1)^k x^{2k}}{(2k)!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \sum_{k=0}^{n} \frac{x^{2k}}{(2k)!}$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!}$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad (-1 \le x < 1)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} \cdots + \frac{2^{2n}\left(2^{2n}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad |x| < \frac{\pi}{2}$$

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \cdots + \frac{E_n x^{2n}}{(2n)!} + \cdots \quad |x| < \frac{\pi}{2}$$

$$\csc x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \cdots + \frac{2\left(2^{2n-1}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad 0 < |x| < \pi$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - \cdots - \frac{2^{2n} B_n x^{2n-1}}{(2n)!} - \cdots \quad 0 < |x| < \pi$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$$

$$\sec x = 1 + \frac{x^2}{\lfloor 2} + \frac{5x^4}{\lfloor 4} + \cdots$$

$$\log(\cos x) = -\frac{x^2}{\lfloor 2} - \frac{2x^4}{\lfloor 4} - \cdots$$

$$\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \cdots$$

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots \quad |x| < 1$$

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$= \frac{\pi}{2} - \left(x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots \right) \quad |x| < 1$$

$$\tan^{-1} x = \begin{cases} x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots & |x| < 1 \\ \pm\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \cdots & \begin{cases} + \text{ if } x \geq 1 \\ - \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$$

$$= \frac{\pi}{2} - \left(\frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots \right) \quad |x| > 1$$

$$\csc^{-1} x = \sin^{-1}(1/x)$$

$$= \frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots \quad |x| > 1$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$= \begin{cases} \frac{\pi}{2} - \left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \right) & |x| < 1 \\ p\pi + \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} + \cdots & \begin{cases} p = 0 \text{ if } x \geq 1 \\ p = 1 \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \ldots\right]$$

$$= 2\sum_{n=1}^{\infty} \frac{1}{2n-1}\left(\frac{x-1}{x+1}\right)^{2n-1} \quad (x > 0)$$

$$\ln x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \ldots$$

$$= \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{x-1}{x}\right)^n \quad (x > \frac{1}{2})$$

$$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}(x-1)^n \quad (0 < x \le 2)$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} x^n \quad (|x| < 1)$$

$$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots\infty \, (-1 \le x < 1)$$

$$\log_e(1+x) - \log_e(1-x) =$$

$$\log_e \frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\infty\right)(-1 < x < 1)$$

$$\log_e\left(1+\frac{1}{n}\right) = \log_e \frac{n+1}{n} = 2 \quad \left[\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \ldots\infty\right]$$

$$\log_e(1+x) + \log_e(1-x) = \log_e(1-x^2) = -2\left(\frac{x^2}{2} + \frac{x^4}{4} + \ldots\infty\right)(-1 < x < 1)$$

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots \quad = \frac{1}{1\cdot 2} + \frac{1}{3.4} + \frac{1}{5.6} + \ldots$$

Important Results

(i) (a) $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x}\,dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x}\,dx$

(b) $\int_0^{\pi/2} \frac{\tan^n x}{1+\tan^n x}\,dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{dx}{1+\tan^n x}$

(c) $\int_0^{\pi/2} \frac{dx}{1+\cot^n x} = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{1+\cot^n x}\,dx$

(d) $\int_0^{\pi/2} \frac{\tan^n x}{\tan^n x + \cot^n x}\,dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{\tan^n x + \cot^n x}\,dx$

(e) $\int_0^{\pi/2} \frac{\sec^n x}{\sec^n x + \mathrm{cosec}^n x}\,dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\mathrm{cosec}^n x}{\sec^n x + \mathrm{cosec}^n x}\,dx$ where, $n \in R$

(ii) $\int_0^{\pi/2} \frac{a^{\sin^n x}}{a^{\sin^n x} + a^{\cos^n x}}\,dx = \int_0^{\pi/2} \frac{a^{\cos^n x}}{a^{\sin^n x} + a^{\cos^n x}}\,dx = \frac{\pi}{4}$

(iii) (a) $\int_0^{\pi/2} \log \sin x\,dx = \int_0^{\pi/2} \log \cos x\,dx = -\frac{\pi}{2}\log 2$

(b) $\int_0^{\pi/2} \log \tan x\,dx = \int_0^{\pi/2} \log \cot x\,dx = 0$

(c) $\int_0^{\pi/2} \log \sec x\,dx = \int_0^{\pi/2} \log \mathrm{cosec}\, x\,dx = \frac{\pi}{2}\log 2$

(iv) (a) $\int_0^{\infty} e^{-ax} \sin bx\,dx = \frac{b}{a^2+b^2}$

(b) $\int_0^{\infty} e^{-ax} \cos bx\,dx = \frac{a}{a^2+b^2}$

(c) $\int_0^{\infty} e^{-ax} x^n\,dx = \frac{n!}{a^n+1}$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left(x + \sqrt{x^2 - a^2}\right) + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left(\frac{x - a}{x + a}\right) + C$$

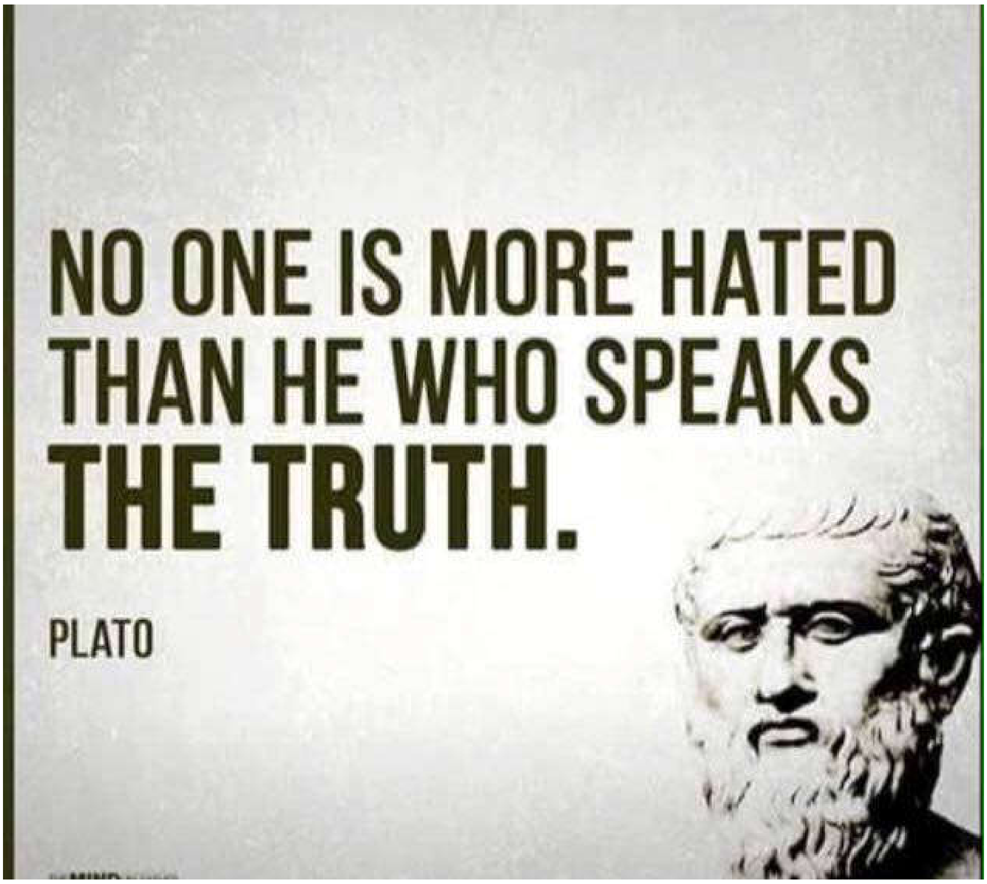
$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left(\frac{a + x}{a - x}\right) + C$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

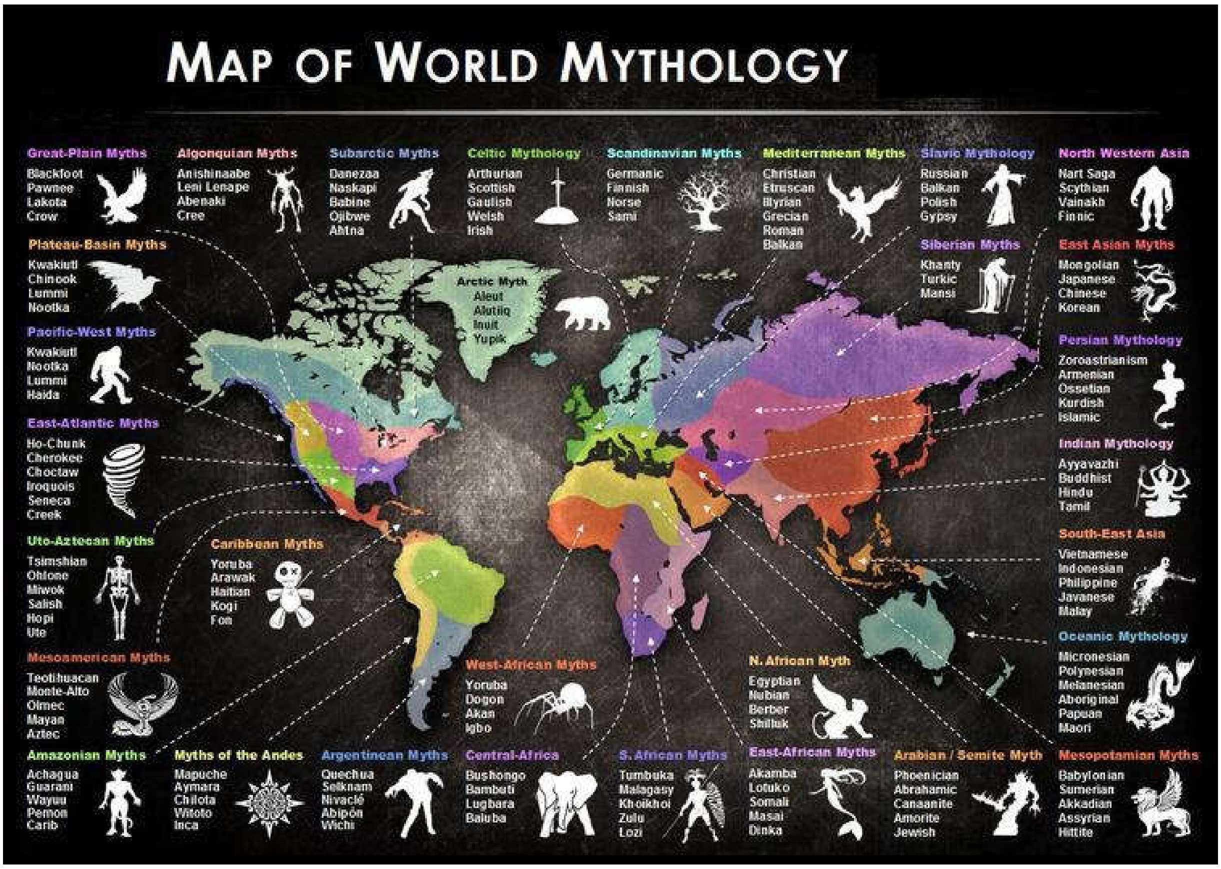
$$\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right) + C$$

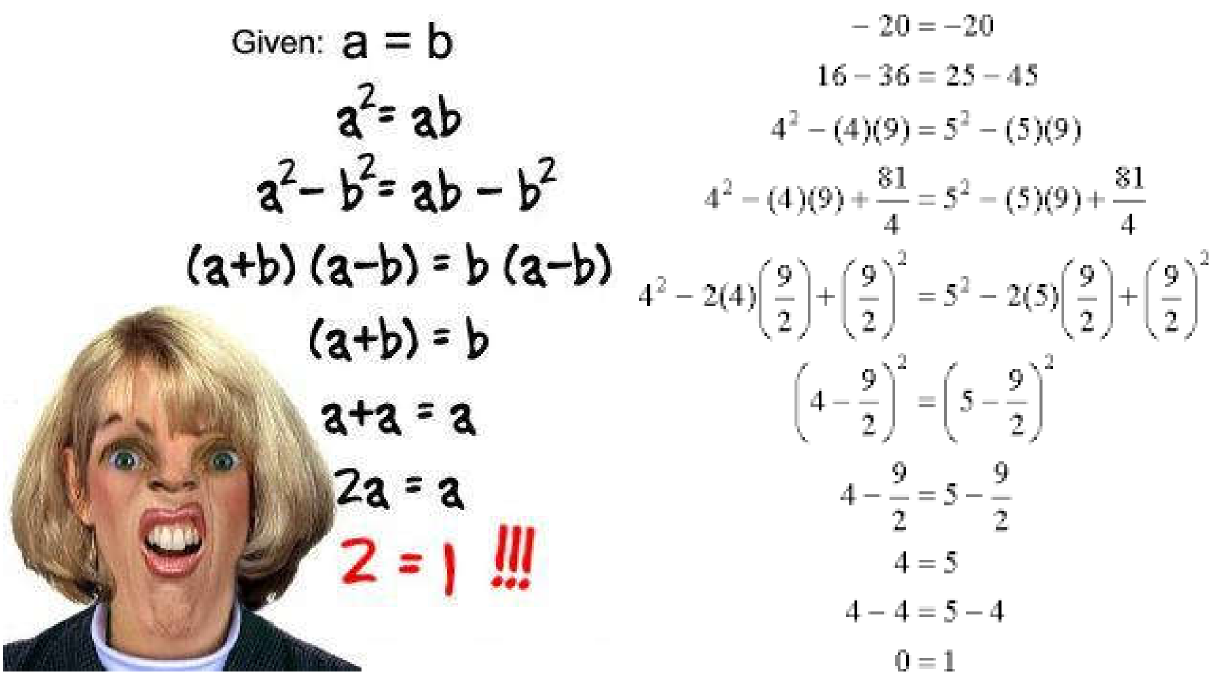
$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right) + C$$

Plato and many others, since long told something about Truth …

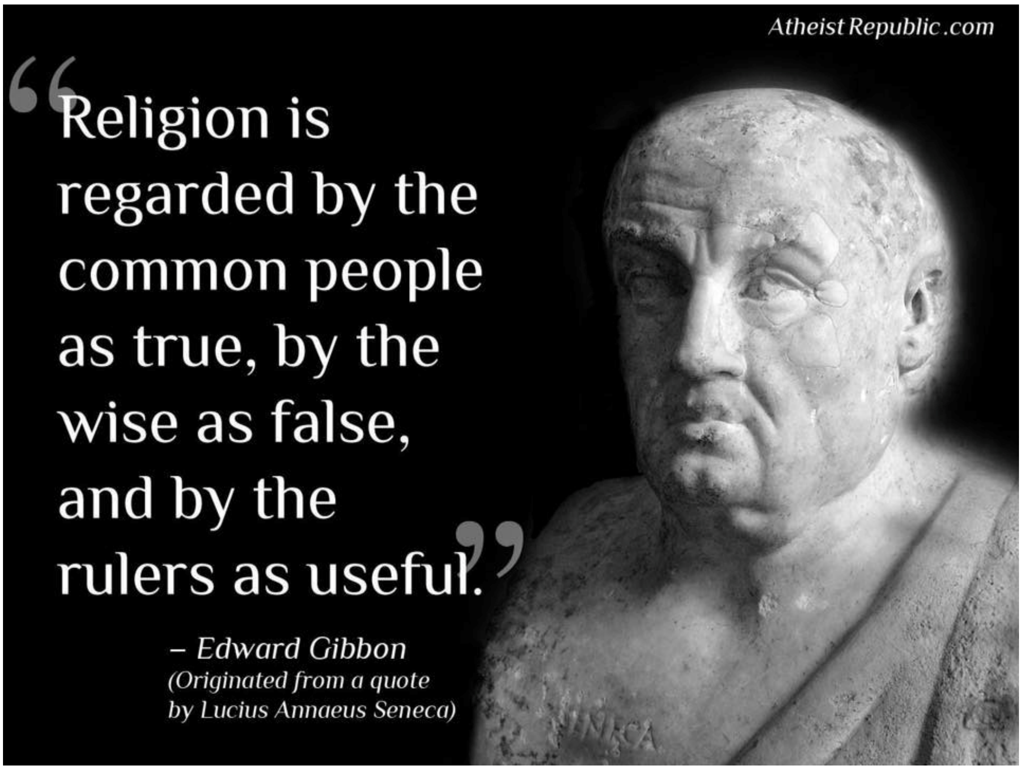


So I "lied" on a few things in this Book ! :-{D





"Logic of Religion and Mythology" is like above …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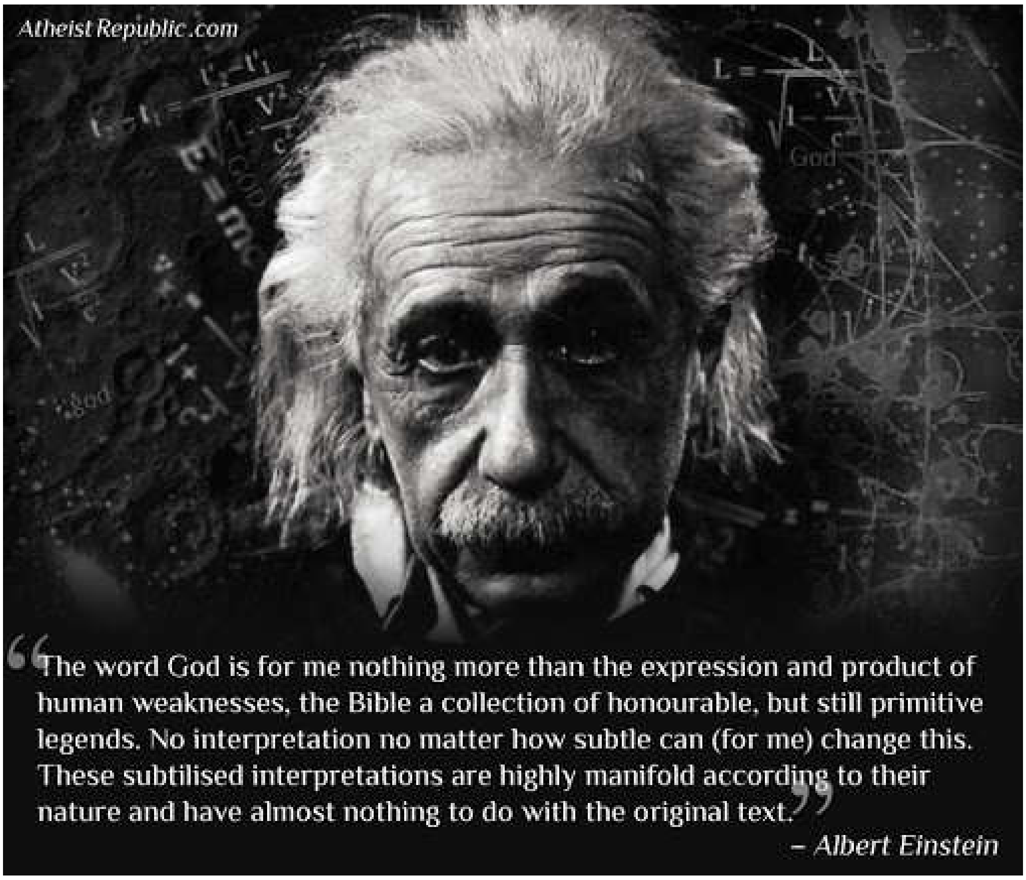
"Religion is regarded by the common people as true, by the wise as false, and by the rulers as useful."
– Edward Gibbon
(Originated from a quote by Lucius Annaeus Seneca)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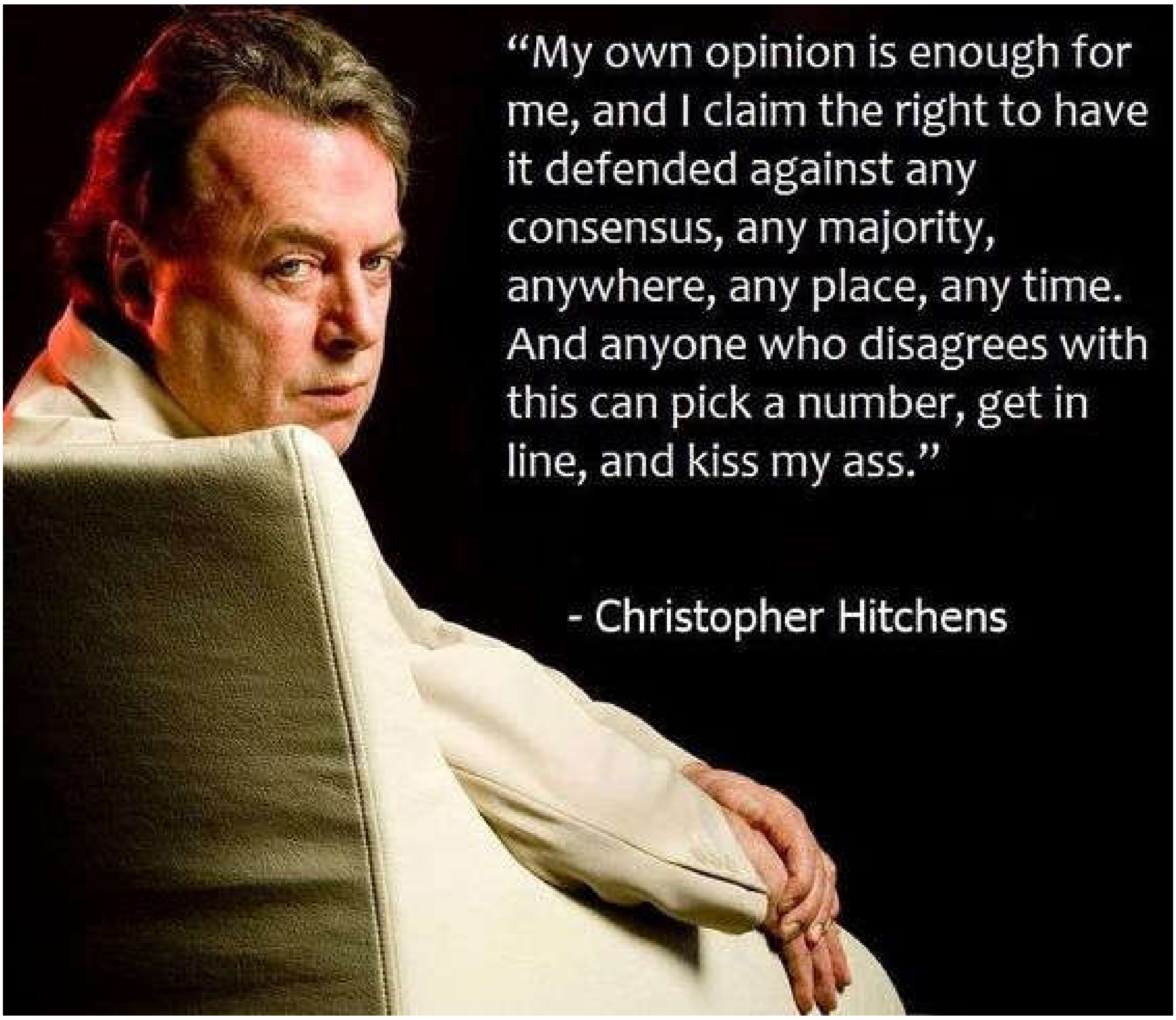
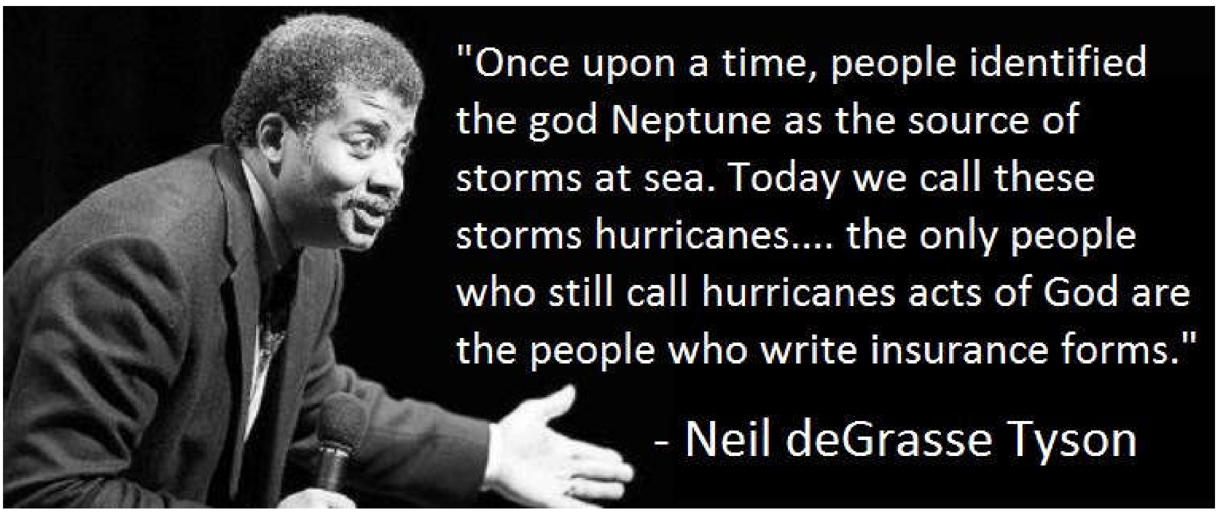
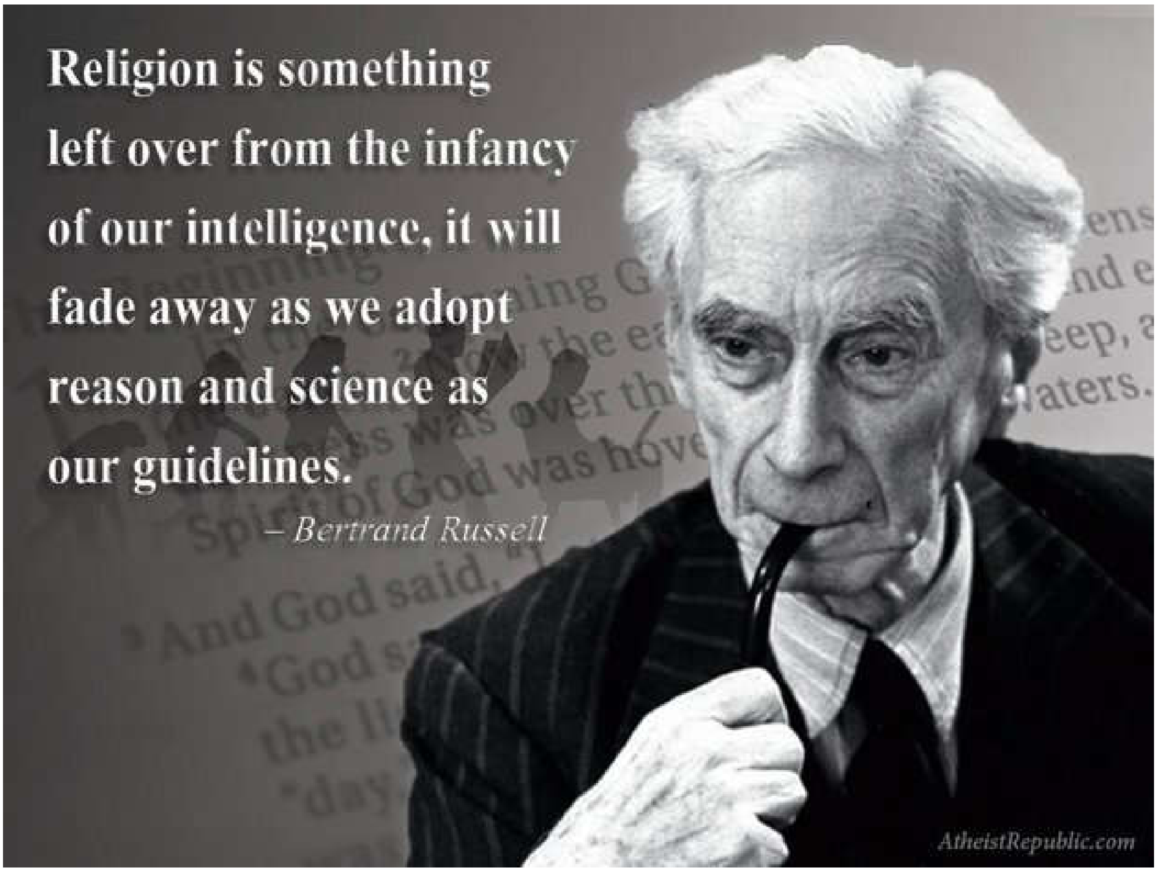
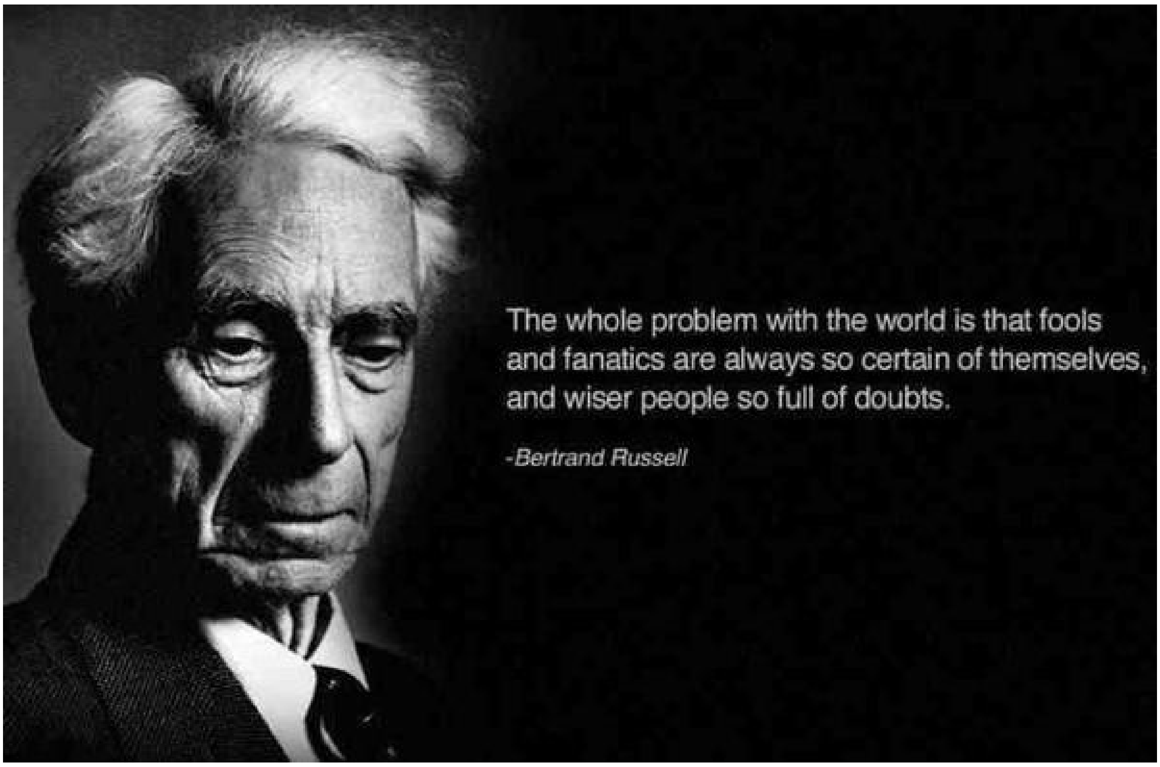
"The word God is for me nothing more than the expression and product of human weaknesses, the Bible a collection of honourable, but still primitive legends. No interpretation no matter how subtle can (for me) change this. These subtilised interpretations are highly manifold according to their nature and have almost nothing to do with the original text."
– Albert Einstein

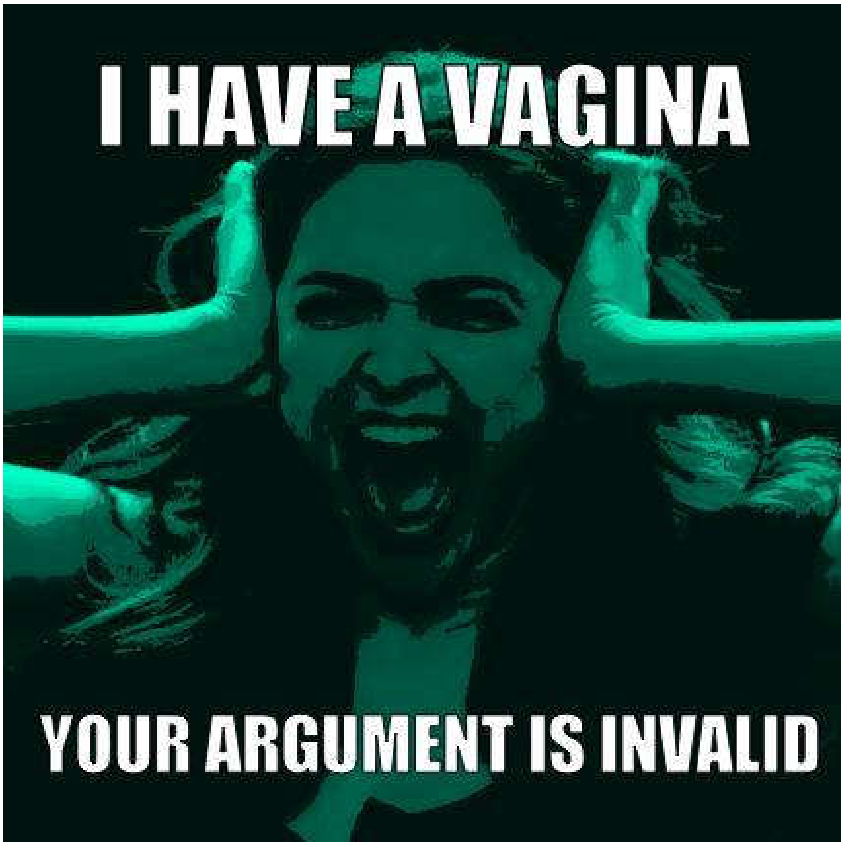


(Some people may agree that I am much more Polite, than Christopher Hitchens … May be I achieved much lesser because of that!)




Religion is something
left over from the infancy
of our intelligence, it will
fade away as we adopt
reason and science as
our guidelines.
– Bertrand Russell
AtheistRepublic.com


The whole problem with the world is that fools
and fanatics are always so certain of themselves,
and wiser people so full of doubts.
-Bertrand Russell


I HAVE A VAGINA
YOUR ARGUMENT IS INVALID

(In 2016 Celebrating 27 years of Excellence in Teaching)

Good Luck to you for your Preparations, References, and Exams

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